CURRENT STATUS OF CONSTRAINTS ON THE ELEMENTS OF THE NEUTRINO MASS MATRIX*

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We analyse the mass matrix of the three light neutrinos in the basis where the charged-lepton mass matrix is diagonal and discuss constraints on its elements for the Majorana and the Dirac case.

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1. Introduction

The recent enormous improvement of our knowledge of the neutrino oscillation parameters suggests a detailed investigation of the current constraints on the neutrino mass matrix. Most of these constraints depend on the assumed nature of neutrinos (Dirac or Majorana).

The structure of this paper is as follows. After a brief general discussion of the light-neutrino mass matrix in Section 2, we will investigate the implications of the currently available data on the Majorana neutrino mass matrix in Section 3. In Section 4 we will discuss constraints on the neutrino mass matrix in the Dirac case. Finally, we will conclude in Section 5.

2. The light-neutrino mass matrix

In this paper, we assume that there are exactly three light neutrino mass eigenstates with masses smaller than $\mathcal{O}(1\ \mathrm{eV})$, *i.e.* we assume that there are no light sterile neutrinos. By the term "neutrino mass matrix" we thus always mean the 3×3 mass matrix of the three light neutrinos.

If neutrinos are *Majorana particles*, we assume that there is a (possibly effective) mass term

$$\mathcal{L} = -\tfrac{1}{2}\overline{\nu^c_{\mathrm{L}}} M_\nu \nu_{\mathrm{L}} + \mathrm{H.c.} = \tfrac{1}{2}\nu_{\mathrm{L}}^T C^{-1} M_\nu \nu_{\mathrm{L}} + \mathrm{H.c.}\,, \tag{1}$$

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where M_ν is a complex symmetric 3×3-matrix. Such a mass term directly arises from the type-II see-saw mechanism and can be effectively generated via the see-saw mechanisms of type I and III.

If neutrinos are Dirac particles, *i.e.* if the total lepton number is conserved, we assume the existence of three right-handed neutrino fields ν_R leading to the mass term

$$\mathcal{L} = -\overline{\nu_\mathrm{R}} M_D \nu_\mathrm{L} + \mathrm{H.c.}\,, \tag{2}$$

where M_D is an arbitrary complex 3×3-matrix.

Before we can discuss any constraints on the neutrino mass matrix, we have to specify a basis in flavour space[1]. In models involving flavour symmetries, the chosen matrix representations of the flavour symmetry group specify the basis. Since we will at this point not assume any flavour symmetries in the lepton sector, we are free to choose a basis. For simplicity, we will always choose a basis in which the charged-lepton mass matrix is given by

$$M_\ell = \mathrm{diag}(m_e,\, m_\mu,\, m_\tau)\,. \tag{3}$$

3. Constraints on the Majorana neutrino mass matrix

3.1. Parametrization of the Majorana neutrino mass matrix

In the basis specified by equation (3), the Majorana neutrino mass matrix has the form

$$M_\nu = U^*_\mathrm{PMNS}\, \mathrm{diag}(m_1,\, m_2,\, m_3)\, U^\dagger_\mathrm{PMNS}\,, \tag{4}$$

where U_PMNS is the lepton mixing matrix and the m_i $(i = 1, 2, 3)$ are the masses of the three light neutrinos. As any unitary 3×3-matrix, U_PMNS can be parametrized by six phases and three mixing angles. We will use the parametrization

$$U_\mathrm{PMNS} = D_1 V D_2 \tag{5}$$

with

$$D_1 = \mathrm{diag}\left(e^{i\alpha},\, e^{i\beta},\, e^{i\gamma}\right) \qquad \text{and} \qquad D_2 = \mathrm{diag}\left(e^{i\rho},\, e^{i\sigma},\, 1\right)\,. \tag{6}$$

The phases α, β and γ are unphysical since they may be eliminated by a suitable redefinition of the charged-lepton fields. On the contrary, ρ and σ

[1] Since the gauge interactions are flavour-blind, we *a priori* have the freedom of performing arbitrary rotations in flavour space.

are physical in the case of Majorana neutrinos and are, therefore, referred to as the Majorana phases. V denotes the well-known unitary matrix

$$V = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}, \quad (7)$$

where $s_{ij} \equiv \sin\theta_{ij}$ and $c_{ij} \equiv \cos\theta_{ij}$ are the sines and cosines of the three mixing angles, respectively. The phase δ is responsible for a possible CP violation in neutrino oscillations (also in the Dirac case) and is, therefore, frequently referred to as the Dirac CP phase.

3.2. Upper and lower bounds on the absolute values of the elements of the neutrino mass matrix

The fact that the neutrino masses are the *singular values* of M_ν allows to derive a generic upper bound on the absolute values $|(M_\nu)_{\alpha\beta}|$. From linear algebra it is known that the absolute value of an element of a matrix is smaller or equal its largest singular value. For the neutrino mass matrix, this implies [1]

$$|(M_\nu)_{\alpha\beta}| \leq \max_k m_k\,. \quad (8)$$

Since this bound is valid for *any* matrix, it holds also for Dirac neutrinos. The strongest bounds on the absolute neutrino mass scale come from cosmology, where the sum of the masses of the light neutrinos is usually constrained to be at most of the order of $\mathcal{O}(1\,\mathrm{eV})$ — see *e.g.* the list of upper bounds in [2]. From this, we deduce the approximate upper bound $m_k \lesssim 0.3\,\mathrm{eV}$ leading to

$$|(M_\nu)_{\alpha\beta}| \lesssim 0.3\,\mathrm{eV}\,. \quad (9)$$

In [1] also an analytical lower bound on the $|(M_\nu)_{\alpha\beta}|$ is provided. Defining $a_k \equiv m_k|V_{\alpha k}||V_{\beta k}|$, one can show that

$$|(M_\nu)_{\alpha\beta}| \geq 2\max_k a_k - \sum_k a_k\,. \quad (10)$$

Note that this lower bound is independent of the Majorana phases ρ and σ. Unlike the generic upper bound discussed before, the lower bound (10) is valid only for Majorana neutrinos. Numerically evaluating this lower bound using the results of the global fits of oscillation data of [3, 4] only for two matrix elements leads to non-trivial lower bounds. The lower bounds in units of eV for these matrix elements are listed in the following table taken from [1].

TABLE I

		1σ	2σ	3σ
$\lvert(M_\nu)_{ee}\rvert$ (inv. spect.)	Forero *et al.* [3]	1.52×10^{-2}	1.36×10^{-2}	1.14×10^{-2}
	Fogli *et al.* [4]	1.62×10^{-2}	1.44×10^{-2}	1.24×10^{-2}
$\lvert(M_\nu)_{\tau\tau}\rvert$ (norm. spect.)	Forero *et al.* [3]	0	0	0
	Fogli *et al.* [4]	1.86×10^{-2}	1.27×10^{-2}	0

For both global fits the only element being bounded from below at the 3σ-level is $|(M_\nu)_{ee}|$ in the case of an inverted neutrino mass spectrum, for which in both cases one finds $|(M_\nu)_{ee}| \gtrsim 10^{-2}$ eV. Unfortunately, this bound is still far from the current upper bound stemming from searches for neutrinoless double beta decay, which is given by [5, 6]

$$m_{\beta\beta} \lesssim 0.4\,\mathrm{eV}\,. \tag{11}$$

3.3. Correlations of the elements of the neutrino mass matrix

In the case of Majorana neutrinos, the absolute values of the elements of M_ν depend on nine real parameters, namely

$$m_0\,, \quad \Delta m^2_{21}\,, \quad \Delta m^2_{31}\,, \quad \theta_{12}\,, \quad \theta_{23}\,, \quad \theta_{13}\,, \quad \delta\,, \quad \rho\,, \quad \sigma\,, \tag{12}$$

where m_0 denotes the mass of the lightest neutrino. Using the experimental/observational constraints on these parameters, one can create plots of the allowed ranges of the $|(M_\nu)_{\alpha\beta}|$ *versus* m_0, which was first done by Merle and Rodejohann in [7]. In [1] the analysis of [7] was repeated using the results of the recent global fits of oscillation data of [3, 4]. It turned out that, at the 3σ-level, the plots of [7] are still in good agreement with the ones of [1].

In addition, in [1] also *correlation plots* of the $|(M_\nu)_{\alpha\beta}|$ were created. Since the Majorana neutrino mass matrix has six independent entries, there are 15 correlations. Taking into account the two possible neutrino mass spectra, there is a total of 30 plots. Among these 30 correlations, one finds only five which are manifest at the 3σ-level, namely [1]:

$$\begin{array}{llll}
|(M_\nu)_{ee}| & \textit{vs.} & |(M_\nu)_{\mu\mu}| & (\text{normal spectrum})\,,\\
|(M_\nu)_{ee}| & \textit{vs.} & |(M_\nu)_{\mu\tau}| & (\text{normal spectrum})\,,\\
|(M_\nu)_{ee}| & \textit{vs.} & |(M_\nu)_{\tau\tau}| & (\text{normal spectrum})\,,\\
|(M_\nu)_{\mu\mu}| & \textit{vs.} & |(M_\nu)_{\mu\tau}| & (\text{normal spectrum})\,,\\
|(M_\nu)_{\mu\tau}| & \textit{vs.} & |(M_\nu)_{\tau\tau}| & (\text{normal spectrum})\,.
\end{array}$$

All of these five correlations may be subsumed as "if one matrix element is small, the other one must be large". An example for such a correlation plot can be found in figure 1. In the case of an inverted neutrino mass spectrum, there are no correlations manifest at the 3σ-level.

It is important to note that while at the 3σ-level the correlation plots based on the global fits of [3] and [4] agree, this is not true at the 1σ-level — for further details see [1].

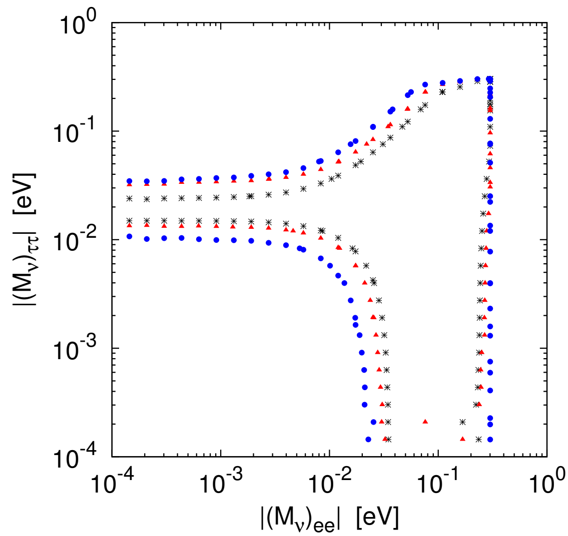

Fig. 1. Correlation plot of $|(M_\nu)_{ee}|$ *vs.* $|(M_\nu)_{\tau\tau}|$ based on the global fit results of Forero *et al.* [3] assuming a normal neutrino mass spectrum and allowing m_0 to vary between zero and 0.3 eV. The boundaries of the allowed areas are depicted by the following symbols: best fit: $\ast$, 1σ : $\blacktriangle$, 3σ : $\bullet$.

4. Constraints on the Dirac neutrino mass matrix

4.1. Parametrization of the Dirac neutrino mass matrix

In analogy to the Majorana case, we will study the 3×3 Dirac neutrino mass matrix M_D in the basis where the charged-lepton mass matrix is diagonal — see equation (3). In this basis, M_D takes the form

$$M_D = V_R \, \mathrm{diag}\,(m_1, \, m_2, \, m_3) \, U^\dagger_{\mathrm{PMNS}} \,, \tag{13}$$

where V_R is a unitary 3×3-matrix. V_R can be eliminated by considering the matrix

$$H_D \equiv M_D^\dagger M_D = U_{\mathrm{PMNS}} \, \mathrm{diag}\, \left(m_1^2, \, m_2^2, \, m_3^2\right) \, U^\dagger_{\mathrm{PMNS}} \,. \tag{14}$$

Since all observables accessible by current experimental scrutiny are contained in H_D, all matrices M_D leading to the same H_D are *indistinguishable* from the experimental point of view. Therefore, the nine parameters of V_D have to be treated as *free* parameters. Consequently, in stark contrast to the Majorana case, in the Dirac case the neutrino mass matrix has at least nine free parameters (even if the mixing matrix and the neutrino masses are known).

This freedom of choosing V_R has important consequences for the analysis of M_D. Obviously it is much harder to put constraints on the elements of M_D than in the Majorana case. The freedom of choosing V_R even allows to set several elements of M_D to zero without changing the physical predictions. This directly follows from the fact that every matrix can be decomposed into a product of a unitary matrix and an upper triangular matrix[2]. Thus, there is a choice of V_R such that

$$M_D = \begin{pmatrix} m_{11} & m_{12} & m_{13} \\ 0 & m_{22} & m_{23} \\ 0 & 0 & m_{33} \end{pmatrix} . \tag{15}$$

Similarly, by multiplication of M_D by one of the six 3×3 permutation matrices generated by

$$\begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 0 \end{pmatrix} \quad \text{and} \quad \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} \tag{16}$$

from the left, one can arbitrarily permute the rows of M_D without changing any physical predictions [8].

However, it is most important to note that the freedom of choosing V_R holds only as long as we do not impose a symmetry in the lepton sector. Namely, a flavour symmetry which acts non-trivially in the neutrino sector imposes constraints on the form of the neutrino mass matrix M_D, *i.e.* not only on the neutrino masses and U_{PMNS} but also on V_R. Consequently, a choice of V_R, *e.g.* such that M_D is upper triangular, will, in general, be incompatible with the flavour symmetry. Nevertheless, if we want to set bounds on the elements of M_D (without introducing flavour symmetries), we indeed have the freedom of arbitrarily choosing the matrix V_R. Therefore, examining bounds and correlations of the elements of M_D is much less elucidating than in the Majorana case. That said, studies of M_D such as the question for the allowed cases of texture zeros in M_D are still of great interest.

[2] This factorization is known as the so-called QR-decomposition.

4.2. Texture zeros in the Dirac neutrino mass matrix

In the following, we will shortly comment on the allowed cases of texture zeros in M_D under the assumption that M_ℓ is diagonal. A detailed analysis has been done by Hagedorn and Rodejohann in [8], which provides a classification of all possible texture zeros in this framework. We repeated the analysis of [8] of the allowed cases of five, four and three texture zeros[3] in M_D based on the global fit results of [4]. Our numerical results are in perfect agreement with the analysis of [8]. However, there are some previously allowed cases of texture zeros which can be excluded due to the new data, namely precisely those which lead to a vanishing or too small value ($\lesssim 10^{-3}$) of $\sin^2\theta_{13}$, *i.e.* A, B, $\tilde{B}$, C, D_1–D_3, $\tilde{D}_1$–$\tilde{D}_3$, E (inverted spectrum) and $\tilde{E}$ (inverted spectrum) in the notation of [8]. Consequently, all cases of five texture zeros in M_D are now excluded, and among the cases of four texture zeros only E (normal spectrum), $\tilde{E}$ (normal spectrum) as well as F_1–F_3 remain valid.

5. Conclusions

In the case of Majorana neutrinos, the absolute values of the elements of the light-neutrino mass matrix M_ν can be described by nine parameters, of which seven are constrained by experiments/observations. The by now very precise knowledge of the oscillation parameters therefore allows detailed studies of the elements of M_ν, including their allowed ranges and their correlations.

The situation is quite different in the case of Dirac neutrinos, where the neutrino mass matrix M_D is by far not uniquely determined, even if the neutrino masses and the mixing matrix are known. Therefore, putting bounds on the elements of M_D is much harder than in the Majorana case. Nevertheless studies of M_D are possible, for example the analysis of texture zeros in M_D. We reinvestigated the allowed texture zeros of M_D in the basis where the charged-lepton mass matrix is diagonal. Our results agree with the original analysis [8], the only difference being that by now we know that $\sin^2\theta_{13} \gg 10^{-3}$, which excludes some previously viable types of texture zeros.

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[3] In [8] also the cases of one and two texture zeros in M_D are investigated, the result being that all these cases are allowed and do not show any relations among the observables.

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