ELECTRIC TRANSITIONS IN HYPOTHETICAL TETRAHEDRAL/OCTAHEDRAL BANDS*

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A collective approach combining zero- and one-phonon excitations in nuclear quadrupole and octupole modes together with the rotational motion up to spin $J = 5$ is used to verify the possibility of reproducing the experimental electric $B(E\lambda)$ probabilities in ^{156}Gd nucleus in presence of the high-rank tetrahedral/octahedral symmetries in collective quadrupole, octupole and rotational states.

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1. Introduction

Systematic knowledge of electromagnetic reduced transition probabilities, $B(E\lambda)$, is essential for discovering possible high-rank symmetries, such as, *e.g.*, tetrahedral or octahedral ones, in nuclear systems. Symmetries are known to be the key factor determining the structure of the wave functions, thus strongly affecting the reduced transition probabilities. In this paper, we apply a harmonic-like model, discussed in details in Ref. [1], to reproduce the experimental $B(E1)$ and $B(E2)$ probabilities in ^{156}Gd, a nucleus supposed to possess low-lying tetrahedral, negative-parity band and for which some data of interest are available from the recent experiments [2, 3]. We thus analyze the behaviour of the electric transition probabilities within and between the ground-state and the negative-parity bands in order to find out to which irreducible representation of the tetrahedral group T_d (or the octahedral group O) the state of interest could belong.

The proposed collective model contains twelve collective variables (α_{20}, $\alpha_{22}, \{\alpha_{3\nu}\}, \{\Omega\},\ \nu = 0, \pm1, \pm2, \pm3$), describing respectively the axial and

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non-axial quadrupole vibrational modes, all the seven octupole vibrational modes and the three rotational modes described by the Euler angles. Nuclear surface in the intrinsic coordinate system is described in terms of those nine shape variables $(\alpha_{20}, \alpha_{22}, \{\alpha_{3\nu}\})$ using the multipole expansion given, *e.g.* in Ref. [4]. In this work, for simplicity of calculations and focusing on our strategical goal, we apply the idea of the adiabatic separation of vibrational and rotational motions. As a consequence, we introduce a factorized wave function of the form

$$\psi \equiv \psi_{\rm vib}^{\mathit{\Gamma}_1 \mathit{\Gamma}_2}(\alpha_{20}, \alpha_{22}, \alpha_{3\nu}) R_{JM}^{\mathit{\Gamma}_3}(\mathit{\Omega}) = \psi_{{\rm vib},2}^{\mathit{\Gamma}_1}(\alpha_{20}, \alpha_{22})\, \psi_{{\rm vib},3}^{\mathit{\Gamma}_2}(\alpha_{3\nu})\, R_{JM}^{\mathit{\Gamma}_3} \tag{1}$$

composed of products of the vibrational quadrupole $\psi_{{\rm vib},2}^{\mathit{\Gamma}_1}(\alpha_{20}, \alpha_{22})$, octupole $\psi_{{\rm vib},3}^{\mathit{\Gamma}_2}(\alpha_{3\nu})$ and rotational $R_{JM}^{\mathit{\Gamma}_3}(\mathit{\Omega})$ solutions corresponding to uncoupled Hamiltonians. Each of those three ψ-functions belongs to only one irreducible representation $\mathit{\Gamma}_i$ of the octahedral group O or the tetrahedral group T_d. One should realize that both those groups are isomorphic, thus having the same set of the irreducible representation matrices. On the other hand, the proposed states (1) span the collective space of our model in which a collective Hamiltonian should be constructed. It is the symmetry of that Hamiltonian which uniquely determines the symmetry properties of its eigensolutions built as the linear combinations of the basis functions (1). Therefore, at this stage, one is not able to unambiguously judge which of those two symmetries the underlying system really possesses.

As often assumed in simplistic collective approaches, the overall behaviour of a low-lying state can be crudely reproduced by the zero- and one-phonon harmonic oscillator eigensolution. Since a collective Hamiltonian, able to reasonably reproduce the transitional probabilities is not known at this moment, we choose the physical states $\psi_{{\rm vib},2}^{\mathit{\Gamma}_1}(\alpha_{20}, \alpha_{22})$, $\psi_{{\rm vib},3}^{\mathit{\Gamma}_2}(\alpha_{3\nu})$ of Eq. (1) to be the specific and complicated linear combinations of the zero- and one-phonon harmonic oscillator solutions transforming according to a given irreducible representation $\mathit{\Gamma}$ of the group O (or T_d) and, in addition, having good parity. Notice however, that inversion operation does not belong, neither to the octahedral group, O, nor to the tetrahedral one, T_d, and that it commutes with all elements of both groups. As a consequence, the parity can be a good quantum number for the tetrahedrally or octahedrally symmetric states. The details of the procedures leading to such states and the way of constructing the rotational states $R_{JM}^{\mathit{\Gamma}_3}(\mathit{\Omega})$ are presented in Refs. [1, 4, 5].

2. Results

The reduced transition probability between the states (1), governed by the intrinsic transition operator $\hat{Q}_{\lambda\mu}$, can be calculated as

$$B\left(E\lambda, J \to J'\right) = \left|\sum_{\mu} \left\langle \psi'^{\Gamma_1' \Gamma_2'}_{\rm vib} \right| \hat{Q}_{\lambda\mu} \left| \psi^{\Gamma_1 \Gamma_2}_{\rm vib} \right\rangle \left\langle R'^{J';\Gamma_3'} \right\| \left| D^{\lambda\star}_{.\mu} \right| \left| R^{J;\Gamma_3} \right\rangle \right|^2, \quad (2)$$

where J and J' are the angular momenta of the initial and final states, respectively. The symbol $.\mu$ as the sub-script of the Wigner function signifies that the considered reduced matrix element is reduced with respect to the first index. The meaning of the Γ-type symbols has been explained after Eq. (1).

Suppose that the initial $|i\rangle$ and final $|f\rangle$ states of a given collective Hamiltonian belong to the representations $\Gamma^{\rm i}$ and $\Gamma^{\rm f}$, respectively, whereas tensor transition operator $\hat{Q}_{\lambda\nu}$ transforms according to irreducible representation Γ^{Q} of the octahedral group. The reduced probability of Eq. (2) can be non-zero if and only if $\Gamma^{\rm i} \otimes \Gamma^{Q} \supset \Gamma^{\rm f}$, where symbol $\otimes$ denotes the Kronecker product of the irreducible representations.

Among all the five irreducible representations of the group O (or T_d) there exist several pairs of representations corresponding to $\Gamma^{\rm i}$ and $\Gamma^{\rm f}$ which do not fulfil the above condition. The problem of the selection rules for the octahedral group O has been discussed in details in Ref. [1].

Aware of those facts, our task, at this stage, is to find out to which irreducible representation a given experimental level could possibly belong. The question becomes even more challenging if one takes into account the symmetrization problem, usually solved by introducing the so-called *symmetrization group*, (for more details, see *e.g.* Ref. [1, 6]).

It can be shown that for the collective space discussed here the symmetrization group is, in fact, the octahedral group, O. In short, the symmetrization or, in other words, the condition assuring that the constructed intrinsic nuclear states are unique in the laboratory coordinate system, requires that each state (both for the ground-state band and for the negative-parity band in question) transforms with respect to the scalar irreducible representation (denoted by A_1) of the symmetrization group. Let us emphasize that, in general, one should clearly distinguish the *intrinsic symmetry group* acting on the intrinsic-component functions of Eq. (1) from the symmetrization group, incidentally they coincide in the present model. Each of those two types of groups influences different aspects of a collective model (more details available *e.g.* in Ref. [4]). Since the intrinsic state (1) by construction is defined as the product of the three components transforming according to Γ_1, Γ_2 and Γ_3 representations respectively, the symmetrization condition requires that $\Gamma_1 \otimes \Gamma_2 \otimes \Gamma_3 \supset A_1$. Keeping in mind that the total collective state should be finally symmetrized with respect to the group O and to avoid ambiguities related to the octahedral and tetrahedral group isomorphism, the conclusions in the following will refer to the octahedrally symmetric states. As demonstrated in details in Ref. [1], one of the consequences of existing a non-trivial symmetrization group is that a symmetrized intrinsic state of given (even or odd) spin J can be usually obtained through more than one scheme of coupling of Γ_1, Γ_2 and Γ_3 irreducible representations of the group O. For the assumed symmetrized, positive- and negative-parity functions, used in this work to approximately reproduce the ground-state and octupole bands, twenty two different coupling schemes are potentially possible, (see, Ref. [1]).

Contrarily to the early simplistic considerations of the high-rank symmetries in atomic nuclei, based mainly on the tetrahedrally deformed mean-field, a realistic collective approach pretending to properly treat the problem of tetrahedral symmetry should comprise, apart from the simplest tetrahedral mode commonly recognized as the one which has zero quadrupole moment and represented in the lowest order by the $Y_{32}(\theta,\varphi)$ spherical harmonic alone, a possibility of incorporating the equivalent tetrahedral "combined modes" given by the characteristic superpositions of other than the Y_{32} octupole modes, as done in Ref. [7]. It turns out that such a mode, coupled of the quadrupole, octupole and rotational motions, (see, Ref. [1]) can have arbitrarily large quadrupole moment, able to produce a substantial $B(E2)'$s.

However, aiming at a construction of octahedrally symmetrized states, corresponding to the experimental rotational octupole band whose states, modelled by the combinations of the functions (1), are connected by strong $B(E2)'$s and, in addition, decaying to the ground-state band by relatively strong $B(E1)'$s, we dispose with significantly reduced number of possible coupling schemes, compared to the all twenty two possible scenarios. The further limitations to select the most suitable coupling scheme can be provided by characteristic experimental branching ratios of the $E1$ transitions. Precisely, the branching ratios of the probabilities of the dipole transitions starting from the odd-spin states to the corresponding probabilities starting from the even-spin states varies, considering the uncertainty of the measurement, from about one to thirty. The latter comes from the analysis of the negative-parity spectrum in ^{156}Gd, measured in the ILL laboratory in a series of experiments using the Bragg spectrometry methods, (see, Ref. [2]).

Having rejected the coupling schemes which do not fulfil the above conditions, *the octahedrally symmetrized*, quadrupole-deformed ground-state band can be built out of only the two sets of irreducible representations $\{\Gamma_1,\ \Gamma_2,\ \Gamma_3\}$, namely: $\{\Gamma_1 = A_1, \Gamma_2 = A_1, \Gamma_3 = A_1\}$ for $J = 0, 4$ and $\{\Gamma_1 = E, \Gamma_2 = A_1, \Gamma_3 = E\}$ for $J = 2, 4$. As seen, the $J = 0^+$ state can be uniquely obtained with $\Gamma_1 = \Gamma_2 = \Gamma_3 = A_1$. We are using the standard labelling of irreducible representations [8], where A_1, A_2 are the one-dimensional, E denotes two-dimensional and T_1, T_2 three-dimensional irreducible representations. In order to reproduce the experimental $B(E2)'$s in ^{156}Gd within the ground-state band with the best accessible in this model accuracy, both the states with $J = 2^+$ and $J = 4^+$ should be generated according to the second scheme.

In turn, the octupole negative-parity band of interest can be composed of the following sets of irreducible representations: $\{\Gamma_1 = A_1, \Gamma_2 = \Gamma_3 = A_2\}$ for $J = 3$, $\{\Gamma_1 = A_1, \Gamma_2 = T_1, \Gamma_3 = T_1\}$ for $J = 1, 3, 4, 5$ and $\{\Gamma_1 = A_1, \Gamma_2 = T_2, \Gamma_3 = T_2\}$ for $J = 2, 3, 4, 5$. However, the most reasonable reproduction of experimental $B(E1)$ as well as $B(E2, 5^- \to 3^-)$ probabilities can be obtained if the octupole $J = 3^-$ and $J = 5^-$ states are constructed according to the second of the above written schemes.

After a number of trials we have found out that for the ground-state band, explicit shapes of the functions $\psi^{\Gamma_1}_{\mathrm{vib},2}(\alpha_{20}, \alpha_{22})$ of Eq. (1) are, in general, complicated linear combinations of two-dimensional zero- and one-phonon oscillator solutions proportional to

$$u_0\left(\eta_2, \alpha_{20} - \overset{\circ}{\alpha}_{20}\right) u_0\left(\sqrt{2}\eta_2, \alpha_{22} - \overset{\circ}{\alpha}_{22}\right) ,$$

$$u_0\left(\eta_2, \alpha_{20} - \overset{\circ}{\alpha}_{20}\right) u_1\left(\sqrt{2}\eta_2, \alpha_{22} - \overset{\circ}{\alpha}_{22}\right) ,$$

$$u_1\left(\eta_2, \alpha_{20} - \overset{\circ}{\alpha}_{20}\right) u_0\left(\sqrt{2}\eta_2, \alpha_{22} - \overset{\circ}{\alpha}_{22}\right) .$$

The linear combination coefficients are determined through the projecting onto the required representation, described in Ref. [4]. Above, u_0 and u_1 are the one-dimensional, zero- and one-phonon oscillator eigenfunctions, respectively, where $\eta_\lambda = \sqrt{\frac{B_\lambda \omega_\lambda}{\hbar}}, \lambda = \{2, 3\}$ with the mass (inertia) parameter B_λ and ω_λ corresponding to the angular oscillator frequency. It can be shown that the resulting (projected) quadrupole functions $\Psi_{\mathrm{vib};2}$ of positive parity belong to the three irreducible representations A_1, A_2, E of the group O.

In the case of the zero-phonon functions, the static deformation parameters $\overset{\circ}{\alpha}_{20}$ and $\overset{\circ}{\alpha}_{22}$ describe, on average, the position of the wave function peaks. These peaks, clearly, should fit the quadrupole potential energy well at the $\{\alpha_{20}, \alpha_{22}\}$ surface.

The octupole $\psi^{\Gamma_2}_{\mathrm{vib},3}(\alpha_{3\nu})$ component of the ground-state members is assumed to be the zero-phonon seven-dimensional oscillator solution transforming with respect to the $\Gamma_2 = A_1$ representation of the group O:

$$\begin{aligned}\psi^{A_1}_{\mathrm{vib},3}(\alpha_3) \;=\; & u_{n_1}\left(\eta_3, \alpha^{\mathrm{r}}_{30}\right) u_{n_2}\left(\sqrt{2}\eta_3, \alpha^{\mathrm{r}}_{31}\right) u_{n_3}\left(\sqrt{2}\eta_3, \alpha^{\mathrm{r}}_{32}\right) u_{n_4}\left(\sqrt{2}\eta_3, \alpha^{\mathrm{r}}_{33}\right) \\ & \times u_{n_5}\left(\sqrt{2}\eta_3, \alpha^{\mathrm{i}}_{31}\right) u_{n_6}\left(\sqrt{2}\eta_3, \alpha^{\mathrm{i}}_{32}\right) u_{n_7}\left(\sqrt{2}\eta_3, \alpha^{\mathrm{i}}_{33}\right) , \end{aligned} \tag{3}$$

where the super-scripts 'r' and 'i' denote the real and imaginary parts of $\alpha_{3\nu}, (\nu = 0, \pm 1, \pm 2, \pm 3)$ deformation parameter, respectively with the condition $\alpha_{\lambda\mu} = (-1)^{\mu}\alpha^{\star}_{\lambda-\mu}$. In this case, the fact that $n_k = 0$ for $k = \{3, 4, \ldots, 7\}$ implies that all the seven one-dimensional oscillator states are of zero-phonon type. Observe that the factor of $\sqrt{2}$ by η_2 and η_3 in Eq. (3) as well as in the previously defined two-dimensional quadrupole oscillator functions appears due to the rewriting the oscillator eigenfunctions, depending initially on the deformation parameters $\{\alpha_{20}, \alpha_{2-2}, \alpha_{22}, \alpha_{3\nu}, \nu = 0, \pm 1, \pm 2, \pm 3\}$, in terms of pure real parameters $\{\alpha_{20}, \alpha_{22}, \alpha^{\mathrm{r}}_{3\mu}, \alpha^{\mathrm{i}}_{3\mu}, \mu = 0, 1, 2, 3\}$. Applying the similar projecting procedure as in the case of the quadrupole function $\psi^{\Gamma_1}_{\mathrm{vib},2}(\alpha_{20}, \alpha_{22})$, the octupole (after projection) one-phonon component $\psi^{\Gamma_2}_{\mathrm{vib},3}(\alpha_{3\nu})$, transforming with respect to desired irreducible representation $\Gamma_2 = T_1$ is the triplet of linear combinations of the zero- and one-phonon, seven-dimensional oscillator solutions of the form (3) with the following substitutions: $\alpha^{\mathrm{i}}_{32} \to \alpha^{\mathrm{i}}_{32} + \overset{\circ}{\alpha}{}^{\mathrm{i}}_{32}$, $n_k = \{0, 1\}, k = \{3, 4, \ldots, 7\}$ and $\sum_{k=3}^{7} n_k = 1$. In the above, the quantity $\overset{\circ}{\alpha}{}^{\mathrm{i}}_{32}$, similarly as in the quadrupole case, is the static deformation parameter describing the position of the octupole peaks of the wave function (1). Finally, as the quadrupole part of the octupole, negative- or positive-parity states, we put the $\psi^{\Gamma_1}_{\mathrm{vib},2}(\alpha_{20}, \alpha_{22}) \equiv u_0(\eta_2, \alpha_{20})u_0(\sqrt{2}\eta_2, \alpha_{22})$ zero-phonon function transforming according to the scalar representation A_1. As shown in Ref. [3], the values of the Q_0 moments of the ground state and the octupole bands are comparable. Therefore, in the following the quadrupole axial deformations of both these bands, $\overset{\circ}{\alpha}{}^{(\mathrm{gs})}_{20}$ and $\overset{\circ}{\alpha}{}^{(\mathrm{oct})}_{20}$ are assumed to be identical whereas their non-axial deformations $\overset{\circ}{\alpha}_{22}$ as equal to zero. With the above assumptions leading to the reduction of the number of adjustable parameters, we have determined through the least-square method the parameters η_2, η_3, $\overset{\circ}{\alpha}{}^{(\mathrm{gs})}_{20}$, $\overset{\circ}{\alpha}{}^{(\mathrm{oct})}_{20}$ and $\overset{\circ}{\alpha}{}^{\mathrm{i}}_{32}$ of the constructed 0^+, 2^+, 4^+, 3^-, 5^- states. This has been achieved by minimizing the difference between the theoretical and experimental of Ref. [3] values of the six selected $B(E1)$ and $B(E2)$ probabilities in ^{156}Gd nucleus. The experimental energies of those states, taken from Ref. [3] are: 0.0 keV, 88.970(1) keV, 288.187(1) keV, 1276.138(2) keV, 1408.133(5) keV, respectively. Such an adjustment has been done in order to verify whereas a reasonable reproduction of the experimental reduced probabilities in this nucleus is, in general, possible in presence of the octahedral symmetry of the above states. In the table below, we write down the set of electric transitions $E\lambda$ of Ref. [3] between the states of spins J_{i} and J_{f} (1st column) and the corresponding, estimated within this work reduced probabilities $B_{\mathrm{th}}(E\lambda)$ (2nd column). The 3rd column represents its experimental values $B_{\mathrm{exp}}(E\lambda)$ for which the adjustment in question has been carried out:

Transition $E\lambda$, $J_{\rm i} \to J_{\rm f}$	$B_{\rm th}(E\lambda)$ [W.u.]	$B_{\rm exp}(E\lambda)$ [W.u.]
$E2, 2^+ \to 0^+$	159	187(5)
$E2, 4^+ \to 2^+$	279	263(5)
$E2, 5^- \to 3^-$	293	293^{+61}_{-134}
$E1, 3^- \to 2^+$	1.10×10^{-3}	$0.98(21) \times 10^{-3}$
$E1, 3^- \to 4^+$	0.63×10^{-3}	$0.77(16) \times 10^{-3}$
$E1, 5^- \to 4^+$	0.75×10^{-3}	$0.85^{+19}_{-38} \times 10^{-3}$

Using the obtained in this way parameters, ($\eta_2 = 12.67$, $\eta_3 = 11.60$, $\overset{\circ}{\alpha}_{22} = 0.0$, $\overset{\circ}{\alpha}{}^{(\rm gs)}_{20} = \overset{\circ}{\alpha}{}^{(\rm oct)}_{20} = 0.34$) we have estimated the $B(E1, 2^- \to 2^+)$ and $B(E1, 4^- \to 4^+)$ probabilities in ^{156}Gd nucleus, measured in the GAMAS experiments, whose results are collected in Ref. [2]. Both the 2^- and 4^- octupole, even-spin states are constructed according to the $\{\mathit{\Gamma}_1 = E, \mathit{\Gamma}_2 = \mathit{\Gamma}_3 = T_2\}$ coupling scheme. The estimated $B(E1)$ probability values are written in the 2$^{\rm nd}$ column of the below table whereas the measured values are placed in the 3$^{\rm rd}$ column.

Transition $E\lambda$, $J_{\rm i} \to J_{\rm f}$	$B_{\rm th}(E1)$ [W.u.]	$B_{\rm exp}(E1)$ [W.u.]
$E1, 2^- \to 2^+$	5.2×10^{-4}	$1.1^{+6}_{-0.7} \times 10^{-4}$
$E1, 4^- \to 4^+$	1.5×10^{-4}	$2.0^{+7}_{-0.8} \times 10^{-4}$

As seen, the above theoretical estimates of the dipole transition probabilities within the octahedrally symmetrized harmonic-like collective solutions differ by a factor of one to five, relative to the corresponding experimentally measured with the uncertainty of about 50% probabilities. Hence, the advocated here hypothesis predicting the existence of the octahedrally symmetrized octupole bands in ^{156}Gd, whose states are described by above studied vibrational–rotational functions is not in contradiction with the empirical facts. The latter indicates a need for further, more involved investigations for the presence of the high-rank symmetries in nuclear systems.

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