

ASSOCIATED HIGGS PRODUCTION AT NLO WITH GoSam*

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Integrand reduction has shown to be an invaluable method for the reduction of scattering amplitudes at one-loop and beyond. The algorithm implemented in **Samurai**, a Fortran library for d -dimensional integrand reduction, is discussed, mainly focusing on its novel extension **Xsamurai** to perform the reduction of higher-rank integrals. **GoSam** has been used to compute processes of associated Higgs production in Gluon Fusion, Vector Boson Fusion and in combination with a $t\bar{t}$ pair. In particular, the calculation of Higgs boson plus up to three jets production in the heavy-top effective theory is discussed, where higher-rank integrals may appear.

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1. Introduction

In July 2012 ATLAS and CMS announced the discovery of a new boson with a mass of approximately 125 GeV at LHC [1, 2]. Further investigation greatly supported the hypothesis that this is indeed the Higgs boson of the electroweak symmetry breaking mechanism. In order to determine this conclusively, several properties of the new boson have to be measured. These include its spin, its CP-properties and its couplings to other particles [3]. Two promising channels to determine some of these properties are Vector Boson Fusion (VBF), in which two vector bosons fuse to form a Higgs and the channel in which the Higgs is produced in association with a top-anti-top pair. The channel with the highest cross section, however, is the one in which two gluons fuse into a Higgs via a top quark loop, the Gluon Fusion (GF). Therefore, the latter is an important background to the other two channels. Here, recent calculations of these three production channels will be discussed.

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In order to make more precise predictions, the leading order (LO) calculations are not sufficient. Rather, next-to-leading order (NLO) calculations are warranted. Not only because they reduce the theoretical error by including a higher order, but also because the LO shows too great a dependence on the renormalization and factorization scale.

2. The GoSam framework

In the past few years, tremendous progress has been made in the automation of NLO calculations. NLO calculations consist of several parts: Besides the LO contribution, there are virtual and real corrections, as well as subtraction terms to regulate infinities that may arise. GoSam [4] is a framework that provides the virtual correction part of this calculation, *i.e.* the loop diagrams. Within GoSam, there is a choice of several reduction algorithms. The default is Samurai [5], a d -dimensional integrand-level reduction algorithm. The other is Golem95C [6, 7], a tensorial reduction algorithm, which is numerically stable and, therefore, used as a rescue system. Recently a third algorithm, Ninja, has been added to the framework, which uses the Laurent expansion to extract the coefficients in the integrand-level reduction [8].

In order to do a full NLO calculation, a One Loop Program like GoSam should be interfaced with a Monte Carlo program. In recent years, a standard for these kind of interfaces has emerged that is known as the Binoth Les Houches Accord (BLHA) [9, 10]. GoSam has been interfaced to several Monte Carlo programs, both via the BLHA and via *ad hoc* interfaces.

3. Integrand reduction and definition of the residues

Within a renormalizable theory, a generic one-loop integral can be decomposed in a set of master integrals times coefficients

$$\begin{aligned}
 \mathcal{A}_n = & \sum_{i < j < k < \ell}^{n-1} \left\{ c_{4,0}^{(ijk\ell)} I_{ijk\ell} + c_{4,4}^{(ijk\ell)} I_{ijk\ell} [\mu^4] \right\} + \sum_{i < j < k}^{n-1} \left\{ c_{3,0}^{(ijk)} I_{ijk} + c_{3,7}^{(ijk)} I_{ijk} [\mu^2] \right\} \\
 & + \sum_{i < j}^{n-1} \left\{ c_{2,0}^{(ij)} I_{ij} + c_{2,1}^{(ij)} I_{ij} [(q + p_i) e_2] + c_{2,2}^{(ij)} I_{ij} [((q + p_i) e_2)^2] \right. \\
 & \left. + ; c_{2,9}^{(ij)} I_{ij} [\mu^2] \right\} + \sum_i^{n-1} c_{1,0}^{(i)} I_i.
 \end{aligned} \tag{1}$$

It can be shown [8, 11, 12] that this expression can be obtained by decomposing the numerator of the integrand as

$$\begin{aligned}
N(q, \mu^2) = & \sum_{i \leq m}^{n-1} \Delta_{ijk\ell m}(q, \mu^2) \prod_{h \neq i, j, k, \ell, m}^{n-1} D_h + \sum_{i \leq \ell}^{n-1} \Delta_{ijk\ell}(q, \mu^2) \prod_{h \neq i, j, k, \ell}^{n-1} D_h \\
& + \sum_{i \leq k}^{n-1} \Delta_{ijk}(q, \mu^2) \prod_{h \neq i, j, k}^{n-1} D_h + \sum_{i < j}^{n-1} \Delta_{ij}(q, \mu^2) \prod_{h \neq i, j}^{n-1} D_h + \sum_i^{n-1} \Delta_i(q, \mu^2) \prod_{h \neq i}^{n-1} D_h
\end{aligned} \tag{2}$$

and integrating over the d -dimensional loop momentum $\bar{q} \equiv q + \mu$. The residues Δ are process-independent polynomial functions of q and μ^2 , containing process-dependent coefficients. Their universal form can be obtained either by direct investigation [11, 13], or by using the Gram relations [14], or by multivariate polynomial division [15]. Each residue contains terms proportional to the coefficients in Eq. (1) as well as spurious terms, which vanish upon loop integration.

In the case of renormalizable theories, the rank in q and μ^2 of each residue cannot exceed the number of propagators. When dealing with effective field theories, such as the large top mass limit employed in the Higgs production via GF, additional powers might appear. This alters the decomposition of Eq. (1) which requires additional terms

$$\begin{aligned}
\delta \mathcal{A}_n = & \sum_{i < j < k}^{n-1} c_{3,14}^{(ijk)} I_{ijk} [\mu^4] \\
& + \sum_{i < j}^{n-1} \left\{ c_{2,13}^{(ij)} I_{ij} [((q + p_i) e_2)^3] + c_{2,10}^{(ij)} I_{ij} [\mu^2 ((q + p_i) e_2)] \right\} \\
& + \sum_i^{n-1} \left\{ c_{1,14}^{(i)} I_i [\mu^2] + c_{1,15}^{(i)} I_i [((q + p_i) e_3)((q + p_i) e_4)] \right\}. \tag{3}
\end{aligned}$$

The form of the residues in Eq. (2) has to be extended as well [8]. All process-dependent coefficients are obtained by sampling the numerator on the l.h.s. of Eq. (2) at the various multiple cuts (fit-on-the cut approach). The algorithm is based on the solution of a triangular system of equations to be solved top-down, from the 5-point to the 1-point coefficients.

The triangular system of equations is solved by expanding q in a cut-dependent basis of massless vectors, $q^\mu = \sum_{i=1}^4 x_i e_i^\mu$. Therefore, for each cut, we get a polynomial in the variables not fixed by the cut conditions. The coefficients can be determined through polynomial fitting. Every polynomial has to be sampled as many times as the number of coefficients to be determined. In the following, we will describe the sampling level-by-level for the case in which the rank exceeds the number of propagators.

4. Sampling strategy

An efficient way to determine the coefficients of a normal polynomial is to use the Discrete Fourier Transformation (DFT). First one evaluates (samples) the polynomial symmetrically in a circle in the complex plane. Then the orthogonality relation is used to project out the coefficients [16].

Quintuple cut. At the level of the quintuple cut, which in d dimensions is a maximum cut [15], no new coefficients appear in the higher rank numerator, which can be parametrized by a single coefficient. The latter can be obtained by sampling on the unique solution fulfilling the cut conditions.

Quadruple cut. In the case of the quadruple cut, there is one new coefficient with respect to the normal rank polynomial. The solutions of the quadruple cut can be cast into two one-parameter families which depend on μ^2 . Therefore, for each value of μ^2 , we still can sample at two different values for q . The coefficients not proportional to μ^2 can be obtained sampling on the two solutions at $\mu^2 = 0$. The two coefficients proportional to μ^2 and the two proportional to μ^4 can be efficiently disentangled by sampling with the two solutions at μ_s^2 and at $-\mu_s^2$, where μ_s^2 can be any sampling value.

TABLE I

The number of samplings per polynomial at the quadruple and triple cut.

Quadruple cut		Triple cut	$C = 0$	$C \neq 0$
$\Lambda(0, q)$	2	$\Lambda(0, x_3, C/x_3)$ $\Lambda(0, C/x_4, x_4)$	5 4	9 0
$\Lambda(+\mu_s^2, q)$	2	$\Lambda(+\mu_s^2, x_3, C/x_3)$	5	5
$\Lambda(-\mu_s^2, q)$	2	$\Lambda(-\mu_s^2, 1, C)$	1	1

Triple cut. At the triple-cut level, the higher-rank polynomial is extended with five new coefficients, to make a total of fifteen. The three cut-conditions completely determine x_1 and x_2 and constrain the product x_3x_4

$$x_3x_4 = C(\mu^2) . \quad (4)$$

This means we can either choose x_3 or x_4 to sample, the other one being fixed by this condition. The easiest way to get the nine coefficients which are not proportional to μ^2 is to choose $\mu^2 = 0$ and to use the DFT with x_3 to sample nine times. The solutions one acquires in this manner are inversely proportional to C and thus unstable if the latter is small. In this case, one can sample five times with x_3 and four times with x_4 , getting solutions inversely proportional to $(1 - C)$. To get the six remaining coefficients, we

sample with $\mu^2 = \mu_s^2$ five times and one additional time with $\mu^2 = -\mu_s^2$ to disentangle the two coefficients proportional to x_3 nor x_4 . The procedure is summarized in Table I.

Double cut. At the double-cut level, going to higher rank doubles the number of coefficients to twenty. The two conditions imposed by the double cut fix x_2 and the product x_3x_4

$$x_3x_4 = F(\mu^2, x_1) = Ax_1^2 + Bx_1 + C(\mu^2). \quad (5)$$

The seven coefficients which are independent of x_1 and μ^2 are computed by setting μ^2 and x_1 to zero and sampling on x_3 . As in the triple cut case, if $F(0,0)$ is small, one has to sample both with x_3 and x_4 . In the next step, one can consider $F(0, x_1)$. Depending on the actual value of A , B and C , $F(0, x_1)$ may vanish for some values of x_1 . Those values should be preferred because if $F(0, x_1) = 0$ either x_3 or x_4 vanishes and the number of the coefficients to be determined simultaneously is reduced. At this level, many branches allow one to take advantage of these simplifications. The coefficients multiplying μ^2 are obtained using $F(\mu_s^2, 0)$. The last step is to determine the coefficient multiplying both μ^2 and x_1 . Again, $F(\mu_s^2, x_1)$ may vanish and we have to branch accordingly. The full procedure is summarized in Table II, where x_{1c} is a random value. x_{1a} and x_{1b} are either the solutions of $F(0, x_1) = 0$, if present, or a random value.

TABLE II

The number of samplings per polynomial at the double and single cut.

Double cut	$F = 0$	$F \neq 0$	Single cut	$G = 0$	$G \neq 0$
$\Lambda(0, 0, x_3, F/x_3)$	4	7	$\Lambda(0, x_1, G/x_1, 0, 0)$	3	5
$\Lambda(0, 0, F/x_4, x_4)$	3	0	$\Lambda(0, G/x_2, x_2, 0, 0)$	2	0
$\Lambda(0, x_{1a}, x_3, F/x_3)$	3	5	$\Lambda(0, 0, 0, x_3, G/x_3)$	3	5
$\Lambda(0, x_{1a}, F/x_4, x_4)$	2	0	$\Lambda(0, 0, 0, G/x_4, x_4)$	2	0
$\Lambda(0, x_{1b}, x_3, F/x_3)$	2	3	$\Lambda(0, x_1, -G/x_1, 1, 0)$	1	2
$\Lambda(0, x_{1b}, F/x_4, x_4)$	1	0	$\Lambda(0, -G/x_2, x_2, 1, 0)$	1	0
$\Lambda(0, x_{1c}, 1, F)$	1	1	$\Lambda(0, x_1, -G/x_1, 0, 1)$	1	2
			$\Lambda(0, -G/x_2, x_2, 0, 1)$	1	0
$\Lambda(\mu_s^2, 0, x_3, F/x_3)$	2	3	$\Lambda(\mu_s^2, 0, 0, 0, 0)$	1	0
$\Lambda(\mu_s^2, 0, F/x_4, x_4)$	1	0	$\Lambda(\mu_s^2, 1, G, 0, 0)$	0	1
$\Lambda(\mu_s^2, 1, 1, F)$	1	1			

Single cut. The number of coefficients increases from five to fifteen when going to higher rank. The single cut condition

$$x_3x_4 - x_1x_2 = G(\mu^2) \quad (6)$$

allows one to choose four out of the five variables. If $G = 0$, we can set all but one variable to zero and perform a DFT on the non-zero one. When $G \neq 0$, we can still choose either x_1 and x_2 or x_3 and x_4 to be zero and sample with one of the remaining two. For the remaining ‘cross term’ coefficients, that multiply x_1 or x_2 with x_3 or x_4 , we can only set one variable to zero.

5. Associated Higgs production at NLO

Higgs plus jets production in GF. We applied the extended rank decomposition, implemented in *Xsamura* as described above, to compute the NLO QCD corrections to the production of a Higgs boson plus jets in GF, in the large m_{top} approximation. This is a challenging project, in particular with an increasing number of jets. In order to test our machinery, we started by calculating the correction to Higgs plus two jets in GF [17], which contains already 926 NLO diagrams. The calculation was performed employing the interface between *GoSam* and *Sherpa* [18], and we found excellent agreement with previous results [19] and *MCFMv6.4* [20].

The next step was the calculation of the NLO QCD corrections to Higgs plus three jets in GF [21]. By adding an additional jet, the number of NLO diagrams increases to 13179, many of which have higher-rank numerators, including 60 rank-7 hexagons (see Fig. 1 for an example). In order to deal with such a computational challenge, *GoSam* has been enhanced by including new features such as the grouping of the numerators, optimization through *FORM4.0* [22], the use of numerical polarization vectors and the possibility to parallelize computations. These upgrades will be publicly available in the next release of the *GoSam* code. We used a hybrid setup that employs *GoSam* for the evaluation of the virtual part and *Sherpa* for the Born and to integrate the virtual contributions. For the real contributions, the subtraction

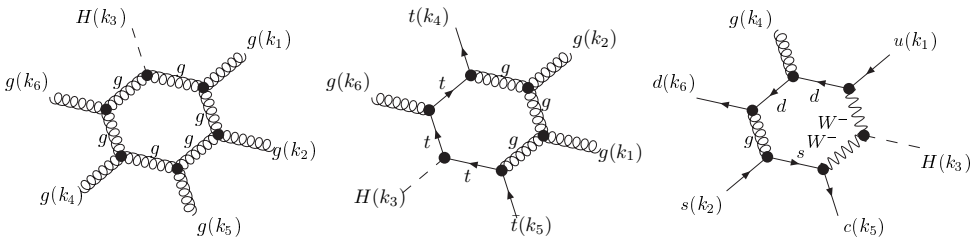


Fig. 1. Example diagrams of (a) Higgs plus three jets in GF, (b) Higgs plus jet in association with a $t\bar{t}$ pair, (c) Higgs plus three jets in VBF.

terms and the integrated dipoles, we used an *ad hoc* interface with MadGraph [23, 24], MadDipole [25, 26] and MadEvent [27]. We validated our hybrid set-up by checking the LO result for Higgs plus three jets against MadGraph and by reproducing the total cross section for Higgs plus two jets computed in [17]. Finally, we verified the independence of the NLO cross-section from the choice of the alpha-parameter.

The amplitudes generated with GoSam for the production of Higgs plus jets at various multiplicities are ready to be paired with available Monte Carlo programs for further phenomenological analyses.

Higgs plus jet production in association with a $t\bar{t}$ pair. The production of Higgs in association with a $t\bar{t}$ pair is important to study the coupling properties of the Higgs boson. In the high p_T region, the addition of a jet can be relevant. This is however known as a difficult process, because of the appearance of two mass scales: m_{top} and m_H . There are also a substantial number of diagrams to be computed, 1895 NLO, especially in the double gluon channel, where 51 massive hexagons need to be calculated. The calculation of this process [28] was the first application of the new reduction algorithm Ninja [29].

Higgs plus jets production in VBF. The other important process in the study of the Higgs properties is the VBF production channel. In Higgs plus two jets production in VBF there are 240 NLO diagrams, if an additional jet is added (recently computed in [30]) there are 2160 NLO diagrams. These processes can also easily be done with GoSam.

6. Conclusions

Xsamurai, a reduction algorithm that is an extension of Samurai for the automated evaluation of one-loop corrections with diagrams that involve a higher rank numerator, has been presented. It has been used for the calculation of Higgs plus two and three jets in GF within the framework of GoSam, which has been interfaced to several Monte Carlo programs to do full NLO calculations. GoSam has also been used to calculate two other Higgs production processes that are important to determine the Higgs properties: The full NLO calculation of the production of a Higgs in association with a $t\bar{t}$ pair plus a jet and the virtual correction to the production of a Higgs in association with two and three jets in VBF.

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