PHENOMENOLOGY OF NEUTRINO OSCILLATIONS AND MIXING*

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We review the status of three-neutrino mixing and the results of global analyses of short-baseline neutrino oscillation data in 3+1 and 3+2 neutrino mixing schemes.

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1. Introduction

Neutrino oscillations have been measured with high accuracy in solar, atmospheric and long-baseline neutrino oscillation experiments. Hence, we know that neutrinos are massive and mixed particles (see [1]). In this short review, we discuss the status of the standard three-neutrino mixing paradigm (Section 2) and the indications in favor of the existence of additional sterile neutrinos given by anomalies found in some short-baseline neutrino oscillation experiments (Section 3).

2. Three-neutrino mixing

Solar neutrino experiments (Homestake, Kamiokande, GALLEX/GNO, SAGE, Super-Kamiokande, SNO, BOREXino) measured $\nu_e \to \nu_\mu, \nu_\tau$ oscillations generated by the solar squared-mass difference $\Delta m^2_{\text{SOL}} \simeq 7 \times 10^{-5}\,\text{eV}^2$

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and a mixing angle $\sin^2\vartheta_{\mathrm{SOL}} \simeq 0.3$. The KamLAND experiment confirmed these oscillations by observing the disappearance of reactor $\bar{\nu}_e$ at an average distance of about 180 km.

Atmospheric neutrino experiments (Kamiokande, IMB, Super-Kamiokande, MACRO, Soudan-2, MINOS) measured ν_μ and $\bar{\nu}_\mu$ disappearance through oscillations generated by the atmospheric squared-mass difference $\Delta m^2_{\mathrm{ATM}} \simeq 2.3\times10^{-3}\,\mathrm{eV}^2$ and a mixing angle $\sin^2\vartheta_{\mathrm{ATM}} \simeq 0.5$. The K2K and MINOS long-baseline experiments confirmed these oscillations by observing the disappearance of accelerator ν_μ at distances of about 250 km and 730 km, respectively.

The two independent solar and atmospheric Δm^2s are nicely accommodated in the standard framework of three-neutrino mixing in which the three active flavor neutrinos ν_e, ν_μ, ν_τ are superpositions of three neutrinos ν_1, ν_2, ν_3 with masses m_1, m_2, m_3: $\nu_\alpha = \sum_{k=1}^{3} U_{\alpha k}\nu_k$, for $\alpha = e, \mu, \tau$. The unitary mixing matrix can be written in the standard parameterization in terms of three mixing angles ϑ_{12}, ϑ_{23}, ϑ_{13} and a CP-violating phase[1] δ

$$U = \begin{pmatrix} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix} , \qquad (1)$$

where $c_{ab} \equiv \cos\vartheta_{ab}$ and $s_{ab} \equiv \sin\vartheta_{ab}$. It is convenient to choose the numbers of the massive neutrinos in order to have

$$\Delta m^2_{\mathrm{SOL}} = \Delta m^2_{21} , \qquad \Delta m^2_{\mathrm{ATM}} = \left|\Delta m^2_{31}\right| \simeq \left|\Delta m^2_{32}\right| , \qquad (2)$$

with $\Delta m^2_{kj} = m^2_k - m^2_j$. Then, there are two possible hierarchies for the neutrino masses: the normal hierarchy (NH) with $m_1 < m_2 < m_3$ and the inverted hierarchy (IH) with $m_3 < m_1 < m_2$.

With the conventions in Eqs. (1) and (2), we have $\vartheta_{\mathrm{SOL}} = \vartheta_{12}$ and $\vartheta_{\mathrm{ATM}} = \vartheta_{23}$. Moreover, the mixing angle ϑ_{13} generates $\overset{(-)}{\nu}_e$ disappearance and $\overset{(-)}{\nu}_\mu \to \overset{(-)}{\nu}_e$ transitions driven by $\Delta m^2_{\mathrm{ATM}}$, which can be observed in long-baseline neutrino oscillation experiments.

In 2011, the T2K experiment reported the first indication of long-baseline $\nu_\mu \to \nu_e$ transitions [2], followed by the MINOS experiment [3]. Recently, the T2K Collaboration reported a convincing 7.5σ observation of $\nu_\mu \to \nu_e$ transitions through the measurement of 28 ν_e events with an expected background of 4.64 ± 0.53 events [4].

[1] For simplicity, we do not consider the two Majorana CP-violating phases which contribute to neutrino mixing if massive neutrinos are Majorana particles, because they do not affect neutrino oscillations (see [1]).

On the other hand, the most precise measurement of the value of ϑ_{13} comes from the measurement of $\bar{\nu}_e$ disappearance in the Daya Bay reactor experiment [5], which has been confirmed by the data of the RENO [6] and Double Chooz [7] reactor experiments

$$\sin^2 2\vartheta_{13} = 0.090^{+0.008}_{-0.009} \qquad [8]\,. \tag{3}$$

Hence, we have a robust evidence of a non-zero value of ϑ_{13}, which is very important, because the measured value of ϑ_{13} opens promising perspectives for the observation of CP violation in the lepton sector and matter effects in long-baseline oscillation experiments, which could allow to distinguish the normal and inverted neutrino mass spectra (see [9]).

As a result of all these observations of neutrino oscillations, the mixing parameters can be determined with good precision by a global fit of the data [10–12]. The most recent result is NuFIT-v1.2 [13]

$$\Delta m^2_{21} = 7.45^{+0.19}_{-0.16} \times 10^{-5}\,\mathrm{eV}^2\,, \qquad \sin^2\vartheta_{12} = 0.306^{+0.012}_{-0.012}\,, \tag{4}$$

$$\Delta m^2_{31} = 2.417^{+0.013}_{-0.013} \times 10^{-3}\,\mathrm{eV}^2\,, \qquad \sin^2\vartheta_{23} = 0.446^{+0.007}_{-0.007} \quad (\mathrm{NH})\,, \tag{5}$$

$$\Delta m^2_{32} = -2.410^{+0.062}_{-0.062} \times 10^{-3}\,\mathrm{eV}^2\,, \qquad \sin^2\vartheta_{23} = 0.587^{+0.032}_{-0.037} \quad (\mathrm{IH})\,, \tag{6}$$

$$\sin^2\vartheta_{13} = 0.0229^{+0.0020}_{-0.0019}\,. \tag{7}$$

Hence, the squared-mass differences are known with good precision: about 2.5% for both Δm^2_{21} and $|\Delta m^2_{31}| \simeq |\Delta m^2_{32}|$. The mixing parameters $\sin^2\vartheta_{12}$, $\sin^2\vartheta_{13}$, $\sin^2\vartheta_{23}$ are known, respectively, with 4%, 9%, 10% precision. Currently, the most puzzling uncertainty is that of the mixing angle ϑ_{23}, which is known to be close to the maximal mixing value of $\pi/4$, but we do not know if it is smaller or larger.

Finally, we conclude this section with the notation of a small tension between reactor and accelerator measurements of the ϑ_{13} angle. It may be reconciled within the 3 neutrino mixing scheme by fitting the phase δ [13]. However, from an experimental point of view, T2K shows an anomalous event vertex distribution of electron like events, with the events concentrated near the border of the detector [4].

3. Beyond three-neutrino mixing: Sterile neutrinos

The completeness of the three-neutrino mixing paradigm has been challenged by the following indications in favor of short-baseline neutrino oscillations, which require the existence of at least one additional squared-mass difference, $\Delta m^2_{\mathrm{SBL}}$, which is much larger than $\Delta m^2_{\mathrm{SOL}}$ and $\Delta m^2_{\mathrm{ATM}}$: (a) The LSND experiment, in which a signal of short-baseline $\bar{\nu}_\mu \to \bar{\nu}_e$ oscillations has been observed with a statistical significance of about 3.8σ [14]. (b) The reactor antineutrino anomaly [15], which is a $\sim 2.8\sigma$ deficit of the rate of $\bar{\nu}_e$ observed in several short-baseline reactor neutrino experiments in comparison with that expected from a new calculation of the reactor neutrino fluxes [16, 17]. (c) The gallium neutrino anomaly [18], consisting in a short-baseline disappearance of ν_e measured in the gallium radioactive source experiments GALLEX and SAGE with a statistical significance of about 2.9σ.

In this review, we consider 3+1 [19, 20] and 3+2 [21] neutrino mixing schemes in which there are one or two additional massive neutrinos at the eV scale and the masses of the three standard massive neutrinos are much smaller. Since from the LEP measurement of the invisible width of the Z boson, we know that there are only three active neutrinos (see [1]), in the flavor basis the additional massive neutrinos correspond to sterile neutrinos [22], which do not have standard weak interactions.

In the 3+1 scheme, the effective probability of $\overset{(-)}{\nu}_\alpha \to \overset{(-)}{\nu}_\beta$ transitions in short-baseline experiments has the two-neutrino-like form

$$P_{\overset{(-)}{\nu}_\alpha \to \overset{(-)}{\nu}_\beta} = \delta_{\alpha\beta} - 4|U_{\alpha 4}|^2 \left(\delta_{\alpha\beta} - |U_{\beta 4}|^2\right) \sin^2\left(\frac{\Delta m^2_{41} L}{4E}\right) , \tag{8}$$

where U is the mixing matrix, L is the source-detector distance, E is the neutrino energy and $\Delta m^2_{41} = m^2_4 - m^2_1 = \Delta m^2_{\mathrm{SBL}} \sim 1\,\mathrm{eV}^2$. The electron and muon neutrino, and antineutrino appearance and disappearance in short-baseline experiments depend on $|U_{e4}|^2$ and $|U_{\mu 4}|^2$, which determine the amplitude $\sin^2 2\vartheta_{e\mu} = 4|U_{e4}|^2|U_{\mu 4}|^2$ of $\overset{(-)}{\nu}_\mu \to \overset{(-)}{\nu}_e$ transitions, the amplitude $\sin^2 2\vartheta_{ee} = 4|U_{e4}|^2\left(1 - |U_{e4}|^2\right)$ of $\overset{(-)}{\nu}_e$ disappearance, and the amplitude $\sin^2 2\vartheta_{\mu\mu} = 4|U_{\mu 4}|^2\left(1 - |U_{\mu 4}|^2\right)$ of $\overset{(-)}{\nu}_\mu$ disappearance.

Since the oscillation probabilities of neutrinos and antineutrinos are related by a complex conjugation of the elements of the mixing matrix (see [1]), the effective probabilities of short-baseline $\nu_\mu \to \nu_e$ and $\bar{\nu}_\mu \to \bar{\nu}_e$ transitions are equal. Hence, the 3+1 scheme cannot explain a possible CP-violating difference of $\nu_\mu \to \nu_e$ and $\bar{\nu}_\mu \to \bar{\nu}_e$ transitions in short-baseline experiments. In order to allow this possibility, one must consider a 3+2 scheme, in which, there are four additional effective mixing parameters in short-baseline experiments: Δm^2_{51}, which is conventionally assumed $\geq \Delta m^2_{41}$, $|U_{e5}|^2$, $|U_{\mu 5}|^2$ and $\eta = \arg\left[U^*_{e4} U_{\mu 4} U_{e5} U^*_{\mu 5}\right]$. Since this complex phase appears with different signs in the effective 3+2 probabilities of short-baseline $\nu_\mu \to \nu_e$ and $\bar{\nu}_\mu \to \bar{\nu}_e$ transitions, it can generate measurable CP violations.

Global fits of short-baseline neutrino oscillation data have been presented recently in Refs. [23, 24]. In the following, we summarize the results of the analysis of short-baseline data in the 3+1 and 3+2 schemes presented in Ref. [23]. The statistical results are listed in Table I. In the LOW fits all the

MiniBooNE data are considered, including the anomalous low-energy bins, which are omitted in the HIG fits. There is also a 3+1-noMB fit without MiniBooNE data and a 3+1-noLSND fit without LSND data.

TABLE I

Results of the fit of short-baseline data [23] taking into account all MiniBooNE data (LOW), only the MiniBooNE data above 475 MeV (HIG), without MiniBooNE data (noMB) and without LSND data (noLSND) in the 3+1 and 3+2 schemes. The first three lines give the minimum χ^2 ($\chi^2_{\rm min}$), the number of degrees of freedom (NDF) and the goodness-of-fit (GoF). The following five lines give the quantities relevant for the appearance–disappearance (APP–DIS) parameter goodness-of-fit (PG). The last three lines give the difference between the χ^2 without short-baseline oscillations and $\chi^2_{\rm min}$ ($\Delta\chi^2_{\rm NO}$), the corresponding difference of number of degrees of freedom ($\mathrm{NDF}_{\rm NO}$) and the resulting number of σs ($n\sigma_{\rm NO}$) for which the absence of oscillations is disfavored.

	3+1 LOW	3+1 HIG	3+1 noMB	3+1 noLSND	3+2 LOW	3+2 HIG
$\chi^2_{\rm min}$	291.7	261.8	236.1	278.4	284.4	256.4
NDF	256	250	218	252	252	246
GoF	6%	29%	19%	12%	8%	31%
$(\chi^2_{\rm min})_{\rm APP}$	99.3	77.0	50.9	91.8	87.7	69.8
$(\chi^2_{\rm min})_{\rm DIS}$	180.1	180.1	180.1	180.1	179.1	179.1
$\Delta\chi^2_{\rm PG}$	12.7	4.8	5.1	6.4	17.7	7.5
$\mathrm{NDF}_{\rm PG}$	2	2	2	2	4	4
$\mathrm{GoF}_{\rm PG}$	0.2%	9%	8%	4%	0.1%	11%
$\Delta\chi^2_{\rm NO}$	47.5	46.2	47.1	8.3	54.8	51.6
$\mathrm{NDF}_{\rm NO}$	3	3	3	3	7	7
$n\sigma_{\rm NO}$	6.3σ	6.2σ	6.3σ	2.1σ	6.0σ	5.8σ

From Table I one can see that in all fits which include the LSND data the absence of short-baseline oscillations is disfavored by about 6σ, because the improvement of the χ^2 with short-baseline oscillations is much larger than the number of oscillation parameters.

In both the 3+1 and 3+2 schemes, the goodness-of-fit in the LOW analysis is significantly worse than that in the HIG analysis and the appearance–disappearance parameter goodness-of-fit is much worse. This result confirms the fact that the MiniBooNE low-energy anomaly is incompatible with neutrino oscillations, because it would require a small value of Δm^2_{41} and a large value of $\sin^2 2\vartheta_{e\mu}$ which are excluded by the data of other experiments (see Ref. [23] for further details). Note that the appearance–disappearance tension in the 3+2-LOW fit is even worse than that in the 3+1-LOW fit, since the $\Delta\chi^2_{\rm PG}$ is so much larger that it cannot be compensated by the additional degrees of freedom. Therefore, we think that it is very likely that the MiniBooNE low-energy anomaly has an explanation which is different from neutrino oscillations and the HIG fits are more reliable than the LOW fits.

The 3+2 mixing scheme was considered to be interesting in 2010 when the MiniBooNE neutrino [25] and antineutrino [26] data showed a CP-violating tension. Unfortunately, this tension reduced considerably in the final MiniBooNE data [27] and from Table I one can see that there is a little improvement of the 3+2-HIG fit with respect to the 3+1-HIG fit, in spite of the four additional parameters and the additional possibility of CP violation. Moreover, since the p-value obtained by restricting the 3+2 scheme to 3+1 disfavors the 3+1 scheme only at 1.2σ [23], we think that considering the larger complexity of the 3+2 scheme is not justified by the data[2].

Figure 1 shows the allowed regions in the $\sin^2 2\vartheta_{e\mu}$–Δm^2_{41}, $\sin^2 2\vartheta_{ee}$–Δm^2_{41} and $\sin^2 2\vartheta_{\mu\mu}$–$\Delta m^2_{41}$ planes obtained in the 3+1-HIG fit of Ref. [23]. These regions are relevant, respectively, for $\overset{(-)}{\nu}_\mu \to \overset{(-)}{\nu}_e$ appearance, $\overset{(-)}{\nu}_e$ disappearance and $\overset{(-)}{\nu}_\mu$ disappearance searches. Figure 1 shows also the region allowed by $\overset{(-)}{\nu}_\mu \to \overset{(-)}{\nu}_e$ appearance data and the constraints from $\overset{(-)}{\nu}_e$ disap-

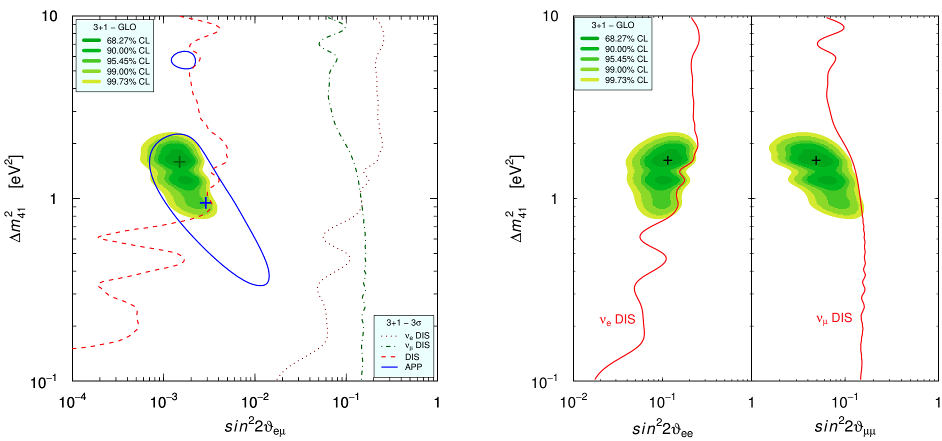

Fig. 1. Allowed regions in the $\sin^2 2\vartheta_{e\mu}$–Δm^2_{41}, $\sin^2 2\vartheta_{ee}$–Δm^2_{41} and $\sin^2 2\vartheta_{\mu\mu}$–$\Delta m^2_{41}$ planes obtained in the global (GLO) 3+1-HIG fit [23] of short-baseline neutrino oscillation data compared with the 3σ allowed regions obtained from $\overset{(-)}{\nu}_\mu \to \overset{(-)}{\nu}_e$ short-baseline appearance data (APP) and the 3σ constraints obtained from $\overset{(-)}{\nu}_e$ short-baseline disappearance data (ν_e DIS), $\overset{(-)}{\nu}_\mu$ short-baseline disappearance data (ν_μ DIS) and the combined short-baseline disappearance data (DIS). The best-fit points of the GLO and APP fits are indicated by crosses.

[2] See, however, the somewhat different conclusions reached in Ref. [24].

pearance and $\overset{(-)}{\nu}_\mu$ disappearance data. One can see that the combined disappearance constraint in the $\sin^2 2\vartheta_{e\mu}$–Δm^2_{41} plane excludes a large part of the region allowed by $\overset{(-)}{\nu}_\mu \to \overset{(-)}{\nu}_e$ appearance data, leading to the well-known appearance–disappearance tension quantified by the parameter goodness-of-fit in Table I.

It is interesting to investigate what is the impact of the MiniBooNE experiment on the global analysis of short-baseline neutrino oscillation data. With this aim, the authors of Ref. [23] performed two additional 3+1 fits: a 3+1-noMB fit without MiniBooNE data and a 3+1-noLSND fit without LSND data. From Table I one can see that the results of the 3+1-noMB fit are similar to those of the 3+1-HIG fit and the exclusion of the case of no-oscillations remains at the level of 6σ. On the other hand, in the 3+1-noLSND fit, without LSND data, the exclusion of the case of no-oscillations drops dramatically to 2.1σ. In fact, in this case, the main indication in favor of short-baseline oscillations is given by the reactor and gallium anomalies which have a similar statistical significance. Therefore, it is clear that the LSND experiment is still crucial for the indication in favor of short-baseline $\bar{\nu}_\mu \to \bar{\nu}_e$ transitions and the MiniBooNE experiment has been rather inconclusive.

4. Conclusions

The current status of our knowledge of three-neutrino mixing is very satisfactory after the recent determination of the smallest mixing angle ϑ_{13}: the two squared-mass differences and the three mixing angles are known with good precision. Future experiments must determine if ϑ_{23} is smaller or larger than $\pi/4$, the value of the Dirac CP-violating phase in the mixing matrix, the mass hierarchy and the absolute scale of neutrino masses. It is also very important to find if neutrinos are Majorana particles and in that case what are the values of the Majorana CP-violating phases.

Anomalies which cannot be explained in the framework of three-neutrino mixing and require the existence of sterile neutrinos have been observed by some short-baseline neutrino oscillation experiments. The results of the global fit of short-baseline neutrino oscillation data presented in Ref. [23] show that the data can be explained by 3+1 neutrino mixing and this simplest scheme beyond three-neutrino mixing cannot be rejected in favor of the more complex 3+2 scheme. The low-energy MiniBooNE anomaly cannot be explained by neutrino oscillations in any of these schemes. Moreover, the crucial indication in favor of short-baseline $\bar{\nu}_\mu \to \bar{\nu}_e$ appearance is still given by the old LSND data and the MiniBooNE experiment has been inconclusive. Hence new better experiments are needed in order to check this signal.

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