THE SURFACE GRAVITATIONAL REDSHIFT OF THE NEUTRON STAR PSR J1614-2230

Xian-Feng Zhao

College of Mechanical and Electronic Engineering, Chuzhou University
Chuzhou 239000, China

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Based on the hadronic level, the value range of the surface gravitational redshift of the neutron star PSR J1614-2230 is tried to be determined in the framework of the relativistic mean field theory. It is found that for the neutron star PSR J1614-2230 the ratio of mass M and radius R is in the range of $\sim$ 0.1692 to 0.1958 and the surface gravitational redshift is in the range of $\sim$ 0.4138 to 0.5397. However, for a small mass neutron star, which is calculated with the hyperon coupling constant obtained by the constituent quark model [SU(6) symmetry], the ratio of mass M and radius R is 0.1221 and the surface gravitational redshift is 0.2507. The surface gravitational redshift of the neutron star PSR J1614-2230 is about one times larger than that of the small mass neutron star.

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1. Introduction

The neutron star PSR J1614-2230 was observed in 2010 and at the same time its mass was determined as $1.97 \pm 0.04\, M_{\odot}$ [1], being almost the largest one up to now.

It is well known that whether the equation of state is soft or stiff would determine the mass of a neutron star being small or large. The stiffer equation of state would give a larger mass of neutron star. Therefore, for the mass of the neutron star PSR J1614-2230, its equation of state is relatively stiff [2] because of its large mass.

Whether the neutron star PSR J1614-2230 is hyperon star or quark star is investigated by constructing an equation of state based on a combination of phenomenological relativistic hyper-nuclear density functional and an effective model of quantum chromodynamics (the Nambu–Jona-Lasinio model). It is found that the neutron star PSR J1614-2230 would be hybrid stars with hyperons and quark color-superconductivity [3].

Based on a microscopic Brueckner–Hartree–Fock approach of hyperonic matter supplemented with additional simple phenomenological density-dependent contact terms, the effect of hyperonic three-body forces on the maximum mass of neutron stars is estimated by Vidaña *et al.* [4]. Their results show that although hyperonic three-body forces can reconcile the maximum mass of hyperonic stars with the current limit of $1.4 \sim 1.5\,M_\odot$, they are unable to provide the repulsion needed to make the maximum mass compatible with the PSR J1614-2230 ($1.97 \pm 0.04\,M_\odot$).

Constraining the hadronic equation of state from the mass of the neutron star PSR J1614-2230, the influence of the hyperon potentials on the stiffness of the equation of state was systematically investigated by Weissenborn *et al.* [5]. It was found that they have but little influence on the maximum mass compared to the inclusion of an additional vector-meson mediating repulsive interaction amongst hyperons. The new mass limit can only be reached with this additional meson regardless of the hyperon potentials.

Although a lot of works have been done on the neutron star PSR J1614-2230, its surface gravitational redshift almost has not been mentioned. The measurement of the gravitational redshift can provide unique information about the compactness M/R of the star [6]. So it is important to study the surface gravitational redshift of the neutron star PSR J1614-2230.

In this paper, employing the relativistic mean field theory, the surface gravitational redshift of the neutron star PSR J1614-2230 is investigated.

2. The relativistic mean field theory and the surface gravitational redshift of a neutron star

The Lagrangian density of hadron matter reads as follows [7]

$$\begin{aligned}\mathcal{L} \;=\; & \sum_B \overline{\Psi}_B \left(i\gamma_\mu\partial^\mu - m_B + g_{\sigma B}\sigma - g_{\omega B}\gamma_\mu\omega^\mu - \tfrac{1}{2}g_{\rho B}\gamma_\mu\tau\,\rho^\mu\right)\Psi_B \\ & +\tfrac{1}{2}\left(\partial_\mu\sigma\partial^\mu\sigma - m_\sigma^2\sigma^2\right) - \tfrac{1}{4}\omega_{\mu\nu}\omega^{\mu\nu} + \tfrac{1}{2}m_\omega^2\omega_\mu\omega^\mu - \tfrac{1}{4}\rho_{\mu\nu}\,\rho^{\mu\nu} \\ & +\tfrac{1}{2}m_\rho^2\rho_\mu\,\rho^\mu - \tfrac{1}{3}g_2\sigma^3 - \tfrac{1}{4}g_3\sigma^4 + \sum_{\lambda=e,\mu}\overline{\Psi}_\lambda\left(i\gamma_\mu\partial^\mu - m_\lambda\right)\Psi_\lambda\,. \qquad (1)\end{aligned}$$

The energy density and pressure of a neutron star are given by

$$\begin{aligned}\varepsilon = & \frac{1}{2}m_\sigma^2\sigma^2 + \frac{1}{3}g_2\sigma^3 + \frac{1}{4}g_3\sigma^4 + \frac{1}{2}m_\omega^2\omega_0^2 + \frac{1}{2}m_\rho^2\rho_{03}^2 \\ & + \sum_B \frac{2J_B+1}{2\pi^2}\int_0^{\kappa_B}\kappa^2 d\kappa\sqrt{\kappa^2+m^{*2}} + \frac{1}{3}\sum_{\lambda=e,\mu}\frac{1}{\pi^2}\int_0^{\kappa_\lambda}\kappa^2 d\kappa\sqrt{\kappa^2+m_\lambda^{*2}}\,, \quad (2)\end{aligned}$$

$$p = -\frac{1}{2}m_\sigma^2\sigma^2 - \frac{1}{3}g_2\sigma^3 - \frac{1}{4}g_3\sigma^4 + \frac{1}{2}m_\omega^2\omega_0^2 + \frac{1}{2}m_\rho^2\rho_{03}^2$$
$$+\frac{1}{3}\sum_B \frac{2J_B+1}{2\pi^2}\int_0^{\kappa_B} \frac{\kappa^4}{\sqrt{\kappa^2+m^{*2}}}d\kappa + \frac{1}{3}\sum_{\lambda=e,\mu}\frac{1}{\pi^2}\int_0^{\kappa_\lambda}\frac{\kappa^4}{\sqrt{\kappa^2+m_\lambda^{*2}}}d\kappa\,, \quad (3)$$

where, m^* is the effective mass of baryons

$$m^* = m_B - g_{\sigma B}\sigma\,. \quad (4)$$

The surface gravitational redshift of a neutron star is given by [7]

$$z = \left(1 - \frac{2M}{R}\right)^{-1/2} - 1\,. \quad (5)$$

3. Parameters

In this work, we choose the GL97 set [7] listed in Table I.

TABLE I

The coupling constants of the nucleons GL97 set.

m	m_σ	m_ω	m_ρ	g_σ	g_ω	g_ρ	g_2
939	500	782	770	7.9835	8.7	8.5411	20.966
g_3	C_3	ρ_0	B/A	K	$a_{\rm sym}$	m^*/m	
−9.835	0	0.153	16.3	240	32.5	0.78	

We define the ratios: $x_\sigma = x_{\sigma h} = \frac{g_{\sigma h}}{g_{\sigma N}}$, $x_\omega = x_{\omega h} = \frac{g_{\omega h}}{g_{\omega N}}$, $x_\rho = x_{\rho h} = \frac{g_{\rho h}}{g_{\rho N}}$, with h denoting hyperons.

Generally, the hyperon coupling constants are in the range of 0.3–1 [8]. In our previous work, we fixed $x_{\rho\Lambda} = 0$, $x_{\rho\Sigma} = 2$, $x_{\rho\Xi} = 1$, which are given by the constituent quark model [SU(6) symmetry] [7], and chose $x_\sigma = 0.33$, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9. For each x_σ, the x_ω are also respectively chosen as 0.33, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9. By the steps above, we determined the value ranges of x_σ and x_ω, by which the mass of the neutron star PSR J1614-2230 can be gotten (see Fig. 1) [9].

In this paper, we would determine the value range of the surface gravitational redshift of the neutron star PSR J1614-2230 according to the values of x_σ and x_ω determined by Ref. [9]. As a comparison, we also calculate the surface gravitational redshift of a small mass neutron star, which is calculated with the hyperon coupling constant obtained by the constituent quark model [SU(6) symmetry].

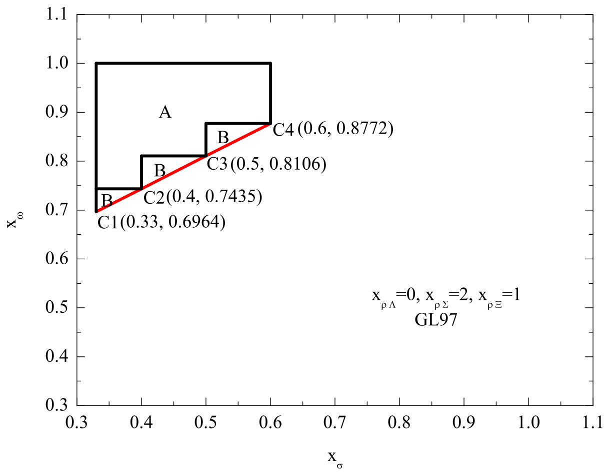

Fig. 1. The value ranges of x_σ and x_ω, determined by Ref. [9], corresponding to the mass of the neutron star PSR J1614-2230.

4. Effect of x_σ and x_ω on the surface gravitational redshift of a neutron star

In order to examine the effect of x_ω on the surface gravitational redshift of a neutron star, we respectively choose $x_\omega = 0.70$, 0.74, 0.78, 0.82, 0.86, 0.90, 0.94 with $x_\sigma = 0.33$, $x_{\rho\Lambda} = 0$, $x_{\rho\Sigma} = 2$ and $x_{\rho\Xi} = 1$ being fixed.

Figure 2 shows the effect of x_ω on the mass and the surface gravitational redshift of a neutron star. From the left graph we see that the mass of the neutron star increases as the x_ω increases. That is to say that the equation of state will become stiffer with the x_ω increase. From the right graph it can be seen that the surface gravitational redshift increases with the increase of the central energy density. As the hyperon coupling constant x_ω increases from 0.70 to 0.94 in step of 0.04, the surface gravitational redshift increases.

In order to examine the effect of x_σ on the surface gravitational redshift of a neutron star, we respectively choose $x_\sigma = 0.33$, 0.37, 0.41, 0.45, 0.49, 0.53, 0.60 with x_ω=0.6964, $x_{\rho\Lambda} = 0$, $x_{\rho\Sigma} = 2$ and $x_{\rho\Xi} = 1$ being fixed.

The influence of x_σ on the mass and the surface gravitational redshift of a neutron star is given in Fig. 3. From the left graph we see that the mass of the neutron star decreases as the x_σ increases. That is to say that the equation of state will become softer with the x_σ increase. From the right graph we see that the surface gravitational redshift decreases as the hyperon coupling constant x_σ increases from 0.33 to 0.60.

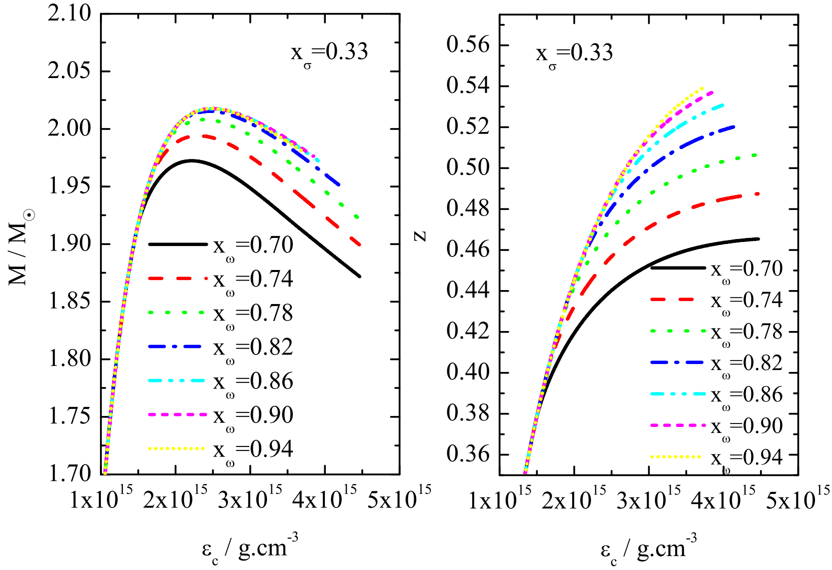

Fig. 2. Effect of x_ω on the mass and the surface gravitational redshift of a neutron star.

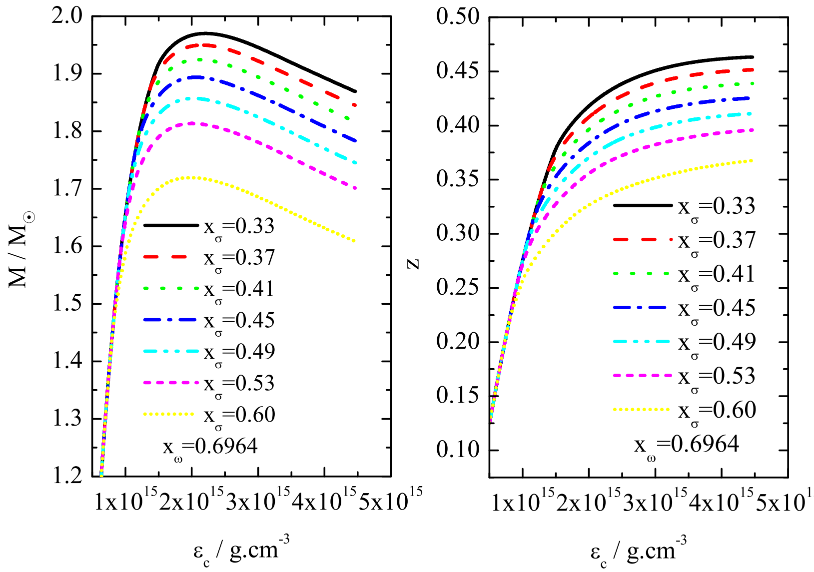

Fig. 3. Effect of x_σ on the mass and the surface gravitational redshift of a neutron star.

In short, the surface gravitational redshift of a neutron star increases with the x_ω increase and decreases with the x_σ increase. Having understood the influence of x_σ and x_ω on the surface gravitational redshift of a neutron star will make us determine the value range of the surface gravitational redshift of the neutron star PSR J1614-2230.

5. The surface gravitational redshift of the neutron star PSR J1614-2230

In order to determine the value range of the surface gravitational redshift of the neutron star PSR J1614-2230, according to Fig. 1, we calculate the eight cases as follows: $(x_\sigma,\ x_\omega) = (0.33,\ 0.7435)$, $(0.33,\ 1)$, $(0.6,\ 1)$, $(0.6,\ 0.8772)$, $(0.5,\ 0.8772)$, $(0.5,\ 0.8106)$, $(0.4,\ 0.8106)$, $(0.4,\ 0.7435)$. Here, we fix $x_{\rho\Lambda} = 0$, $x_{\rho\Sigma} = 2$, $x_{\rho\Xi} = 1$ and choose the nucleon coupling constant GL97.

The maximum mass of neutron star, corresponding to every equation of state is given in Table II. The surface gravitational redshift as a function of central energy density is shown in Fig. 4. We can see that the surface gravitational redshift increases with the central energy density increasing. Since $x_\sigma = 0.33$ is the smallest in the eight cases calculated, $x_\sigma = 0.33$ corresponds the maximum value of the surface gravitational redshift. On the other hand, for $x_\omega = 1$ is the largest value of the eight cases, so $x_\omega = 1$ corresponds the maximum value of the surface gravitational redshift. Therefore, the case of $x_\sigma = 0.33$ and $x_\omega = 1$ corresponds the maximum value of the surface gravitational redshift of the neutron star PSR J1614-2230. For a similar reason, the case of $x_\sigma = 0.33$ and $x_\omega = 0.7435$ corresponds to the minimum value. The shadow in Fig. 4 represents the value range of the surface gravitational redshift of the neutron star PSR J1614-2230 determined by us.

TABLE II

The maximum mass of neutron star corresponding to every equation of state calculated in this work. Here, we fix $x_{\rho\Lambda} = 0$, $x_{\rho\Sigma} = 2$, $x_{\rho\Xi} = 1$ and choose the nucleon coupling constant GL97.

x_σ	x_ω	Maximum mass $M_\odot$	x_σ	x_ω	Maximum mass $M_\odot$
0.33	0.7435	1.9957	0.50	0.8772	2.0047
0.33	1.0000	2.0177	0.50	0.8106	1.9700
0.60	1.0000	2.0165	0.40	0.8106	2.0039
0.60	0.8772	1.9700	0.40	0.7435	1.9700

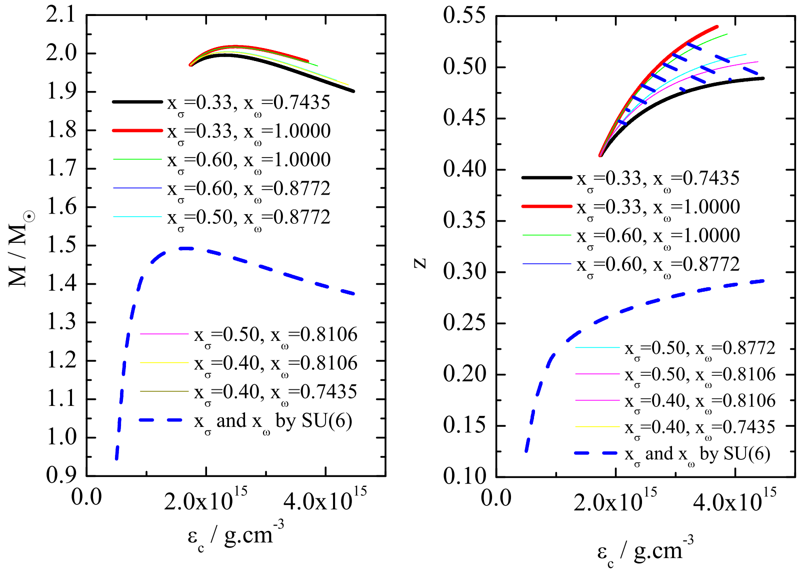

Fig. 4. The mass and the surface gravitational redshift value range, determined by this work, corresponding to the neutron star PSR J1614-2230.

Table III gives the values of ε_c, R, M/R and z corresponding to the mass $1.970\,M_{\odot}$. From Fig. 4 and Table III it can be seen that the surface gravitational redshift of the neutron star PSR J1614-2230 is in the range of $\sim$ 0.4138 to 0.5397. The dashed bottom (navy) line in figure 4 expresses the surface gravitational redshift of a small mass neutron star calculated with the hyperon coupling constant obtained by the constituent quark model [SU(6) symmetry]. We see that the surface gravitational redshift of the small mass neutron star is far smaller that that of the neutron star PSR J1614-2230. For example, as the central energy density being $2.7384 \times 10^{15}\,\mathrm{g.cm}^{-3}$, the surface gravitational redshift of the small neutron star is only 0.2744 while that of the neutron star PSR J1614-2230 is in the range of $\sim$ 0.4668 to 0.5026. Thus the surface gravitational redshift of the neutron star PSR J1614-2230 calculated by us is about one times larger than that of the small neutron star.

Figure 5 shows the value range of the surface gravitational redshift, which is determined by us in this work, of the neutron star PSR J1614-2230 as a function of the ratio of mass M and radius R. The curve segment between the two solid (red) circles represents the value range of the surface gravitational redshift of the neutron star PSR J1614-2230. The five-pointed (navy) star represents the surface gravitational redshift of the small mass neutron star. We see that for the neutron star PSR J1614-2230 the ratio of mass M and radius R is in the range of $\sim$ 0.1692 to 0.1958 and the surface grav-

TABLE III

The values of ε_c, R, M/R and z corresponding to the mass $1.970\,M_\odot$. For the case of SU(6), the values correspond to the maximum mass of the neutron star calculated.

x_σ	x_ω	ε_c $\times 10^{15}$ g.cm^{-3}		R km		M/R $M_\odot$/km		z	
0.33	0.7435	1.7518	3.1741	11.6400	10.7520	0.1692	0.1832	0.4139	0.4761
0.33	1.0000	1.7350	3.6943	11.6420	10.1080	0.1692	0.1958	0.4138	0.5397
0.60	1.0000	1.7350	3.8164	11.6420	10.1410	0.1692	0.1943	0.4138	0.5316
0.60	0.8772	2.2760	2.2760	11.2670	11.2670	0.1748	0.1748	0.4379	0.4379
0.50	0.8772	1.7394	3.4286	11.6410	10.4640	0.1692	0.1883	0.4138	0.5007
0.50	0.8106	2.2457	2.2457	11.3300	11.3300	0.1739	0.1739	0.4337	0.4337
0.40	0.8106	1.7357	3.4040	11.6420	10.5440	0.1692	0.1868	0.4138	0.4937
0.40	0.7435	2.2220	2.2220	11.3820	11.3820	0.1731	0.1731	0.4302	0.4302
by SU(6)	by SU(6)	1.6532	1.6532	12.2210	12.2210	0.1221	0.1221	0.2507	0.2507

itational redshift is in the range of ~ 0.4138 to 0.5397. But for the small mass neutron star the ratio of mass M and radius R is 0.1221 and the surface gravitational redshift is 0.2507. The surface gravitational redshift of the neutron star PSR J1614-2230 is about one times larger than that of the small mass neutron star calculated with the hyperon coupling constant obtained by the constituent quark model [SU(6) symmetry].

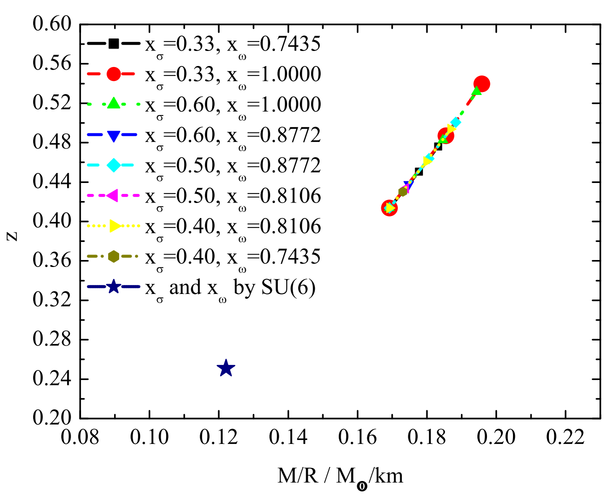

Fig. 5. The surface gravitational redshift value range, determined by this work, corresponding to the neutron star PSR J1614-2230.

6. Summary

In this paper, based on the hadronic level, the value range of the surface gravitational redshift of the neutron star PSR J1614-2230 is tried to be determined. It is found that the surface gravitational redshift of a neutron star increases with the x_ω increase and decreases with the x_σ increase. For the neutron star PSR J1614-2230, the ratio of mass M and radius R is in the range of ~ 0.1692 to 0.1958 and the surface gravitational redshift is in the range of ~ 0.4138 to 0.5397. But for the small mass neutron star, which is calculated with the hyperon coupling constant obtained by the constituent quark model [SU(6) symmetry], the ratio of mass M and radius R is 0.1221 and the surface gravitational redshift is 0.2507. The surface gravitational redshift of the neutron star PSR J1614-2230 is about one times larger than that of the small mass neutron star.

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