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QED RADIATIVE CORRECTIONS FOR $\gamma\gamma$-PRODUCTION OF HADRONS IN e^+e^- SCATTERING*

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The theoretical calculation of the leading quantum electrodynamics radiative corrections to the processes of hadron production via the photon–photon interaction in the electron–positron scattering is briefly reviewed.

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1. Introduction

We consider the gamma–gamma production of $\mathcal{P}$-hadrons as a subprocess in $e^+e^- \to e^+e^-\gamma^*\gamma^* \to e^+e^-\mathcal{P}$ reaction with virtual photons being exchanged in t-channel. The relevant phenomenology of our interest corresponds to the low $\sqrt{s}$ physics case from the threshold to a few GeV; the hadrons $\mathcal{P}$ in the final state are several neutral particles — π^0, η, η' meson or a meson pair, say, $\pi^0\pi^0$. The aim of our ongoing research is to have the description of this process at the next-to-leading order (NLO) in electromagnetic coupling α and to create a computer program which would allow for an efficient simulation appropriate for the needs of modern gamma–gamma physics experiments such as:

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- KLOE-2 with $\sqrt{s}$ around the ϕ meson mass and $\mathcal{P} = \pi^0$ (see, *e.g.*, [1]) or $\mathcal{P} = \pi^0\pi^0$, η (for a recent review, see I. Prado Longhi in [2]).

- BES-III with $\sqrt{s} \approx \psi(2S)$, $\psi(3770)$, $\mathcal{P} = \pi^0$, η, η' and, hopefully, $\pi^0\pi^0$ (see some details in C. Redmer in [2]).

The physics contents of these experiments is very challenging and exciting, and, among other issues, includes the study of the two-photon transition form factors of hadrons. Lately, the theoretical work in this field has put a lot of emphasis on the necessity of new precise measurements. The key properties are the shape and the slope of the form factors, see, *e.g.*, [3] and references therein, and the asymptotic behavior of the form factors [4–6]. Precise measurements are expected to have an impact on the calculation of the muon anomalous magnetic moment $(g-2)_\mu$ [7, 8], see also [1] for discussion of the possible impact of KLOE-2 experiment. Recently, we have calculated the two-photon transition form factors of π^0, η and η' mesons [3] in the framework of effective chiral theory with resonances and implemented the formulae in the Monte Carlo generator EKHARA [9][1]. The radiative corrections, discussed in this paper, are being implemented in the same generator.

2. Radiative corrections

At the leading order (LO), the squared matrix element is given by the tree level diagrams shown in Fig. 1 and in this form, with various parameterizations of the two-photon form factors, it is implemented in the version 2.1 of the Monte Carlo generator EKHARA [3] for $\mathcal{P} = \pi^0$, η, η'. Typically, the experimental conditions are chosen such that the s-channel contributions can be neglected. Therefore, one usually considers only the t-channel diagram. The parts of radiative corrections are classified as:

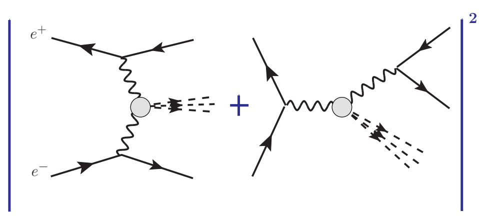

Fig. 1. The leading order squared matrix element in terms of the t-channel $T_{\rm LO}$ (left) and s-channel $S_{\rm LO}$ (right) diagrams.

[1] For details on the Monte Carlo generator EKHARA, the source code and the manual, please visit `http://prac.us.edu.pl/~ekhara`

Soft photon correction, accounts for the emission of very low-energetic (soft) photons from positron and electron lines, never observed in a realistic experiment. The infrared divergent part (IR) of this correction gets canceled by the corresponding terms in virtual corrections. The energy scale M_0 gives a separation of hard and soft photons. The contribution to the squared matrix element is proportional to that of the LO t-channel amplitude squared.
Virtual correction to the vertex, Fig. 2. The IR part of this correction cancels that of the soft corrections coming from the interference of the photon radiation from the same lepton line. The contribution to the squared matrix element is proportional to the interference of the LO t-channel diagram and the similar diagram with the virtual correction to the vertex.

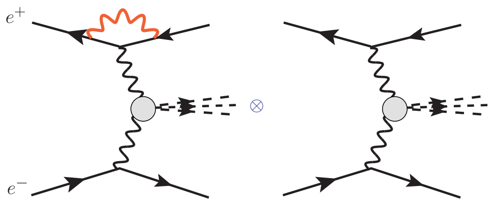

Fig. 2. Virtual correction to the positron line. The similar correction is accounted for also for the electron line.

Self energy/vacuum polarization, Fig. 3. This correction accounts for e^-, μ^-, τ^- loops and the hadronic vacuum polarization. The contribution to the squared matrix element is proportional to the interference of the LO t-channel diagram and similar diagram with the self energy insertion.

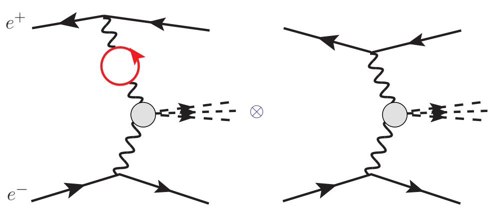

Fig. 3. Virtual correction which accounts for the vacuum polarization. The similar correction is accounted for also for the electron line.

Hard photon emission, Fig. 4, accounts for the emission of energetic photon (above the separation scale M_0), which, potentially, can be observed in the experiment. Therefore, this type of radiative correction introduces one extra particle in the final state.

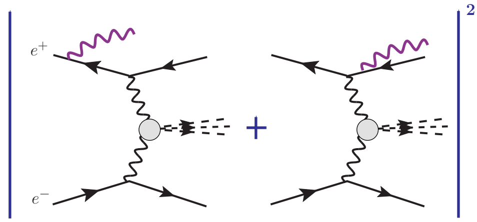

Fig. 4. Hard photon emission from positron line. The similar correction is accounted for also for the electron line.

Box diagrams, Fig. 5, accounts for possible extra photon exchange. The conclusions of [14] are that "contributions of the five-point functions will always be negligible or irrelevant". The IR part of this correction cancels that of the soft corrections coming from the interference of the photon radiation from different lepton lines. For the moment, we neglect the box diagrams, leaving the detailed consideration for future.

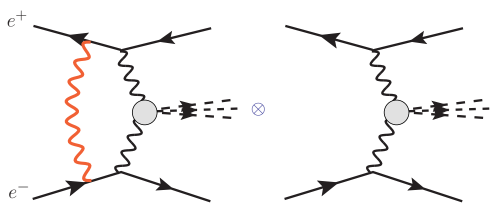

Fig. 5. Pentabox contribution.

The standard (analytical) approach relies on choosing an appropriate approximation for the "Soft+Virtual" part of the radiative corrections (small t or large t case) and integrating the hard photon emission such that $|T_{\rm Hard}|^2 \Rightarrow \delta_{\rm Hard} \times |T_{\rm LO}|^2$. Then, one obtains the single-differential cross section in the form $\frac{d\sigma}{dQ^2} \Rightarrow \left(1+\delta(Q^2)\right) \times \frac{d\sigma_0}{dQ^2}$, where Q^2 gives the virtuality of the tagged photon; or at the level of total cross section: $\sigma \Rightarrow (1+\delta) \times \sigma_0$ in terms of the LO cross section σ_0. The well-known example of application of the approximate radiative corrections for the gamma–gamma physics is that by Ong and Kessler [15, 16]. In a modified form, it was implemented in the Monte Carlo generator GGRESRC [13] and used in data analyses by BaBar [17–19]. The final formula of Ong and Kessler for $\delta(Q^2)$ in a single-tag case depends on Q^2 (via logs of type $L \equiv \ln \frac{Q^2}{m^2}$) and $r_{\rm max}$ (the maximum energy of undetected ISR hard photon normalized to the beam energy; typically, $r_{\rm max} \ll 1$ and is restricted by the event selection in the experiment): $\delta = \delta_{\rm V+S} + \delta_{\rm HI} + \delta_{\rm HF} = -\,\frac{\alpha}{\pi}\left(\left(\ln\frac{1}{r_{\rm max}} - \frac{17}{12}\right)(L-1) + \frac{25}{36}\right)$, where HI and HF correspond to hard photon emission from the lepton in the initial and the final state. If, instead of taking an approximate "Soft+Virtual" part, one uses the exact formulae, the total radiative correction factor is given only by the vacuum polarization contributions (while the rest gets fully canceled) $\delta = \delta_{\rm vac.pol.}$ [12] (the hard-photon spectrum is integrated over the full phase space).

Let us stress that the radiative corrections affect the shapes of the various sub-distributions, not only the value of the (integrated) cross section. While integrated form is good for analytical exercises, for a Monte Carlo simulation one needs un-integrated expressions. For the precise Monte Carlo simulation, one needs to understand the impact of the radiative corrections on the event distributions at the event-by-event level. For example, the authors of GGRESRC [13] had to create an "un-integrated" formulae, which would reproduce the results of Ong and Kessler [15] after integration.

In our approach, we use exact QED formulae, do not perform any analytic integration of hard photon spectrum, implement the formulae in the Monte Carlo generator and make it numerically efficient.

The exact formulae for the virtual correction to the electromagnetic vertex can be found in [11], they contain the infrared regulator ("fictious photon mass") λ, which has to be matched with the soft corrections. The formulae and the code for soft photon emission can be found in [10], they contain the infrared regulator λ and the separation scale M_0 between soft and hard photon. For hard photon emission, we perform the LO QED calculation (with both the trace method and helicity amplitudes, for cross-check) and use the exact, fully differential, phase space requiring only that the hard photon energy is above M_0.

On the technical side, the work in progress now is focused on checking the independence of result from the Soft–Hard matching scale M_0. Works on a development of efficient mappings for the hard-photon emission events, in a similar manner as it was done for the LO case in EKHARA [9], following the ideas of [20, 21], are well advanced.

After the radiative corrections are fully implemented in EKHARA and verified, we plan to perform a comparison with GGRESRC.

3. Summary

We have briefly reviewed the QED leading radiative corrections for $e^+e^- \to e^+e^-\gamma^*\gamma^* \to e^+e^-\mathcal{P}$. The references are given to the standard, though approximate, solutions used in the various researches in this field and also implemented in the numerical simulations. Considering the state-of-the-art experiments and having in mind the theory issues, which are tackled by the two-photon physics nowadays, we argue that the possible lack of precision of theoretical formulae used in the data analysis can lead to a misinterpretation of the transition form factors extracted from data. All necessary ingredients for using the exact and un-integrated expressions for the leading corrections are available and should be used in the simulation. We merge the well-known exact expressions for the vertex corrections and hard photon emission with the exact calculations of the soft photon emission and implement in EKHARA Monte Carlo generator for simulation in the full phase space.

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