

PROGRESS IN THE PARAMETRISATION OF THE NEUTRINO SECTOR*

THOMAS GAJDOSIK, ANDRIUS JUODAGALVIS, DARIUS JURČIUKONIS
TOMAS SABONIS

Vilnius University, Universiteto 3, 01513, Vilnius, Lithuania

(Received October 24, 2013)

Adding gauge singlets to the original Standard Model allows an explanation for the observed smallness of the neutrino masses using the see-saw mechanism. Following our plans presented in the last conference of this series, we present the results for the non-standard setting, when the number of the singlets is smaller than the number of the SM generations.

DOI:10.5506/APhysPolB.44.2347

PACS numbers: 11.30.Rd, 13.15.+g, 14.60.St

1. Continuing Ustroń'11

In [1], we described our plans to parametrise the n_L -generations Standard Model equipped with n_R additional gauge singlet fermions N_R and $n_H \geq 1$ Higgs doublets ϕ_k [2]. The Grimus–Lavoura ansatz [3] gives the masses and mixing parameters in terms of the parameters of the Lagrangian

$$\mathcal{L} = \mathcal{L}_{\text{SM},\nu} - \tilde{\phi}_k^\dagger \bar{N}_R Y_\nu^k L_L - \frac{1}{2} N_R^\top C^{-1} M_R N_R + \text{h.c.}, \quad (1)$$

where $\tilde{\phi}_k = i\tau_2 \phi_k^*$ is the SU(2)-conjugated Higgs doublet and Y_ν^k is the $n_R \times n_L$ neutrino Yukawa matrix for the k th Higgs doublet.

Electroweak symmetry breaking triggered by the vacuum expectation values of the neutral Higgs fields $(0, \frac{1}{\sqrt{2}} v_k)^\top = \langle \phi_k \rangle_0^\top$ gives an effective mass term to all Standard Model particles: the vector bosons, the Higgs bosons, and the charged fermions. It also generates the $n_R \times n_L$ dimensional Dirac mass term

$$M_D = \sum_{k=1}^{n_H} \frac{1}{\sqrt{2}} v_k Y_\nu^k \quad (2)$$

* Presented at the XXXVII International Conference of Theoretical Physics “Matter to the Deepest” Ustroń, Poland, September 1–6, 2013.

that enters the symmetric $(n_L + n_R) \times (n_L + n_R)$ neutrino mass matrix

$$M_\nu = \begin{pmatrix} M_L & M_D^\top \\ M_D & M_R \end{pmatrix}, \quad (3)$$

where M_R is the Majorana mass matrix from Eq. (1) and $M_L = 0$ at tree level, as such a term violates the $U(1)_Y \times SU(2)_{\text{weak}}$ gauge symmetry of the Standard Model.

The most convenient diagonalisation of the mass matrix M_ν , Eq. (3), for arbitrary n_L and n_R is the Grimus–Lavoura ansatz [3], as it reduces the $(n_L + n_R)^2$ parameters of the unitary diagonalisation matrix to only $2n_L n_R$ parameters in the complex matrix B (see [3]) that go into the ansatz.

2. Reverse engineering the Grimus–Lavoura ansatz

The Grimus–Lavoura ansatz determines the masses and mixing matrices of the physical particles from the parameters of the Lagrangian. Our idea was to determine the parameters of the Lagrangian from the masses and mixings.

The Casas–Ibarra parametrisation [4], used in [5], does this determination for $n_L = n_R$ and solves the leading order see-saw [6] equation

$$M_\ell = -M_D^\top M_h^{-1} M_D \quad (4)$$

by the ansatz

$$M_D = iM_h^{1/2} \times O \times M_\ell^{1/2} \quad (5)$$

with an arbitrary (complex) orthogonal matrix O . This is the most general parametrisation for the case $n_L = n_R$. Our investigation for the case $n_L > n_R$ showed that it is always possible to reduce the problem of diagonalising the $(n_L + n_R) \times (n_L + n_R)$ dimensional M_ν to diagonalising an effective $2n_R$ dimensional M'_ν using unitary matrices:

$$U^\top M_\nu U = U^\top \begin{pmatrix} 0 & M_D^\top \\ M_D & M_R \end{pmatrix} U = \begin{pmatrix} 0 & 0 \\ 0 & M'_\nu \end{pmatrix}. \quad (6)$$

This was argued before in [2]. We construct the explicit matrices for this reduction. We presented the case $n_L = 3$ and $n_R = 1$ at the conference [7].

In the case $n_L = 3$ and $n_R = 2$, we can define the unitary matrix U as a product $U = U_{12} \times U_{13}$ with the unitary matrices defined as

$$(U_{1n})_{jk} = \delta_{jk} - \left(1 - \sqrt{1 - |s_n|^2}\right) (\delta_{j1}\delta_{k1} + \delta_{jn}\delta_{kn}) + s_n \delta_{j1}\delta_{kn} - s_n^* \delta_{jn}\delta_{k1}, \quad (7)$$

where angles and phases are given by

$$\frac{s_2^*}{\sqrt{1 - |s_2|^2}} = \frac{(M_\nu)_{14}(M_\nu)_{35} - (M_\nu)_{15}(M_\nu)_{34}}{(M_\nu)_{24}(M_\nu)_{35} - (M_\nu)_{25}(M_\nu)_{34}} \tag{8}$$

and

$$\frac{s_3^*}{\sqrt{1 - |s_3|^2}} = -\sqrt{1 - |s_2|^2} \frac{(M_\nu)_{14}(M_\nu)_{25} - (M_\nu)_{15}(M_\nu)_{24}}{(M_\nu)_{24}(M_\nu)_{35} - (M_\nu)_{25}(M_\nu)_{34}}. \tag{9}$$

3. Numerical evaluations

The analytic analysis using angles and phases gives a deeper insight into the problem. However, the rotation matrices defined by the angles can lead to numerical instabilities and more time consuming operations in the numerical calculations than using the parameters of the Lagrangian directly. Therefore, we just use a direct parametrisation of the Yukawa matrices and not our result from the reverse engineering.

3.1. The case $n_L = 3$ and $n_R = 1$

We parametrise the Yukawa coupling as

$$Y_\nu^k = \frac{\sqrt{2}}{v} m_D \vec{a}_k^\top \quad \text{with} \quad v^2 = \sum_{k=1}^{n_H} v_k^2 = \frac{2m_W^2}{g^2}, \tag{10}$$

where m_D is the singular value of M_D , and the vectors \vec{a}_k describe the relative coupling strength of the Higgs doublets. At tree level, we get the mass relations

$$m_D^2 = m_\ell m_h \quad \text{and} \quad m_R = (m_h - m_\ell) \sim m_h \tag{11}$$

with m_ℓ the only non-vanishing light neutrino mass and m_h the heavy mass. The other two light states stay massless. Using the single Higgs doublet of the Standard Model and calculating the loop corrections to the masses of the neutrinos does not change this qualitative picture.

Including more Higgs doublets allows the radiative generation of a mass for one of the massless neutrinos [2]. In our numerical example, we take two Higgs doublets with the Yukawa couplings, Eq. (10). We ignore the effects of the Higgs sector that do not influence the neutrino masses, only the lightest neutral Higgs is required to have 125 GeV. Loop corrections are calculated following [8].

The Monte Carlo sampling is used to get the numerical result. We generate random sets of the parameters $\{m_h, m_{H_2}, m_{H_3}, \vec{a}_1, \vec{a}_2\}$ which determine the Yukawa couplings and the size of the loop corrections. If the generated

one-loop neutrino masses fulfil the measured Δm_{\odot}^2 and Δm_{atm}^2 , we consider the set an allowed point. Figure 1 (a) shows that the multitude of the heavy Higgs masses and the different Yukawa couplings allows a band of neutrino masses, that can fulfil the experimental constraint as long as the heavy singlet is heavier than 830 GeV.

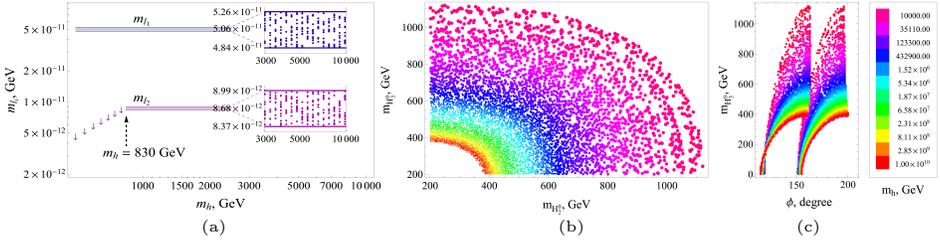


Fig. 1. Results for the case $n_R = 1$. The plot (a) shows the light neutrino masses in dependence on the mass of the heavy singlet. The middle (right) scatterplot (*i.e.* (b) and (c)) shows allowed parameter points depending on the mass of the heavy singlet m_h through the colour code and on the masses of the heavier neutral Higgses $m_{H_{2,3}^0}$ (the parameter ϕ describing the relative phase of the Yukawa couplings).

Figure 1 (b) shows the correlation of the masses of the heavy Higgses with the scale of the heavy singlet. The distribution of the allowed points suggest that only the size of the Higgs mass matters, but its type, whether it is CP-conserving or CP-violating, is less important. The figure also tells us that for a small scale of the heavy singlet, the masses of the heavy Higgses have to become very big, suggesting a decoupling limit.

Figure 1 (c) shows the tight correlation between the alignment of the Yukawa couplings and the required size of the Higgs masses. In order to show this correlation in a single plot, we restrict the vectors to $\vec{a}_1^\top = (0, 0, 1)$ and $\vec{a}_2^\top = (0, 1, e^{i\phi})$. We get valid parameter points only for $|\tan(\phi + 30^\circ)| < 1.2$ and then we can get a rather tight constraint between the value of ϕ , the scale of the heavy singlet and the masses of the heavy Higgses.

3.2. The case $n_L = 3$ and $n_R = 2$

We parametrise the Yukawa couplings as

$$Y_\nu^k = \frac{\sqrt{2}}{v} \begin{pmatrix} m_{D_2} \vec{a}_k^\top \\ m_{D_1} \vec{b}_k^\top \end{pmatrix} \quad \text{with} \quad m_{D_i}^2 = m_{\nu_i} m_{h_i}, \quad (12)$$

where we order the masses as $m_{h_1} > m_{h_2}$ and $m_{\nu_1} > m_{\nu_2} > m_{\nu_3} = 0$ (at tree level). The vectors \vec{a}_k and \vec{b}_k describe the relative coupling strength of the Higgs doublets. At tree level, we can reduce the $(3 + 2) \times (3 + 2)$ mass

matrix M_ν according Eq. (6) to a 4×4 dimensional M'_ν , which can be solved by the Grimus–Lavoura ansatz, with M_D parametrized by the Casas–Ibarra ansatz, Eq. (5).

Although we can get both mass differences, Δm_\odot^2 and Δm_{atm}^2 , already at tree level we perform our numerical analysis with the loop corrections for the masses of the light neutrinos included. We fix $m_{H_1} = 125$ GeV and $m_{h_2} = 100$ GeV and vary the parameters $\{m_{h_1}, \vec{a}_k, \vec{b}_k, m_{H_2}, m_{H_3}\}$ with the constraint $m_{H_{2,3}} > 200$ GeV and check if the mass differences between the light neutrinos give Δm_\odot^2 and Δm_{atm}^2 . In this case, the influence of the heavier Higgses is much smaller than for the case $n_R = 1$. This can be easily understood as the Higgs masses only influence the mass corrections to the light neutrinos. But the mass differences of the light neutrinos, Δm_\odot^2 and Δm_{atm}^2 , are already mostly determined by the tree level.

Figure 2(a) shows the solutions for the normal hierarchy of the neutrino masses. The shaded bands show the most probable values of the masses. We pick a very light scale for the heavy singlet, namely just 100 GeV. An interesting observation is the reduction of the loop generated light neutrino mass with the increase of the mass of the heavier singlet beyond 10^5 GeV. The obtained masses of the light neutrinos do not saturate the cosmological bound of $\sum_i m_{\nu_i} < 1$ eV.

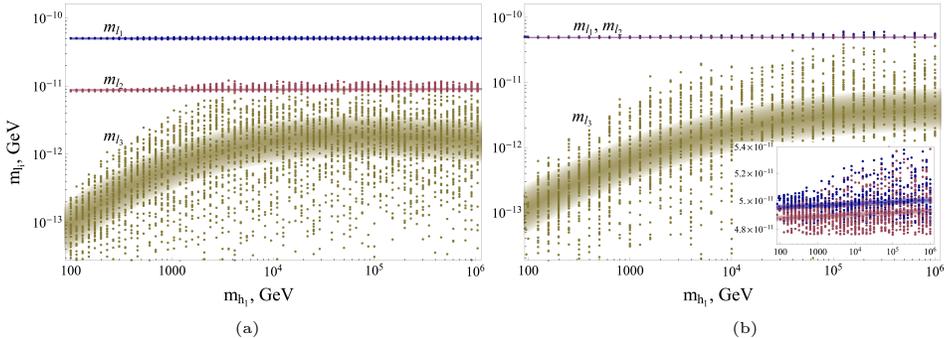


Fig. 2. Results for the case $n_R = 2$ showing the light neutrino masses in dependence on the mass of the heavier singlet. The mass of the lighter heavy singlet m_{h_2} is set to 100 GeV. Normal (inverted) neutrino mass ordering is shown on the left (right) side.

Figure 2(b) shows the solutions for the inverted hierarchy of the neutrino masses. Again, we pick a very light scale for the heavy singlet, namely just 100 GeV. In this case, the light neutrino mass increases monotonically with the mass of the heavier singlet. Even though the sum of the masses gets higher than in the normal hierarchy, we still cannot saturate the cosmological bound. So cosmological estimates do not restrict the parameter space of our model.

4. Conclusions and outlook

Our study of the cases $n_R = 1$ and $n_R = 2$ shows that both cases are not excluded by simple considerations of the measured neutrino mass differences. To use the data of the neutrino mixing matrix, we have to assume something about the charged lepton mass matrices, which was not our goal. The case $n_R = 1$ predicts a tight correlation between the scale of the see-saw, masses of the heavy Higgses and the values of the Yukawa coupling, suggesting a fine tuning of the Higgs sector in order to allow this scenario. Further investigation into the required Higgs sector and the allowed Yukawa couplings is necessary to rule out this scenario.

The case $n_R = 2$ still has too many free parameters to give any tight predictions. As we did not consider the charged lepton mass matrix, we could not use the neutrino mixing matrix as a constraint to our model.

We saw in our analysis that a treatment of the extended Higgs sector is needed. We plan to look for a source that can easily give experimental limits on the parameters of the Higgs sector, including the Yukawa couplings. With this tool equipped, we hope to support or rule out our $n_R = 1$ scenario and to sensibly restrict the case $n_R = 2$.

T.G., A.J., and D.J. thank the Lithuanian Academy of Sciences for the support (the project TauPol2013, No. CERN-VU-2013-3).

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