NEWS ON THE LOOP-TREE DUALITY*

Isabella Bierenbaum

II. Institut für Theoretische Physik, Universität Hamburg
Luruper Chaussee 149, 22761 Hamburg, Germany

Petros Draggiotis

Institute of Nuclear Physics, NCSR Demokritos, 15310, Athens, Greece

Sebastian Buchta, Grigorios Chachamis, Ioannis Malamos
Germán Rodrigo†

Instituto de Física Corpuscular, Universitat de València
Consejo Superior de Investigaciones Científicas
Parc Científic, 46980 Paterna, Valencia, Spain

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We summarize recent developments of the loop-tree duality method at one-loop and higher orders.

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1. Introduction

The last decade has seen an extraordinary progress in the analytical and numerical computation of cross sections in the Standard Model at higher orders [1]. Today, $2 \to 4$ processes at next-to-leading order (NLO), either from Unitarity based methods [2–4] or from a more traditional Feynman diagrammatic approach [5, 6], are state of the art, and even higher multiplicities [7] are affordable. There has been also a lot of progress concerning next-to-next-to-leading order (NNLO) calculations [8–12].

The loop-tree duality method [13] establishes that, after applying directly the Cauchy residue theorem in the loop momentum space, one-loop integrals and scattering amplitudes can be represented by single cut Feynman

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† german.rodrigo@csic.es

diagrams integrated over a modified phase-space. The Duality theorem has been extended in Ref. [14] beyond the one-loop level, to two- and three-loops and it was shown how to extend it to an arbitrary number of loops. Likewise, higher order poles have been treated in Ref. [15]. The main feature and advantage of this approach is that at any number of loops, loop integrals and scattering amplitudes can be written as a sum of tree-level objects, obtained after making all possible cuts to the lines of the corresponding Feynman diagrams, one cut per loop and integrated over a measure that closely resembles the phase-space of the corresponding real corrections. This modified phase-space raises the intriguing possibility that virtual and real corrections can be brought together under a common integral and treated with Monte Carlo techniques at the same time. In this paper, we review the actual state of development of the duality method and anticipate preliminary results on the singular behaviour and numerical implementation of this method.

2. Duality relation at one-loop

A general one-loop N-leg, scalar integral (see Fig. 1) is given by

$$L^{(1)}(p_1, p_2, \ldots, p_N) = \int_{\ell_1} \prod_{i=1}^{N} G_{\rm F}(q_i) \,, \tag{1}$$

where

$$G_{\rm F}(q_i) = \frac{1}{q_i^2 - m_i^2 + i0} \tag{2}$$

are Feynman propagators, with $i \in \alpha_1 = \{1, 2, \ldots, N\}$. The four-momenta of the external legs are denoted p_i. All are taken as outgoing and ordered clockwise. The momenta of the internal lines q_i are defined as

$$q_i = \ell_1 + p_{1,i} \,, \qquad p_{1,i} = p_1 + \ldots + p_i \,, \tag{3}$$

where the loop momentum is ℓ_1 and flows anti-clockwise. We use the shorthand notation

$$\int_{\ell_i} \bullet = -i \int \frac{d^d \ell_i}{(2\pi)^d} \bullet \,, \qquad \tilde{\delta}\,(q_i) \equiv 2\pi\, i\, \theta(q_{i,0})\, \delta\left(q_i^2 - m_i^2\right) = 2\pi\, i\, \delta_+\left(q_i^2 - m_i^2\right) \,, \tag{4}$$

where δ_+ selects the on-shell mode with positive definite energy, $q_{i,0} \geq 0$, such that the phase-space measure of the physical, *i.e.* on-shell, momentum q_i reads

$$\int \frac{d^d q_i}{(2\pi)^{d-1}}\, \theta(q_{i,0})\, \delta\left(q_i^2 - m_i^2\right) \equiv \int_{q_i} \tilde{\delta}\,(q_i) = \int_{\ell_1} \tilde{\delta}\,(q_i) \,. \tag{5}$$

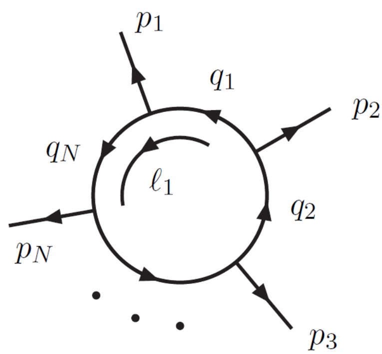

Fig. 1. Momentum configuration of the one-loop N-point scalar integral.

Following Refs. [13, 14], the scalar one-loop integral in Eq. (1) can be rewritten in the form

$$L^{(1)}(p_1,p_2,\dots,p_N) \;=\; -\sum_i \int_{\ell_1} \tilde{\delta}\,(q_i) \prod_{\substack{j=1\\ j\neq i}}^{N} G_{\rm D}(q_i;q_j)\,, \tag{6}$$

where

$$G_{\rm D}(q_i;q_j) = \frac{1}{q_j^2 - m_j^2 - i0\,\eta k_{ji}}\,, \tag{7}$$

with $k_{ji} = q_j - q_i$, are the so-called dual propagators, as defined in Ref. [13], with η a *future-like* vector, $\eta^2 \geq 0$, with positive definite energy $\eta_0 > 0$. The result in Eq. (6), contrary to the Feynman Tree Theorem (FTT) [16, 17], contains only single-cut integrals. Multiple-cut integrals, like those that appear in the FTT, are absent thanks to modifying the original $+i0$ prescription of the uncut Feynman propagators by the new prescription $-i0\,\eta k_{ji}$. At one-loop, k_{ji} depends on external momenta only. The dual $i0$ prescription arises from the fact that the original Feynman propagators $G_{\rm F}(q_j)$ are evaluated at the *complex* value of the loop momentum ℓ_1, which is determined by the location of the pole at $q_i^2 - m_i^2 + i0 = 0$. The $i0$ dependence of the pole of $G_{\rm F}(q_i)$ modifies the $i0$ dependence in the Feynman propagators $G_{\rm F}(q_j)$, $j \neq i$, leading to the total dependence as given by the dual $i0$ prescription. The presence of the vector η_μ is a consequence of using the residue theorem and the fact that the residues at each of the poles are not Lorentz-invariant quantities. The Lorentz-invariance of the loop integral is recovered after summing over all the residues.

3. Duality relation at two-loops and beyond

The extension of the Duality theorem to two-loops and beyond has been discussed in detail in Ref. [14]. It is convenient to define the following functions combining different Feynman and dual propagators

$$G_{\rm F}(\alpha_k) = \prod_{i\in\alpha_k} G_{\rm F}(q_i)\,, \qquad G_{\rm D}(\alpha_k) = \sum_{i\in\alpha_k} \tilde{\delta}\,(q_i) \prod_{\substack{j\in\alpha_k \\ j\neq i}} G_{\rm D}(q_i;q_j)\,, \tag{8}$$

where α_k is used to denote any set of internal momenta that depend on the same loop momentum or the sum of several independent loop momenta. At the one-loop order, α_k is naturally given by all the internal momenta, $\alpha_1 = \{1,2,\ldots,N\}$. At higher orders, several integration loop momenta are needed, and thus several *loop lines* α_k to label all the internal momenta. In the two-loop case, which is illustrated in Fig. 2, we have

$$\alpha_1 \equiv \{0,1,\ldots,r\}\,, \quad \alpha_2 \equiv \{r+1,r+2,\ldots,l\}\,, \quad \alpha_3 \equiv \{l+1,l+2,\ldots,N\}\,. \tag{9}$$

By definition, $G_{\rm D}(\alpha_k) = \tilde{\delta}\,(q_i)$, when $\alpha_k = \{i\}$ and thus consists of a single four-momentum. We also define

$$G_{\rm D}(-\alpha_k) = \sum_{i\in\alpha_k} \tilde{\delta}\,(-q_i) \prod_{\substack{j\in\alpha_k \\ j\neq i}} G_{\rm D}(-q_i;-q_j)\,, \tag{10}$$

where the sign in front of α_k indicates that we have reversed the momentum flow of all the internal lines in α_k.

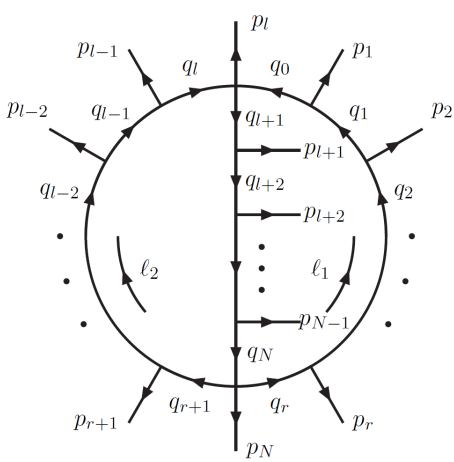

Fig. 2. Momentum configuration of the two-loop N-point scalar integral.

The key ingredient necessary to extend the Duality theorem to higher orders is the following relationship relating the dual and Feynman functions of two subsets

$$G_{\rm D}(\alpha_1 \cup \alpha_2) = G_{\rm D}(\alpha_1)\, G_{\rm D}(\alpha_2) + G_{\rm D}(\alpha_1)\, G_{\rm F}(\alpha_2) + G_{\rm F}(\alpha_1)\, G_{\rm D}(\alpha_2)\,, \quad (11)$$

which can be generalized as well to the union of an arbitrary number of loop lines [14]. The application of the Duality theorem at higher orders proceeds in a recursive way. For the two loop case, one starts by one of the loops

$$\begin{aligned} L^{(2)}(p_1, p_2, \ldots, p_N) &= \int_{\ell_1} \int_{\ell_2} G_{\rm F}(\alpha_1 \cup \alpha_2 \cup \alpha_3) \\ &= -\int_{\ell_1} \int_{\ell_2} G_{\rm F}(\alpha_2)\, G_{\rm D}(\alpha_1 \cup \alpha_3)\,. \end{aligned} \quad (12)$$

As the Duality theorem applies to the Feynman propagators only, we use Eq. (11) to re-express the dual propagators entering the second loop as Feynman propagators. The application of the Duality theorem to the second loop with momentum ℓ_2 also requires to reverse the momentum flow in some of the loop lines. The final dual representation of a two-loop scalar integral reads

$$\begin{aligned} L^{(2)}(p_1, p_2, \ldots, p_N) = \int_{\ell_1} \int_{\ell_2} \{ & -G_{\rm D}(\alpha_1)\, G_{\rm F}(\alpha_2)\, G_{\rm D}(\alpha_3) \quad (13) \\ & + G_{\rm D}(\alpha_1)\, G_{\rm D}(\alpha_2 \cup \alpha_3) + G_{\rm D}(\alpha_3)\, G_{\rm D}(-\alpha_1 \cup \alpha_2)\}\,, \end{aligned}$$

which is given by double cut contributions opening the loop diagram to a tree-level object.

4. Duality relation for multiple poles

The appearance of identical propagators or powers of propagators can be avoided at one-loop by a convenient choice of gauge [13], but not at higher orders. Identical propagators possess higher than single poles and the Duality theorem developed so far, which is based on assuming single poles, must be extended to accommodate for this new feature. Two different strategies have been proposed in Ref. [15] to deal with this problem. The first one consists of extending the Duality theorem by using the Cauchy residue theorem for higher order poles. The second one consists of using the Integration by Parts (IBP) [18, 19] to reduce integrals with multiple poles to integrals with single poles, where the original duality method can be applied directly. It is important to stress that in that case it is not necessary to perform a full reduction to a particular integral basis. Explicit examples at two- and three-loops have been presented in Ref. [15].

5. Singular behaviour of dual integrals and numerical implementation

The loop-momentum space approach is attractive because it allows a rather direct physical interpretation of the singularities of loop quantities. These singularities can arise when subsets of internal lines go on-shell. Although the existence of singular points of the integrand in the loop-momentum space is not enough to ensure the presence of singularities, it is important to isolate properly all the singular regions as even integrable singularities require a careful treatment by contour deformation in numerical methods [20].

The duality method sets at least one of the internal lines on-shell. More specifically, it restricts the loop integration to the forward light cones of the Feynman propagators. Besides restricting the loop integration to a three-dimensional space, each dual integral shows automatically well identified soft and collinear singularities. Moreover, one might expect a partial cancellation of infrared and threshold singularities among the dual components of the same loop integral [21], something that is obviously not plausible with a single integrand. This represents a big advantage for a numerical implementation.

6. Conclusions

The loop-tree duality method presents quite attractive features for the calculation of multipartonic cross-sections at higher orders. Initially developed for the one-loop calculations, it has been extended to an arbitrary number of loops. Also, some more technical aspects such as the treatment of identical propagators have been successfully analysed. This research program cannot be complete without a numerical implementation. Preliminary results show that there is a partial cancellation of singularities among different dual contributions to the same loop integral.

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