FISSION FRAGMENT MASS DISTRIBUTION AS A PROBE OF THE SHAPE-DEPENDENT CONGRUENCE ENERGY TERM IN THE MACROSCOPIC MODELS* **

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The time evolution of nuclei at high excitation energy and angular momentum is studied by means of three-dimensional Langevin calculations performed for two different parameterizations of the macroscopic potential: the Finite Range Liquid Drop Model (FRLDM) and the Lublin–Strasbourg Drop (LSD) prescription. Depending on the mass of the system, the topology of the potential energy surface (PES), in the deformation space, is observed to be different within these two approaches. When incorporated in the dynamical calculation, the FRLDM and LSD models are observed to give similar results in the heavy mass region, whereas the predictions can be strongly dependent on the PES for fission of medium-mass nuclei. The shape-dependent congruence energy is only slightly modulated on the top of the bulk LD part although it changes the PES and the barrier height.

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1. The method

The dynamical evolution of the hot rotating system in the three-dimensional potential-energy landscape is obtained solving the following coupled Langevin equations (repeated subscripts follow Einstein's summation convention)

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$$
\begin{aligned}
\frac{dq_i}{dt} &= \sum_j \mu_{ij}(\vec{q})p_j\,, \\
\frac{dp_i}{dt} &= -\frac{1}{2}\sum_{j,k}\frac{d\mu_{ij}(\vec{q})}{dq_i}p_j p_k - \frac{dF(\vec{q})}{dq_i} - \sum_{j,k}\gamma_{ij}(\vec{q})\mu_{ij}(\vec{q})p_k + \sum_j \theta_{ij}(\vec{q})\mathit{\Gamma}_j(t)\,,
\end{aligned}
$$

where $\vec{q} = (q_1, q_2, q_3)$ is the vector of collective coordinates and $\vec{p}$ the corresponding conjugate momentum. In the present model, the "Funny-Hills" (c, h, α) parametrization of the nuclear shape [1] was adopted as it has shown to be able to describe in a realistic way the large variety of shapes that the system may take along its path to fission. The coordinates (q_1, q_2, q_3) are connected to the elongation, neck thickness and mass asymmetry of the nucleus, respectively [2]. To estimate the mass of the fragment $A_{\rm frag}$ in a very simple way, we propose *ad hoc* formula (1) dependent on the q_3 deformation parameter

$$
A_{\rm frag}(q_3) = \frac{A_{\rm CN}}{2} + (0.47 + 0.55\, A_{\rm CN}) {\rm tg}\, h(1.6\, q_3)\,. \tag{1}
$$

The mathematical conditions for the present shape parametrization allow to establish well the scission point coordinates which define the nuclear shape having two fragments with negligible neck. The mass of the bigger fragment $A_{\rm frag}$ has been obtained by the integration over volume of the nuclear matter for one part of fissioning nucleus. For $q_1 > 1$, there exists the values q_2 which gives the shape with the neck vanishing and such a point is the geometrical scission point. It has been check then the volume of the fragments depends only on the q_3 parameter ($q_3 = 0$ — two mass-symmetric fragments, $q_3 \approx 1$ — nucleon and $(A_{\rm CN} - 1)$ fragment). Assuming the uniformity of the nuclear matter in the nucleus the mass number of the fragment proportional to the fragment volume, has been verified.

The calculations have been done for various nuclei but the results are independent of the mass of the compound nucleus as it was expected. The formula (1) allows to predict the static fission fragment mass distribution as a row estimation based on the properties of the PES.

The driving potential is given by the Helmholtz free energy $F(\vec{q}) = V(\vec{q}) - a(\vec{q})T^2$ with $V(\vec{q})$ being the potential energy. The Fermi-gas model is assumed for the determination of the temperature according to $T = \sqrt{E_{\rm int}/a(\vec{q})}$ where $E_{\rm int}$ and $a(\vec{q})$ are the intrinsic excitation energy and level-density parameter, respectively.

2. Potential-energy landscapes

The macroscopic potential energy $V(\vec{q})$, and hence the driving free energy $F(\vec{q})$, is calculated at every point of the three dimensional $\{q_1, q_2, q_3\}$ mesh. Figure 1 shows the energy landscape in the (q_1, q_3) plane assuming $h = 0$ (which corresponds to the most probable shapes in the path to fission) for a ^{111}In compound nucleus. The potential-energy maps for the FRLDM [3] and LSD [4] models are observed to be not so different for this medium-mass system, while they are found rather similar for heavy nuclei (not shown here) [5]. The equilibrium deformation and the scission point are very close in both models, but the barrier is about 5 MeV higher within the LSD parametrization. The difference in the energy landscape obtained within the FRLDM and LSD models which include the shape-dependent congruence term and without it, is expected to sizable affect the time evolution of the compound nucleus. The mass asymmetric shapes have $q_3 \neq 0$ and the energy barrier for the FRLDM model for necked-in shapes is smaller by about 2 MeV as compared to LSD. The congruence term increases (decreases) the saddle point energy by 2 (4) MeV in case of the FRLDM (LSD) potential energy prescription.

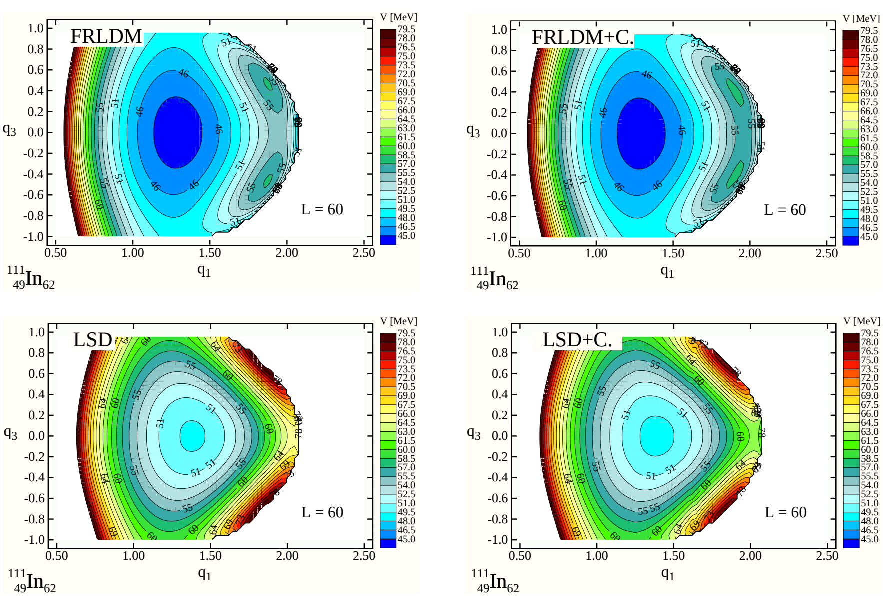

Fig. 1. Two-dimensional $\{q_1, q_3\}$ potential energy surface with $h = 0$ for ^{111}In at spin $L = 60\ \hbar$ calculated within the FRLDM (top) and LSD (bottom) without (left) and with (right) congruence energy term.

The congruence energy term, is parametrized as in [6], *i.e.* $E_{\mathrm{Cong.}} \stackrel{\mathrm{df}}{=} W_0(Z,N)\cdot(2-\frac{R_{\mathrm{neck}}}{R_{\mathrm{frag}}})$, but the shape-dependence is described by two different formulas for liquid drop such as LSD model and for FRLDM

$$W_0(Z,N)^{\mathrm{FRLDM}} = -C_0 + W_2|I|\,, \tag{2}$$
$$W_0(Z,N)^{\mathrm{LSD}} = -C_0\exp(-W_1|I|/C_0)\,, \tag{3}$$

where $I \equiv (N-Z)/A$, $C_0 = 10$ MeV, $W_1 = 42$ MeV and $W_2 = 30$ MeV. The right panels of Fig. 1 are obtained after adding the congruence energy to the plots on the left panels. The main difference is visible at the saddle point.

3. Charge distributions

Dynamical Langevin calculations have been performed in [5] for the decay of excited ^{118}Ba produced in the reaction: ^{78}Kr(429 MeV)+^{40}Ca. The emphasis was put there on the influence of the choice FRLDM or LSD model for the potential-energy landscape and the impact of the viscosity and level density parameters have been tested. On the contrary, in the present draft, the emphasis is on the very modeling of a specific term of the PES, *i.e.* the congruence term. FRLDM and LSD namely use divergent prescriptions. Figure 2 shows the results for the aforementioned two parameterizations of the potential energy and also the influence of the shape-dependent congruence energy included to FRLDM and LSD. The fission-fragment mass-distributions obtained for the FRLDM and the LSD models with and without congruence energy are observed not to be very different.

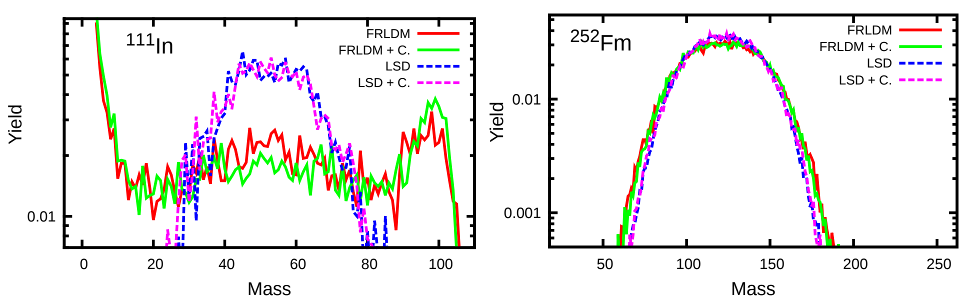

Fig. 2. Fission-fragment mass distribution and influence of the congruence energy included in PES obtained with the FRLDM (solid black/red — no congruence, solid grey/green — with congruence) and the LSD model (dashed black/blue — no congruence, dashed grey/pink — with congruence) for fissioning ^{111}In (left) and ^{252}Fm (right) nuclei.

Various choices of the viscosity and level-density parameters, set to $k_s = 0.6$ and $a = A/12$ (^{252}Fm) and $a = A/10$ (^{111}In) presently, do not affect this observation. The narrow A-distribution predicted by the LSD model, as well as the larger fission probability [5], is due to the stiffer profile of the LSD landscape as compared to the FRLDM one.

4. Conclusions

The results of the present work demonstrate the critical role played by the potential energy landscape used in dynamical calculations in medium mass nuclei. The fission fragment mass distribution for two system: ^{111}In and ^{252}Fm is not so sensitive on the adding the shape-dependent congruence energy as it was expected. Instead, the main contribution is coming from the macroscopic model and the congruence energy changes it slightly only.

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