EXTERNAL NOISE EFFECTS IN SILICON MOS INVERSION LAYER*

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The effect of the addition of an external source of correlated noise on the electron transport in silicon MOS inversion layer, driven by a static electric field, has been investigated. The electron dynamics is simulated by a Monte Carlo procedure which takes into account non-polar optical and acoustic phonons. In our modelling of the quasi-two-dimensional electron gas, the potential profile, perpendicular to the MOS structure, is assumed to follow the triangular potential approximation. We calculate the changes in both the autocorrelation function and the spectral density of the velocity fluctuations, at different values of noise amplitude and correlation time. The findings indicate that, the presence of a fluctuating component added to the static electric field can affect the total noise power, *i.e.* the variance of the electron velocity fluctuations. Moreover, this effect critically depends on both the amplitude of the driving electric field and the noise parameters.

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1. Introduction

The presence of noise in semiconductor materials is generally considered a disturbance, since strong fluctuations affect performance and reliability of semiconductor-based devices. For this reason, in order to fully understand the complex scenario of nonlinear phenomena involved in the devices response, several studies have investigated the hot-electron transport dinamics in bulk and semiconductor structures by analyzing the electronic noise in systems driven by external static or oscillating electric fields [1–12].

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In the last decade an increasing interest has been devoted towards possible constructive aspects of noise and fluctuations in physical and biological systems [13–27]. Previous studies have shown that, under specific conditions, an external noise can constructivelly interact with an intrinsically nonlinear system, characterized by the presence of intrinsic noise, giving rise to positive effects [13–15] such as stochastic resonance (SR) [16–19], resonant activation (RA) [20, 21] and noise enhanced stability (NES) [22–26]. In particular, the possibility to suppress the intrinsic noise in n-type GaAs and Si bulk, driven by a static electric field, with the addition of a Gaussian correlated noise source, has been theoretically investigated [27]. By using a Monte Carlo approach and including energetic considerations in the theoretical analysis, Varani *et al.* have concluded that the most favourable condition to obtain noise suppression should be found in GaAs for a static electric field around 6.5 kV/cm, corresponding to the maximum variance. Detailed studies of diffusion noise in low-doped GaAs bulk, subjected to a driving periodic electric field containing time-correlated fluctuations, have revealed the possibility of a suppression of electronic noise [24, 25]. Under specific conditions, the intrinsic noise in n-type GaAs crystals can be also reduced by adding a two-level random telegraph noise source to the driving high-frequency oscillating electric field [26]. The suppression arises from the fact that the transport dynamics of electrons in the semiconductor material receives a benefit by the constructive interplay between the fluctuating electric field and the intrinsic noise of the system. This effect can be viewed as a NES phenomenon [24–26]. Theoretical works which investigate the possibility to improve the ultra-fast magnetization dynamics of magnetic spin systems by including correlated noise have been published in Refs. [28, 29]. Moreover, very recently it has been shown that also the electron spin relaxation length in semiconductor materials is significantly affected by the addition of an external correlated noise source in the applied voltage [30]. To the best of our knowledge, an investigation of the role of external fluctuations on electronic intrinsic noise in silicon MOS inversion layer is still missing.

The aim of this work is the investigation of the effects of the addition of an external correlated source of noise on the carrier velocity fluctuations in quasi-two-dimensional silicon semiconductor structures, operating under static conditions. The electron dynamics at a kinetic level is simulated by a Monte Carlo procedure which takes into account scattering processes with acoustic and non-polar optical phonons. In particular, in our modelling of the quasi-two dimensional electron gas: *(i)* the potential profile, perpendicular to the MOS structure is assumed in the triangular potential approximation, *(ii)* the lowest three energy subbands are taken into account and *(iii)* non-degenerate conditions are simulated. The carrier intrinsic noise is obtained by computing the velocity fluctuation correlation function and its spectral density. The modifications caused by the addition of an external source of correlated noise are investigated by analysing the noise integrated spectral density (ISD), which coincides with the variance of the electron velocity fluctuations, as a function of the characteristic parameters of the added fluctuations. The results are discussed and compared with those obtained in the absence of the external noise source. Our numerical results show that the effects due to the addition of external correlated fluctuations critically depend on the amplitude of the driving field and the noise parameters.

2. MOS inversion layer model, simulation details and noise calculation method

Quantum-confined semiconductor-based devices, such as silicon MOS, are the cornerstone of current electronics. In these structures, spatial confinement leads to the formation of discrete low-dimensional subbands: energy levels are quantized in each direction of confinement, while the momentum remains a good (continuous) quantum number in the unconfined directions [31]. If carriers are confined along one direction and free to move in the two-dimensional plane perpendicular to it, the structure is known as a quasi-two-dimensional electron gas (Q2DEG). At room temperature, transport within each subband and transitions among subbands can be semiclassically described by using the Boltzmann transport equation [31–33]. In the effective-mass approximation, the Hamiltonian for an electron has the following form

$$H_0\left(\vec{R}\right) = -\frac{\hbar^2}{2}\sum_{j=x,y,z}\frac{1}{m_j^\star}\frac{\partial^2}{\partial j^2} + U(z) = H_{0\perp}(\vec{r}) + H_{0\parallel}(z)\,, \tag{1}$$

where $\vec{R} = (\vec{r}, z)$, $U(z)$ is the effective potential energy profile of the confining potential along the z-direction, $H_{0\parallel}$ is the parallel part of H_0 (associated with the motion in the xy-plane, perpendicular to the confinement direction), and the transverse part is defined as

$$H_{0\perp}(z) = -\frac{\hbar^2}{2m_z^\star}\frac{\partial^2}{\partial z^2} + U(z)\,. \tag{2}$$

The basis-states of the unperturbed Hamiltonian are assumed to be of the form

$$\psi_n\left(\vec{R}\right) = \frac{1}{\sqrt{S}}\exp^{i\vec{k}\cdot\vec{r}}\psi_n\,, \tag{3}$$

where $\vec{k}$ is a wavevector in the xy-plane and S is the area of the sample interface. Here, the silicon inversion layer is modelled as a triangular potential well, *i.e.* $U(z) = ezE$, $z \geq 0$ [33, 34]. Then, the subband wavefunctions $\psi_n(z)$ satisfy the one-dimensional Schrödinger equation

$$-\frac{\hbar^2}{2m_z^\star}\frac{\partial^2\psi_n(z)}{\partial z^2} + ezE_w\psi_n(z) = \epsilon_n\psi_n(z)\,, \tag{4}$$

where ϵ_n is the subband energy.

We have determined both energy levels and wave functions by using the variational method based on the minimization of the energy eigenvalue of the following approximate wavefunctions [33]

$$\psi_1(z) = k_1 z \exp\left(-\frac{c_1 z}{2}\right)\,, \tag{5}$$

$$\psi_2(z) = k_2\left(z + a_2 z^2\right)\exp\left(-\frac{c_2 z}{2}\right)\,, \tag{6}$$

$$\psi_3(z) = k_3\left(z + a_3 z^2 + b_3 z^3\right)\exp\left(-\frac{c_3 z}{2}\right)\,, \tag{7}$$

where a_n, b_n, c_n and k_n (with $n = 1,\ 2,\ 3$) are the variational constants to be determined, among which k_n is the normalization constant. More details about the resolution of the one-dimensional Schrödinger equation (4) in a triangular potential well can be found, for example, in [33]. The employed Monte Carlo procedure takes into account the scattering mechanism with both intra- and inter-subbands non-polar optical and acoustic phonons. We assume field-independent scattering probabilities. Accordingly, the influence of the external fields is only indirect through the field-modified electron velocities [35]. In our simulations, the strength of the electric field E_w along the z-axis of the well is kept constant and equal to 100 kV/cm and the lattice temperature is $T = 300$ K.

The Q2DEG is driven by a fluctuating electric field directed along the x-axis

$$E(t) = E + \eta(t)\,, \tag{8}$$

where E is the amplitude of the deterministic part and $\eta(t)$ is the random term.

The random component $\eta(t)$ is modelled by an Ornstein–Uhlenbeck (OU) stochastic process, which obeys the following stochastic differential equation

$$\frac{d\eta(t)}{dt} = -\frac{\eta(t)}{\tau_D} + \sqrt{\frac{2D}{\tau_D}}\xi(t)\,, \tag{9}$$

where τ_D and D are, respectively, the correlation time and the variance of the OU process, $\xi(t)$ is a Gaussian white noise with zero mean and autocorrelation function $\langle\xi(t)\xi(t^{'}\rangle = \delta(t - t^{'})$ [36]. The OU correlation function is $\langle\eta(t)\eta(t^{'})\rangle = D\exp(-|t - t^{'}|/\tau_D)$.

The electron motion along the direction of an applied electric field is characterized by an average velocity dependent on the amplitude of the electric field E. The fluctuations of the drift velocity with respect to the statistical average $\langle v(t) \rangle$ correspond to the intrinsic noise of the system. The noise-induced modifications of the intrinsic noise features are investigated by a statistical analysis of the correlation function of the velocity fluctuations and its mean spectral density. The autocorrelations function $C(\tau)$ of the velocity fluctuation is calculated as usually

$$C(\tau) = \langle \delta v(t) \delta v(t+\tau) \rangle = \langle v(t) v(t+\tau) \rangle - \langle v(t) \rangle^2 \qquad (10)$$

in which τ is the correlation time and the average is done over a sequence of an enough long time interval $[0;T]$. According to the Wiener–Khintchine theorem, the spectral density is calculated as the Fourier transform of $C(\tau)$.

3. Results and discussion

As analytically shown in Ref. [27], the electron average velocity is modified by the presence of an external source of noise. In particular, it has been shown that the intrinsic noise of the system can be reduced if the term $d^2S_0(E)/dE^2$ is negative, being $S_0(E)$ the spectral density of the velocity fluctuations, and the characteristic time of the fluctuations of the electric field τ_D small enough. With the goal of finding the most favourable conditions for noise suppression in silicon Q2DEG, we have preliminarily calculated S_0. In Fig. 1, we can see that S_0 exhibits a maximum at zero field, thus implying a negative $d^2S_0(E)/dE^2$ in the low field region, which is

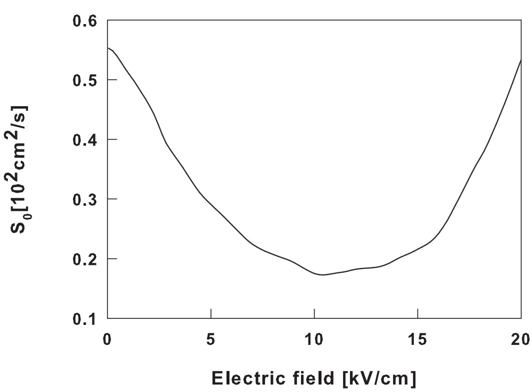

Fig. 1. Spectral density of the velocity fluctuations at zero frequency (S_0) as a function of the electric field calculated by the Monte Carlo simulation at temperature $T = 300$ K in Q2DEG Si MOS.

a necessary condition to obtain the noise suppression. Moreover, the spectral density S_0 exhibits a minimum at $E = 10$ kV/cm. With the purpose of confirming the validity of the analytical theory also in silicon Q2DEG and investigating the role of τ_D in the suppression of the noise, initially we have investigated the transport electron dynamics in the presence of a static electric field having amplitude $E = 10$ kV/cm.

In order to elucidate the effects of the correlated noise source on the intrinsic noise properties, we have performed 100 different realizations and evaluated both average values and error bars for the calculated integrated spectral densities.

Figure 2 shows the spectral densities of the velocity fluctuations obtained by adding a source of correlated noise, characterized by $\tau_D = 2$ ps and three different values of noise amplitude $D^{1/2}$ (namely $D^{1/2} = 1$, 2 and 4 kV/cm). Figure 3 shows the integral spectral density (ISD) of the velocity fluctuations obtained with $E = 10$ kV/cm and a source of correlated noise having $D^{1/2} = 2$ kV/cm for different values of the noise characteristic time τ_D. In Fig. 3 (and in the following Fig. 4) the error bars represent the maximum range of ISD variation.

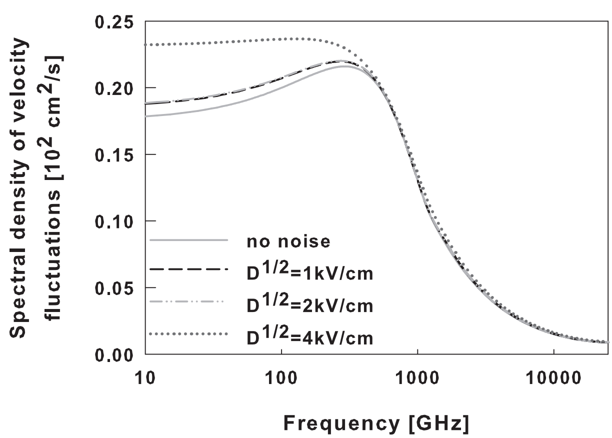

Fig. 2. Spectral densities of the velocity fluctuations obtained for a static electric field $E = 10$ kV/cm and the addition of a correlated source of noise, characterized by $\tau_D = 2$ ps for three different values of noise intensity D.

In the investigated range of the noise correlation times and amplitudes, our findings show that with the addition of an external source of correlated fluctuations to the driving field, the spectrum of the velocity fluctuations changes in an appreciable way only in the low-frequency region ($f < 500$ GHz). In accordance with the analytical theory, no possibility to suppress the intrinsic noise has been found, although the ISD reduces with the de-

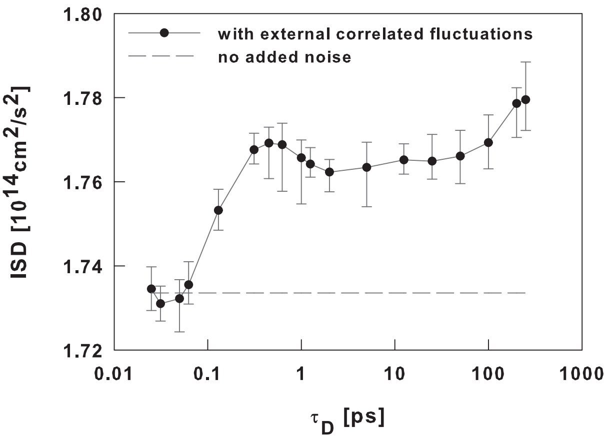

Fig. 3. ISD of the velocity fluctuations obtained with a static electric field $E = 10$ kV/cm and the addition of a source of correlated noise having amplitude $D^{1/2} = 2$ kV/cm for different values of the noise characteristic time τ_D. Dashed line represents the ISD obtained in the absence of external noise.

creasing of the noise correlation time τ_D. Furthermore, a non-monotonic interesting behaviour of the ISD with the noise correlation time has been found. Further studies are needed to learn more about the interplay between the scattering processes time scales and the noise correlation time in the possible suppression of the intrinsic noise.

Subsequently, we have investigated the transport electron dynamics in the silicon MOS inversion layer, driven by a static electric field having amplitude $E = 4$ kV/cm. This is because, for field strengths lower than 5 kV/cm, $d^2S_0(E)/dE^2$ is negative (see Fig. 1). In Fig. 4 (a) we report the ISD obtained with the addition of a source of correlated noise, having characteristic time $\tau_D = 10$ ps, as a function of the ratio between the noise amplitude $D^{1/2}$ and the static electric field E.

The circumstance that the ISD, *i.e.* the variance of the velocity fluctuations, remains nearly constant in the whole investigated range of the noise amplitude, implies that if the noise is suppressed in a certain frequency range it must be enhanced in another range in order to keep a constant value for the total power of the fluctuating signal. In this case, we obtain a noise redistribution, rather than a noise suppression.

In Fig. 4 (b) we show that the ISD of the velocity fluctuations, obtained for $E = 4$ kV/cm and a correlated noise with amplitude $D^{1/2} = 0.8$ kV/cm (20% of the amplitude of the field), remains almost constant in spite of the fact that the noise correlation time varies of about five order of magnitude.

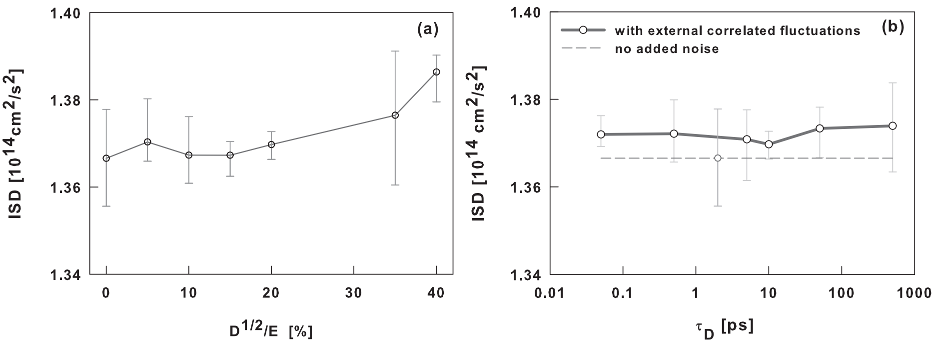

Fig. 4. ISD of the velocity fluctuations as a function of: (a) the ratio between the noise amplitude $D^{1/2}$ and the deterministic component of the electric field E, obtained for $E = 4$ kV/cm and $\tau_D = 10$ ps; (b) the noise correlation time τ_D, for $E = 4$ kV/cm and $D^{1/2} = 0.8$ kV/cm.

4. Conclusions

Previous detailed studies of the electron transport dynamics in a GaAs bulk, working under static or cyclostationary conditions, have revealed that, under specific conditions, the addition of a fluctuating component to the driving field can reduce the total noise power. On the other hand, to the best of our knowledge, no conditions have been found to suppress the electronic intrinsic noise in silicon bulk. On the wake of these findings, we have investigated the effect on the intrinsic noise caused by the addition of external correlated fluctuations in a silicon MOS inversion layer, driven by a static electric field, for different values of both noise amplitude and correlation time.

In the presence of an external source of noise, since the effective electric field experienced by the carrier becomes different, the electron average velocity is modified. Our numerical results show that, if the amplitude of the driving field is $E = 10$ kV/cm, the spectrum of velocity fluctuations changes in an appreciable way only in the low-frequency region ($f < 500$ GHz). Moreover, in the investigated range of the amplitudes and correlation times of the external noise, the transport dynamics of the Q2DEG does not receive a benefit, in terms of a total noise power reduction. This happens also by varying the noise characteristic time. If, instead, the amplitude of the applied field is reduced to $E = 4$ kV/cm, also in the presence of correlated noise, the variance of velocity fluctuations remains nearly constant. This result implies that if the noise is suppressed in a certain frequency range it must be enhanced in another range in order to keep a constant value of the total power of the fluctuating signal. Moreover, the performance of semiconductor-based devices, working on a noisy environment, is not significantly affected. The interplay between the semiconductor characteristic time scales and the noise correlation time is a very interesting topic deserving further investigations.

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