

INFRARED SELF-CONSISTENT SOLUTIONS
OF SPINOR QED₃

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Three-dimensional electrodynamics in the spinor (*i.e.* two-component) version is considered. With the use of the so-called Salam's vertex, the infinite hierarchy of Dyson–Schwinger equations is turned into a set of four self-consistent equations for four parameters describing the infrared behavior of fermion and boson propagators. It is shown numerically, that this set of equations has solutions, at least for certain values of gauge parameter. For weak coupling (*i.e.* for heavy fermions), the values of all these quantities are found analytically. In the case of massless bare fermion, masses of both particles are generated leading thereby to parity breaking.

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1. Introduction

Studying gauge field theories in low-dimensional space-time allowed for investigations both perturbative and nonperturbative aspects of quantum field theory. Among Abelian models of this kind, the particular mention deserve quantum electrodynamics in two dimensions — the so-called Schwinger Model (SM) [1] — and in three dimensions. The former, due to its full solvability, at least in the massless case, has become particularly fruitful and many nonperturbative effects have been discovered and analyzed. One can mention here the generation of the gauge boson mass, via the so-called *Schwinger mechanism*, fermion confinement, chiral symmetry breaking, the presence of instantons and the θ -vacuum (see, for instance, [2–6]), bound state formation [7] but also the status of the perturbation series [8–10]. Due to these nontrivial features, the Schwinger Model has become a testing laboratory for various aspects of the theory of strong interactions. The massive version of this model is no longer solvable but is still attractive thanks to its relative simplicity and remarkable physical content [11, 12].

Quantum electrodynamics in three dimensions — the so-called Planar QED — plays an important role in understanding some aspects of QFT too. One can recall here confinement [13–17], chiral symmetry breaking [15, 17–20], bound states [21–23] and the analysis of the gauge dependence of the nonperturbative results with various approximations [24, 25]. Contrary to the SM, the photon becomes in QED₃ a dynamic particle and corresponds to real degree of freedom, although it has some peculiar features connected with the fact that the magnetic field is now a (pseudo) scalar and electric one a two-component vector restricted by the Gauss law. The number of dimensions attributes to this model also other specific features. The model may be formulated in two inequivalent versions: the fundamental fermion field may be chosen as four- or two-component one. In the latter — which we concentrate on in the present work — the most interesting is the appearance of the topological photon mass term [26]. This term may be, just in three dimensions, added to the standard Lagrangian from the very beginning, since it exhibits both Lorentz and gauge (up to the surface terms) invariance — such a theory is called the Chern–Simons QED₃ — but even if absent, it is generated by the interactions with fermions.

Both low-dimensional models find their applications also beyond QFT: for instance, in condense matter physics [27–29].

Many of the above mentioned phenomena, as confinement, bound states or topological aspects, require for their study nonperturbative language. The problems with perturbation calculations in QED₃ manifest themselves also through ambiguity in proper regularization [26]. The set of Dyson–Schwinger (DS) equations — which, as one believes, contains the whole information about the quantum system — constitutes the main tool in nonperturbative investigations of QFT. The common problem in this approach is, that these equations form an infinite hierarchy, the solution of which seems a hopeless task. To deal with this set, one is forced to truncate this hierarchy at some level. In two dimensions (in SM), thanks to two gauge symmetries of the theory, it turned out to be possible to express the three-point (vertex) function by fermion propagator (*i.e.* two-point function) and obtain the close equation for the latter. Not a whole hierarchy, but just one equation, which turned out to be easily solvable [30, 31]. Similar method, but now applied to higher functions (five-point), allowed to write a self-consistent equation for two-fermion Green’s function [32].

In greater number of dimensions, *i.e.* in QED₃ and QED₄, gauge symmetry is, however, not sufficient to fully express fermion–photon vertex function by lower ones and, therefore, we are obliged to assume a certain ansatz for $\Gamma^\mu(p+k, p)$. Gauge covariance constitutes here a hint but does not solve the whole problem of truncating the hierarchy. Various such ansätze have been proposed based on kinematic properties, renormalizability and requirement

of satisfying the Ward–Takahashi identity. One can mention in this context the Ball–Chiu [33], Burden–Roberts [34] and Curtis–Pennington [35] vertices with further improvements [36, 37]. In the present work, however, we pay particular attention to the vertex introduced by Salam [38] and applied afterwards in the so-called gauge technique [39–42]. The advantage of using Salam’s ansatz in the form (16), apart from its elegance, is that Feynman integrations in DS equations, are similar to those of perturbation calculations for which the computational technique is well elaborated.

The main idea of the present work is similar to that of our previous paper [43] on QED₄, where it was successfully applied and the correct infrared form of propagators was established without the need of infinite renormalization. Here again we assume the possible infrared behavior of propagators up to a set of certain unknown constants and try to determine these constants from the requirement of self-consistency. One has to emphasize that in this approach loop integrations are performed with infrared forms of Green’s functions as integrands. Since we do not wish to extend the results beyond the infrared domain, this procedure is fully acceptable. Firstly, we have to note that right-hand sides of DS equations (18) and (27) have the form of convolutions, so the infrared behavior of the integrals should be dictated by the infrared behavior of integrands, secondly, our method should not be worse than ordinary perturbation calculations, where *free* functions are used as integrands. Finally, one has to stress that Salam’s ansatz (16) in the infrared domain becomes exact.

This work is organized as follows. In Sec. 2, we collect the main definitions and properties of the model. In Sec. 3, we formulate the assumptions as to the infrared behavior of the photon propagator, fermion propagator and fermion–photon vertex. The following two sections are devoted to obtaining, from the DS equations, the set of four nonlinear equations for four parameters: β , τ , δm and Z_3 . In the last section, we present the numerical results and the dependence of all the parameters on the strength of the interaction. We find also the analytical results for the case of weak coupling.

2. Basic definitions and identities

Below, we briefly summarize conventions used in this work and specify basic properties of Dirac gamma matrices in three dimensions. As to the set of matrices γ^μ , we use the two-dimensional, *i.e.* spinor representation in which all gammas may be expressed by Pauli matrices in the following way

$$\begin{aligned}\gamma^0 &= \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, & \gamma^1 &= i\sigma_1 = \begin{pmatrix} 0 & i \\ i & 0 \end{pmatrix}, \\ \gamma^2 &= i\sigma_2 = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix},\end{aligned}\tag{1}$$

and matrix γ^5 does not exist. Due to this nonexistence, the phenomenon of chiral symmetry breaking can be investigated only in four-dimensional representation, unless we introduce an even number of flavors in the model.

The metric tensor $g^{\mu\nu}$ and the totally antisymmetric tensor $\varepsilon^{\mu\nu\alpha}$ are defined as follows

$$\begin{aligned} g^{00} &= -g^{11} = -g^{22} = 1, \\ \varepsilon^{012} &= -\varepsilon^{102} = 1, \end{aligned} \quad (2)$$

with all other nonzero elements of the latter obtained by cyclic permutations.

In three dimensions, the peculiar feature of the choice (1) is that, apart from ordinary relations

$$\begin{aligned} \{\gamma^\mu, \gamma^\nu\} &= g^{\mu\nu}, \quad \text{Tr } \gamma^\mu = 0, \quad \text{Tr } [\gamma^\mu \gamma^\nu] = 2g^{\mu\nu}, \\ \text{Tr } [\gamma^\mu \gamma^\nu \gamma^\rho \gamma^\sigma] &= 2(g^{\mu\nu} g^{\rho\sigma} - g^{\mu\rho} g^{\nu\sigma} + g^{\mu\sigma} g^{\nu\rho}), \end{aligned} \quad (3)$$

also the trace of the product of three (*i.e.* odd number) gamma matrices has a nonzero value

$$\text{Tr } [\gamma^\mu \gamma^\nu \gamma^\rho] = -2i\varepsilon^{\mu\nu\rho}. \quad (4)$$

This identity is (mathematically) responsible for the photon mass generation by the vacuum polarization loop.

In Secs. 4 and 5, we will also make use of other identities, which can be easily derived

$$\begin{aligned} \varepsilon^{\mu\nu\alpha} \gamma_\mu \gamma^\beta \gamma_\nu &= 2\varepsilon^{\mu\beta\alpha} \gamma_\mu + 2i\gamma^\alpha \gamma^\beta, \\ \varepsilon^{\mu\nu\alpha} \gamma_\mu \gamma_\nu k_\alpha &= -2i \not{k}, \\ \gamma^\mu \not{k} \gamma_\mu &= -\not{k}. \end{aligned} \quad (5)$$

The Lagrangian density of three-dimensional electrodynamics with gauge fixing term has the following form

$$\begin{aligned} \mathcal{L}(x) &= \bar{\Psi}(x) (i\gamma^\mu \partial_\mu - m_0 - e_0 \gamma^\mu A_\mu(x)) \Psi(x) \\ &\quad - \frac{1}{4} F^{\mu\nu}(x) F_{\mu\nu}(x) - \frac{\lambda}{2} (\partial_\mu A^\mu(x))^2, \end{aligned} \quad (6)$$

where λ is the gauge parameter. The quantities m_0 and e_0 are the *bare* fermion mass and the *bare* coupling constant (charge), respectively, and the former may eventually be put equal to zero leading to the parity invariant theory (at least on the classical level). It is worth mentioning that coupling constant has a dimension of $\sqrt{\text{mass}}$ and, as a result, the quantum theory is superrenormalizable. The strength tensor $F^{\mu\nu}$ has only three independent elements, two of which (F^{10} and F^{20}) constitute the two-component vector of electric field, and one (F^{12}) pseudoscalar magnetic field.

It is well known [26], that in three space-time dimensions to this Lagrangian density may be added the so-called Chern–Simons (CS) term, proportional to $\varepsilon^{\mu\nu\alpha} A_\mu \partial_\nu A_\alpha$. Although it is not invariant under gauge transformations, its change reduces to a total derivative inessential for physical observables. Due to this term, the photon field acquires a nonzero mass. In this work, however, we do not include explicitly CS term into the Lagrangian and the photon mass will be generated dynamically.

3. Assumptions for the fundamental Green’s functions

Underneath the low momenta forms of the dressed two-point Green’s functions and three-point vertex are given. They reflect their analytic structures suggested by the perturbation calculation.

3.1. Boson propagator

For the free propagator of massless vector particle, we have the usual formula

$$D^{(0)\mu\nu}(k) = \frac{1}{k^2} \left(-g^{\mu\nu} + \frac{k^\mu k^\nu}{k^2} \right) - \frac{1}{\lambda} \frac{k^\mu k^\nu}{(k^2)^2}. \quad (7)$$

It is well known from the perturbative approach [26, 44], that in three space-time dimensions „photon” field acquires a (topological) mass, due to the vacuum polarization process. Even if CS term is initially absent in the Lagrangian, it is generated through the interaction with fermion loop. In the first approximation, we can then assume for the dressed propagator

$$D^{\mu\nu}(k) = \frac{Z_3}{k^2 - \tau^2} \left(-g^{\mu\nu} + \frac{k^\mu k^\nu}{k^2} \right) + \frac{iZ_3\tau}{k^2(k^2 - \tau^2)} \varepsilon^{\mu\nu\rho} k_\rho - \frac{1}{\lambda} \frac{k^\mu k^\nu}{(k^2)^2}, \quad (8)$$

where τ is the photon mass to be determined from the consistency conditions, with the renormalization constant Z_3 present only in the transverse part of $D^{\mu\nu}(k)$. This is a known fact by virtue of the gauge invariance and the Ward–Takahashi identity

$$k_\mu D^{\mu\nu}(k) = k_\mu D^{(0)\mu\nu}(k) = -\frac{1}{\lambda} \frac{k^\nu}{k^2}, \quad (9)$$

which ascertains that only transverse part is modified by the interactions.

By the reason of unitarity, we expect the nonperturbative value of Z_3 to satisfy the condition $0 < Z_3 \leq 1$ [45]. In our work on QED₄, we obtained this value to be equal to $\frac{7}{9}$ [43]. As we will see in Sec. 6, the above bounds will also be satisfied in the present work.

We treat the denominator $k^2 - \tau^2$ in (8) as the first term in its Taylor expansion around $k^2 = \tau^2$ and potentially admit also higher order polynomial in k^2 .

The inverse of $D^{\mu\nu}$ is

$$D^{-1}(k)^{\mu\nu} = Z_3^{-1} [(-k^2 g^{\mu\nu} + k^\mu k^\nu) - i\tau \varepsilon^{\mu\nu\rho} k_\rho] - \lambda k^\mu k^\nu. \quad (10)$$

3.2. Fermion propagator

The free fermion propagator has the standard form

$$S^{(0)}(p) = \frac{1}{\not{p} - m_0} \quad (11)$$

with a pole in the bare mass m_0 . Contrary to that, the perturbative calculations [26] (at least in certain gauges) show that, even if initially massless “photon” acquires a mass τ , the “dressed” fermion propagator should have, as a result of infrared divergences, a branch point at $p^2 = m^2$, where m is a physical mass. Therefore, we assume for further calculations the following form

$$S(p) = \frac{1}{(\not{p} - m)(1 - p^2/m^2)^\beta}, \quad (12)$$

with exponent β to be determined (together with mass renormalization constant $\delta m = m - m_0$) by the later requirement of consistency. The additional power 2β of p in denominator improves the high momentum behavior of loop integrals (after the analytical continuation of (12)), making thereby the theory free of any ultraviolet divergences.

To get the spectral representation of (12), which will be needed for the construction of the vertex, we apply the Cauchy integral formula for the function

$$f(z) = \frac{1}{(m^2 - z^2)^\beta},$$

choosing the contour as shown in Fig. 1 deformed to sandwich both cuts on the real axis. This is possible for $0 < \beta < 1$, since otherwise the integrals over the small circles around branch points at $z = \pm m$ diverge. We have then

$$\begin{aligned} \frac{1}{(m^2 - w^2)^\beta} &= \frac{1}{2\pi i} \oint_C \frac{dz}{(m^2 - z^2)^\beta (z - w)} \\ &= \frac{\sin \pi\beta}{\pi} \left[\int_m^\infty \frac{dx}{(x^2 - m^2)^\beta (x - w)} - \int_{-\infty}^{-m} \frac{dx}{(x^2 - m^2)^\beta (x - w)} \right]. \end{aligned} \quad (13)$$

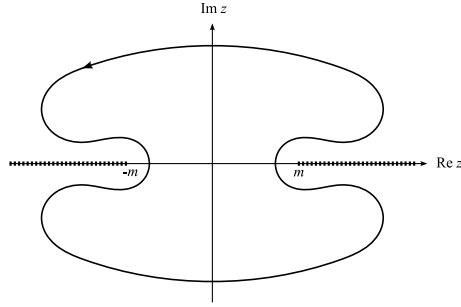


Fig. 1. The contour of integration in formula (13), which leads to the spectral function (14).

Applying this formula for $w = \not{p}$ and taking into account the additional factor $(\not{p} - m)$ in denominator of (12), we get that way (cf. [43])

$$\rho(M) = \frac{\sin(\pi\beta)}{\pi} \frac{1}{(M-m)(M^2/m^2 - 1)^\beta} [\Theta(M-m) - \Theta(-M-m)], \quad (14)$$

where Θ is the Heaviside step function. We found it useful to separate out in the above definition of $\rho(M)$ a term, that describes free propagation of fermion so that $S(p)$ is represented as

$$S(p) = \int dM \rho(M) \left(\frac{1}{\not{p} - m} - \frac{1}{\not{p} - M} \right). \quad (15)$$

The equations, we obtain in the following sections, turn out to be self-consistent without assuming any nontrivial value of the fermion field renormalization constant Z_2 and, therefore, in the present work, we have just put it equal to unity. Eventual inclusion of this constant (*i.e.* an additional unknown variable) would require to generate more equations by adjusting higher Green's functions. This complicates our method, but we keep it in mind and leave for the next paper as a possible following step.

The problem with properly defining constant Z_2 raised in [26] does not arise in our approach, since we do not require the full fermion propagator to have a pole at mass m . On the contrary, we know that such a pole does not exist in (12) due to the infrared divergences and only the additional coefficient in numerator come into play. Its value would be fixed by self-consistency and not by normalization requirement of the residue.

3.3. Fermion-boson vertex

Having expressed the full propagator S through the spectral function, we can now make use of the slightly modified (by inclusion of the second term) Salam's ansatz for the vertex function $\Gamma^\mu(p+k, p)$

$$S(p+k)\Gamma^\mu(p+k,p)S(p) = \int dM\rho(M) \left[\frac{1}{\not{p} + \not{k} - m} \gamma^\mu \frac{1}{\not{p} - m} - \frac{1}{\not{p} + \not{k} - M} \gamma^\mu \frac{1}{\not{p} - M} \right]. \quad (16)$$

By acting on it with a vector k_μ , one automatically obtains the appropriate WT identity, which must be incorporated in any method, that is expected to satisfy gauge invariance

$$\begin{aligned} k_\mu S(p+k)\Gamma^\mu(p+k,p)S(p) &= \int dM\rho(M) \left[\frac{1}{\not{p} + \not{k} - m} \not{k} \frac{1}{\not{p} - m} - \frac{1}{\not{p} + \not{k} - M} \not{k} \frac{1}{\not{p} - M} \right] \\ &= \int dM\rho(M) \left[\frac{1}{\not{p} - m} - \frac{1}{\not{p} + \not{k} - m} - \frac{1}{\not{p} - M} + \frac{1}{\not{p} + \not{k} - M} \right] \\ &= S(p) - S(p+k). \end{aligned} \quad (17)$$

It is a necessary condition but does not guarantee by itself the invariance of physical quantities.

The longitudinal part of the vertex is then correctly fixed, and transverse part (obviously (16) is not purely longitudinal) is postulated in an elegant way, naturally being a subject of further improvements [41, 42].

The expressions (10), (15) and (16), together with (14) will be now required to satisfy DS equations for small momenta.

4. Dyson–Schwinger equation for the boson propagator

The Dyson–Schwinger equation for the photon propagator may be written as

$$\begin{aligned} D^{\mu\nu}(k) &= \frac{1}{k^2} \left(-g^{\mu\alpha} + \frac{k^\mu k^\alpha}{k^2} - \frac{1}{\lambda} \frac{k^\mu k^\alpha}{k^2} \right) \\ &\times \left[\delta_\alpha^\nu - ie_0^2 \text{Tr} \gamma_\alpha \int \frac{d^3p}{(2\pi)^3} S(p) \Gamma_\beta(p, p-k) S(p-k) D^{\beta\nu}(k) \right], \end{aligned} \quad (18)$$

and has the graphical representation shown in Fig. 2.



Fig. 2. The Dyson–Schwinger equation for the gauge boson propagator $D^{\mu\nu}(k)$. Light lines represent free propagators and heavy ones dressed propagators. The full circle stands for the full fermion-boson vertex.

This equation, rewritten for inverse propagator, will be more suitable for deriving self-consistency relations since $D^{\mu\nu}$ decouples from other functions. If we make use of the ansatz (16), it can be given in the form

$$\begin{aligned} D^{-1}(k)^{\mu\nu} = & -k^2 g^{\mu\nu} + k^\mu k^\nu - \lambda k^\mu k^\nu + ie_0^2 \text{Tr} \gamma^\mu \\ & \times \int dM \rho(M) \int \frac{d^3 p}{(2\pi)^3} \left(\frac{1}{\not{p} - m + i\varepsilon} \gamma^\nu \frac{1}{\not{p} - \not{k} - m + i\varepsilon} \right. \\ & \left. - \frac{1}{\not{p} - M + i\varepsilon} \gamma^\nu \frac{1}{\not{p} - \not{k} - M + i\varepsilon} \right). \end{aligned} \quad (19)$$

The expression under the integral is well known from one-loop perturbation calculation and its evaluation is straightforward, for instance, by passing into the Euclidean space. It is needless to show any details, so we give only final (Minkowski) result. If we denote

$$\Pi_m^{\mu\nu}(k) = ie_0^2 \text{Tr} \gamma^\mu \int \frac{d^3 p}{(2\pi)^3} \frac{1}{\not{p} - m + i\varepsilon} \gamma^\nu \frac{1}{\not{p} - \not{k} - m + i\varepsilon}, \quad (20)$$

and introduce Feynman parameters, we find

$$\begin{aligned} \Pi_m^{\mu\nu}(k) = & \frac{e_0^2}{2\pi} \left[(-k^2 g^{\mu\nu} + k^\mu k^\nu) \int_0^1 dx \frac{x(1-x)}{(m^2 - k^2 x(1-x))^{1/2}} \right. \\ & \left. - \frac{i}{2} m \varepsilon^{\mu\nu\rho} k_\rho \int_0^1 dx \frac{1}{(m^2 - k^2 x(1-x))^{1/2}} \right], \end{aligned} \quad (21)$$

where x is the Feynman parameter. When $k^2 < 4m^2$, both integrals are well defined.

The tensor $\Pi_m^{\mu\nu}(k)$ turns out to be transverse with respect to k_μ , as is required by gauge invariance, and particularly by WT identity (9). What is important, it the appearance of the novel term, peculiar for three dimensions, which is proportional to $\varepsilon^{\mu\nu\rho}$ — a result of the nonzero trace of the product of three gamma matrices (4). This property is responsible for the gauge field acquiring a mass.

To find the full form of the polarization tensor, we have to calculate — according to (16) — the following integral involving the spectral function $\rho(M)$

$$\Pi^{\mu\nu}(k) = \int dM \rho(M) (\Pi_m^{\mu\nu}(k) - \Pi_M^{\mu\nu}(k)). \quad (22)$$

Various integrals of that kind, containing spectral function $\rho(M)$, are collected in Appendix A. When we apply formulae (A.3) and (A.4), the only

integrals that are left are those over Feynman parameter x

$$\begin{aligned} \Pi^{\mu\nu}(k) = & \frac{e_0^2 \Gamma(\beta + 1/2)}{2\pi^{3/2} \Gamma(\beta + 1) m} \\ & \times \left[(-k^2 g^{\mu\nu} + k^\mu k^\nu) \int_0^1 dx \frac{x(1-x)}{1 - k^2/m^2 x(1-x))^{\beta+1/2}} \right. \\ & \left. - \frac{i}{2} m \varepsilon^{\mu\nu\rho} k_\rho \int_0^1 dx \frac{1}{1 - k^2/m^2 x(1-x))^{\beta+1/2}} \right]. \end{aligned} \quad (23)$$

Both turn out to be expressed through the hypergeometric (Gauss) function (see Appendix B, formulae (B.1) and (B.2)). These results allow us to bring the whole Dyson–Schwinger equation (19) to the form

$$\begin{aligned} D^{-1}(k)^{\mu\nu} = & -\lambda k^\mu k^\nu + (-k^2 g^{\mu\nu} + k^\mu k^\nu) \\ & \times \left[1 + \frac{e_0^2 \Gamma(\beta + 1/2)}{12\pi^{3/2} \Gamma(\beta + 1) m} {}_2F_1(2, \beta + 1/2; 5/2; k^2/4m^2) \right] \\ & - i \varepsilon^{\mu\nu\rho} k_\rho \frac{e_0^2 \Gamma(\beta + 1/2)}{4\pi^{3/2} \Gamma(\beta + 1)} {}_2F_1(1, \beta + 1/2; 3/2; k^2/4m^2) \end{aligned} \quad (24)$$

and adjust the first two terms in the infrared domain, *i.e.* close to the photon mass ($k^2 \approx \tau^2$). In that way, we get two equations for Z_3 and τ

$$Z_3^{-1} = 1 + \frac{e_0^2 \Gamma(\beta + 1/2)}{12\pi^{3/2} m \Gamma(\beta + 1)} {}_2F_1(2, \beta + 1/2; 5/2; \tau^2/4m^2), \quad (25)$$

$$\tau Z_3^{-1} = \frac{e_0^2 \Gamma(\beta + 1/2)}{4\pi^{3/2} m \Gamma(\beta + 1)} {}_2F_1(1, \beta + 1/2; 3/2; \tau^2/4m^2), \quad (26)$$

which will be solved in Sec. 6 together with the other two derived from the DS equation for the fermion propagator.

5. Dyson–Schwinger equation for the fermion propagator

The Dyson–Schwinger equation for the fermion propagator has the form

$$S(p) = \frac{1}{\not{p} - m_0} \left[1 + i e_0^2 \gamma^\mu \int \frac{d^3 k}{(2\pi)^3} S(p+k) \Gamma^\nu(p+k, p) S(p) D_{\mu\nu}(k) \right], \quad (27)$$

and may be represented as shown in Fig. 3.



Fig. 3. The Dyson–Schwinger equation for the fermion propagator. As in Fig. 2, heavy lines stand for full functions and light for free ones.

It can be rewritten in a simpler manner

$$(\not{p} - m_0)S(p) = 1 + \Sigma(p)S(p), \quad (28)$$

where

$$\Sigma(p)S(p) = ie_0^2 \gamma^\mu \int \frac{d^3 k}{(2\pi)^3} S(p+k) \Gamma^\nu(p+k, p) S(p) D_{\mu\nu}(k). \quad (29)$$

Now, we have to substitute (16) for the vertex, and (8) for $D^{\mu\nu}$, obtaining

$$\Sigma(p)S(p) = [\Sigma(p)S(p)]_A + [\Sigma(p)S(p)]_B + [\Sigma(p)S(p)]_C + [\Sigma(p)S(p)]_D, \quad (30)$$

where, to avoid lengthy expressions, we separated the contributions coming from different tensor structures in the photon propagator: $g^{\mu\nu}$, $k^\mu k^\nu$, $\varepsilon^{\mu\nu\rho} k_\rho$, and from gauge-dependent longitudinal part

$$[\Sigma(p)S(p)]_A = -iZ_3 e_0^2 \int dM \rho(M) \int \frac{d^3 k}{(2\pi)^3} \times \left[\gamma^\mu \frac{1}{\not{p} + \not{k} - m + i\varepsilon} \gamma^\mu \frac{1}{(k^2 - \tau^2 + i\varepsilon)(\not{p} - m + i\varepsilon)} - (m \rightarrow M) \right], \quad (31)$$

$$[\Sigma(p)S(p)]_B = iZ_3 e_0^2 \int dM \rho(M) \int \frac{d^3 k}{(2\pi)^3} \times \left[\not{k} \frac{1}{\not{p} + \not{k} - m + i\varepsilon} \not{k} \frac{1}{(k^2 + i\varepsilon)(k^2 - \tau^2 + i\varepsilon)(\not{p} - m + i\varepsilon)} - (m \rightarrow M) \right], \quad (32)$$

$$[\Sigma(p)S(p)]_C = -Z_3 e_0^2 \tau \int dM \rho(M) \int \frac{d^3 k}{(2\pi)^3} \times \left[\varepsilon^{\mu\nu\rho} \gamma_\mu \frac{1}{\not{p} + \not{k} - m + i\varepsilon} \gamma_\nu \frac{k_\rho}{(k^2 + i\varepsilon)(k^2 - \tau^2 + i\varepsilon)(\not{p} - m + i\varepsilon)} - (m \rightarrow M) \right], \quad (33)$$

$$[\Sigma(p)S(p)]_D = -\frac{ie_0^2}{\lambda} \int dM \rho(M) \int \frac{d^3 k}{(2\pi)^3} \times \left[\not{k} \frac{1}{\not{p} + \not{k} - m + i\varepsilon} \not{k} \frac{1}{(k^2 + i\varepsilon)^2(\not{p} - m + i\varepsilon)} - (m \rightarrow M) \right]. \quad (34)$$

Each of the above momentum integrals can be performed in an ordinary way by performing Wick's rotation and introducing Feynman parameters. In the symmetric integration, no infinities arise even if the simple power counting indicate an apparent logarithmic divergence. Disregarding all details of the computation, as a result, we have (again in the Minkowski space)

$$\begin{aligned}
 I_A &= ie_0^2 \int \frac{d^3k}{(2\pi)^3} \gamma^\mu \frac{1}{\not{p} + \not{k} - m + i\varepsilon} \gamma^\mu \frac{1}{k^2 - \tau^2 + i\varepsilon} \\
 &= \frac{e_0^2}{8\pi} \int_0^1 dx \frac{\not{p}(1-x) - 3m}{(m^2x - p^2x(1-x) + \tau^2(1-x))^{1/2}}, \quad (35)
 \end{aligned}$$

$$\begin{aligned}
 I_B &= ie_0^2 \int \frac{d^3k}{(2\pi)^3} \not{k} \frac{1}{\not{p} + \not{k} - m + i\varepsilon} \not{k} \frac{1}{(k^2 + i\varepsilon)(k^2 - \tau^2 + i\varepsilon)} \\
 &= \frac{e_0^2}{8\pi} \int_0^1 dx \left[\frac{1}{(m^2x - p^2x(1-x) + \tau^2(1-x))^{1/2}} + \frac{\not{p}(\not{p} + m)}{\tau^2} \right. \\
 &\quad \times \left(\frac{x}{(m^2x - p^2x(1-x))^{1/2}} - \frac{x}{(m^2x - p^2x(1-x) + \tau^2(1-x))^{1/2}} \right) \Big] \\
 &\quad \times (\not{p} - m), \quad (36)
 \end{aligned}$$

$$\begin{aligned}
 I_C &= e_0^2 \int \frac{d^3k}{(2\pi)^3} \varepsilon^{\mu\nu\rho} \gamma_\mu \frac{1}{\not{p} + \not{k} - m + i\varepsilon} \gamma_\nu \frac{k_\rho}{(k^2 + i\varepsilon)(k^2 - \tau^2 + i\varepsilon)} \\
 &= \frac{e_0^2}{4\pi} \int_0^1 dx \left[\frac{-1}{(m^2x - p^2x(1-x) + \tau^2(1-x))^{1/2}} + \frac{\not{p}(\not{p} - m)}{\tau^2} \right. \\
 &\quad \times \left(\frac{x}{(m^2x - p^2x(1-x) + \tau^2(1-x))^{1/2}} - \frac{x}{(m^2x - p^2x(1-x))^{1/2}} \right) \Big], \quad (37)
 \end{aligned}$$

$$\begin{aligned}
 I_D &= ie_0^2 \int \frac{d^3k}{(2\pi)^3} \not{k} \frac{1}{\not{p} + \not{k} - m + i\varepsilon} \not{k} \frac{1}{(k^2 + i\varepsilon)^2} \\
 &= \frac{e_0^2}{8\pi} \int_0^1 dx \left[\frac{1}{(m^2x - p^2x(1-x))^{1/2}} + \frac{\not{p}(\not{p} + m)}{2} \frac{x(1-x)}{(m^2x - p^2x(1-x))^{3/2}} \right] \\
 &\quad \times (\not{p} - m). \quad (38)
 \end{aligned}$$

Now, the contributions to $\Sigma(p)S(p)$ may be written as

$$[\Sigma(p)S(p)]_A = -Z_3 \int dM \rho(M) \left[I_A \frac{1}{\not{p} - m + i\varepsilon} - (m \rightarrow M) \right], \quad (39)$$

$$[\Sigma(p)S(p)]_B = Z_3 \int dM \rho(M) \left[I_B \frac{1}{\not{p} - m + i\varepsilon} - (m \rightarrow M) \right], \quad (40)$$

$$[\Sigma(p)S(p)]_C = -Z_3 \tau \int dM \rho(M) \left[I_C \frac{1}{\not{p} - m + i\varepsilon} - (m \rightarrow M) \right], \quad (41)$$

$$[\Sigma(p)S(p)]_D = -\frac{1}{\lambda} \int dM \rho(M) \left[I_D \frac{1}{\not{p} - m + i\varepsilon} - (m \rightarrow M) \right], \quad (42)$$

and the appropriate spectral integrals, after a small rearrangement, are carried out according to formulae of Appendix A.

Before performing x integrations, it is important to notice, that if one substitutes (12) into the left-hand side of (27) and multiply both sides by $(\not{p} + m_0)$, there appear two singular terms, when $p^2 \rightarrow m^2$

$$-\frac{m^{2\beta} \delta m (\not{p} + m)}{(m^2 - p^2)^{\beta+1}} + \frac{m^{2\beta}}{(m^2 - p^2)^\beta}. \quad (43)$$

For our goal, it is sufficient to pick out the identical terms from the right-hand side of (27). It is a very pleasant observation that it actually displays identical analytical structure, which is a strong sign of self-consistency. Consequently, we do not need to preform explicitly all parameter integrations, and can limit ourselves to those terms that contain singularities of the kind (43). The appropriate results are given below

$$[\Sigma(p)S(p)]_A \approx -\frac{e_0^2 Z_3}{8\pi} \mathcal{K}_1(p^2, \tau^2) \frac{\not{p}(\not{p} + m)}{(m^2 - p^2)(1 - p^2/m^2)^\beta}, \quad (44)$$

$$[\Sigma(p)S(p)]_B \approx 0, \quad (45)$$

$$[\Sigma(p)S(p)]_C \approx -\frac{e_0^2 Z_3 \tau}{4\pi} \mathcal{K}_2(p^2, \tau^2) \frac{\not{p} + m}{(m^2 - p^2)(1 - p^2/m^2)^\beta}, \quad (46)$$

$$[\Sigma(p)S(p)]_D \approx \frac{e_0^2}{8\pi \lambda m} \left(\frac{-1}{(m^2 - p^2)(1 - p^2/m^2)^\beta} + \frac{1 - \beta}{2\beta} \frac{1}{m^2(1 - p^2/m^2)^\beta} \right) \times \not{p}(\not{p} + m), \quad (47)$$

where ≈ 0 means that the expression does not diverge, when $p^2 \rightarrow m^2$. The functions \mathcal{K}_1 and \mathcal{K}_2 have the following form

$$\mathcal{K}_1(p^2, \tau^2) = \int_0^1 dx \frac{x+2}{(p^2 x^2 + \tau^2(1-x))^{1/2}}, \quad (48)$$

$$\mathcal{K}_2(p^2, \tau^2) = \int_0^1 dx \frac{1}{(p^2 x^2 + \tau^2(1-x))^{1/2}}. \quad (49)$$

Equating identical divergent terms on both sides of the Dyson–Schwinger equation (28), we obtain two relations for parameters δm and β

$$\delta m = \frac{e_0^2 Z_3}{8\pi} \left(\mathcal{S}_1(m, \tau) + \frac{1}{\lambda Z_3} \right), \quad (50)$$

$$1 = \frac{e_0^2 Z_3}{8\pi} \left(\mathcal{S}_2(m, \tau) + \frac{1}{\lambda Z_3 m \beta} \right), \quad (51)$$

where auxiliary quantities $\mathcal{S}_1(m, \tau)$ and $\mathcal{S}_2(m, \tau)$ are defined as

$$\mathcal{S}_1 = m \mathcal{K}_1(m^2, \tau^2) + 2\tau \mathcal{K}_2(m^2, \tau^2), \quad (52)$$

$$\mathcal{S}_2 = \mathcal{K}_1(m^2, \tau^2) + 2m^2 \frac{\partial \mathcal{K}_1(m^2, \tau^2)}{\partial m^2} + 4m\tau \frac{\partial \mathcal{K}_2(m^2, \tau^2)}{\partial m^2}, \quad (53)$$

and found in Appendix B.

6. Results and conclusions

Below, we rewrite the whole set of equations using *renormalized* quantities: fermion mass m , gauge coupling constant $e = Z_3^{1/2} e_0$ and gauge parameter $\lambda_R = Z_3 \lambda$. We additionally introduce a dimensionless parameter $\zeta = \frac{e^2}{4\pi m}$, obtaining

$$\frac{\delta m}{m} = \frac{\zeta}{2} \left[1 - \frac{\tau}{m} + \left(2 + \frac{2\tau}{m} + \frac{\tau^2}{2m^2} \right) \ln(2m/\tau + 1) + \frac{1}{\lambda_R} \right], \quad (54)$$

$$1 = \frac{\zeta}{2} \left[2 + \frac{2\tau}{m} - \left(\frac{2\tau}{m} + \frac{\tau^2}{m^2} \right) \ln(2m/\tau + 1) + \frac{1}{\lambda_R \beta} \right], \quad (55)$$

$$Z_3 = 1 - \frac{\zeta}{3} \frac{\Gamma(\beta + 1/2)}{\sqrt{\pi} \Gamma(\beta + 1)} {}_2F_1(2, \beta + 1/2; 5/2; \tau^2/4m^2), \quad (56)$$

$$\frac{\tau}{m} = \zeta \frac{\Gamma(\beta + 1/2)}{\sqrt{\pi} \Gamma(\beta + 1)} {}_2F_1(1, \beta + 1/2; 3/2; \tau^2/4m^2). \quad (57)$$

Now, the question arises whether this set of highly nonlinear equations for unknown $\delta m/m$, β , Z_3 and τ/m has a certain domain of solvability. The answer is positive for a large range of values of gauge parameter λ_R and for weak coupling. For exemplary value $\lambda_R = 1$, numerically found solutions of all equations as functions of the parameter ζ are presented below.

In the first graph of Fig. 4, we show the dependence of the topological photon mass τ (in units of m) on the parameter ζ . One can see, that for small values of parameter it approaches the value $e^2/4\pi$ [44]. In spite of initial zero value of the photon mass, it reappears as an outcome of interactions with fermions.

In the second graph, the dependence of power β on ζ is presented. As we remember, our method is reliable only for $0 < \beta < 1$. This corresponds roughly to $0 < \zeta < 1$.

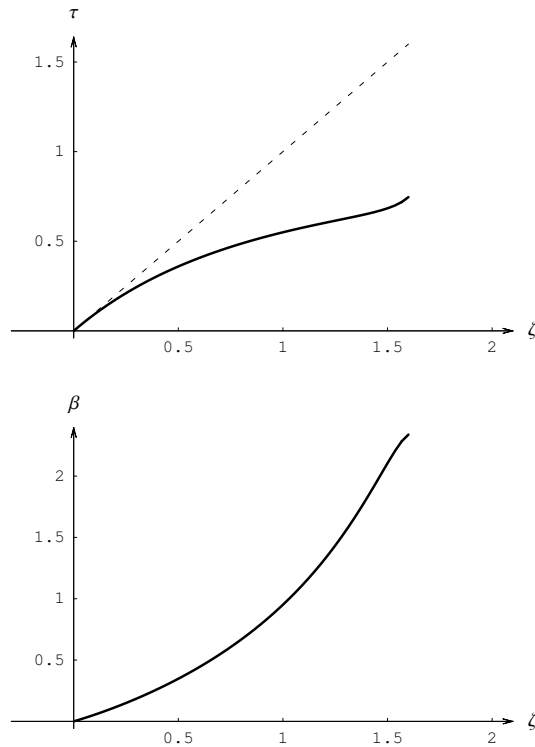


Fig. 4. The dependence of photon mass τ in units of m (upper plot) and power β (lower plot) on the parameter ζ . The dashed line corresponds to $\tau/m = \zeta$.

Figure 5 shows the fermion mass renormalization δm (again in units of m) and coupling constant renormalization constant Z_3 as functions of ζ . Please note, that $\delta m < m$ (for small ζ) and $0 < Z_3 < 1$ and it decreases with increasing coupling.

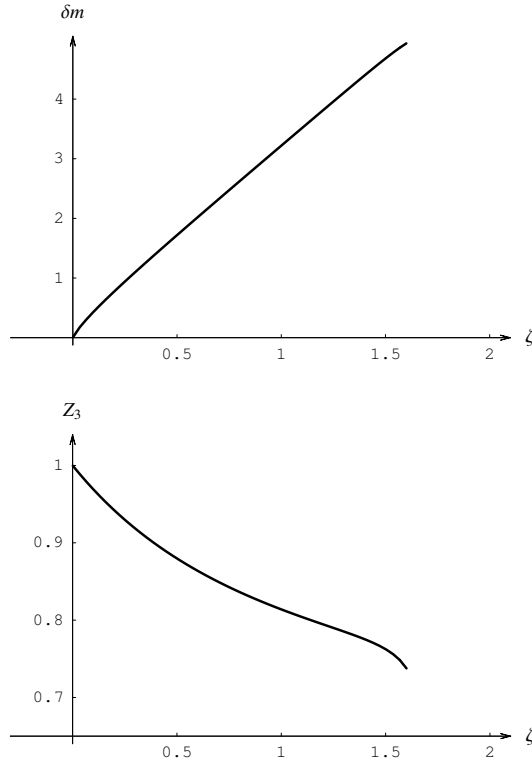


Fig. 5. The dependence of mass renormalization δm in units of m (upper plot) and charge renormalization constant Z_3 (lower plot) on the parameter ζ .

The comparison of all numerical results suggests that it might be possible to find analytical solutions in the case of weak coupling (which corresponds to heavy fermion), *i.e.* when $\zeta \ll 1$. From Fig. 4, we conclude that in such a case, we should assume $\tau/m \ll 1$. We suppose also that $\beta \ll 1$. All these assumptions will be justified *a posteriori*, when the whole set of parameters is determined.

For small values of τ , the equation (55) may be given the approximate form (considering gauges, where λ_R is of the order of unity)

$$1 = \frac{\zeta}{2} \frac{1}{\lambda_R \beta}, \quad (58)$$

since all other terms, particularly those containing a quotient τ/m , are small compared to $1/\beta$ and have been neglected. Consequently, we have

$$\beta = \frac{\zeta}{2\lambda_R}, \quad (59)$$

which is in agreement with the assumption $\beta \ll 1$.

For small values of τ and β , the equation (57) reduces to (recall that $\Gamma(1/2) = \sqrt{\pi}$ and ${}_2F_1(a, b; c; 0) = 1$)

$$\frac{\tau}{m} = \zeta, \quad (60)$$

which is the Coleman's and Hill's result [44]. The first plot of Fig. 4 shows, however, that for larger values of ζ it deviates from the straight line. This is well understood since in the Coleman's and Hill's paper the value of the photon mass was defined as $\tau^2 = \Pi(0)$, where Π is a polarization scalar. In our work, the position of the photon mass is determined by the equation $\tau^2 = \Pi(\tau^2)$. One should also keep in mind that our solution is only approximate.

From (60), we find again that the assumption $\tau/m \ll 1$ is justified. In a similar manner, we get the following result for Z_3

$$0 < Z_3 = 1 - \frac{\zeta}{3} < 1, \quad (61)$$

and for δm

$$\frac{\delta m}{m} = \frac{\zeta}{2} \left[1 - 2 \ln \left(\frac{\zeta}{2} \right) + \frac{1}{\lambda_R} \right]. \quad (62)$$

The unpleasant feature is the gauge dependence of the mass renormalization, which might indicate the similar gauge dependence of the physical fermion mass. It is, however, a common feature of perturbative and nonperturbative results in QED₃ [46, 47].

In the weak coupling regime, one can obtain also the analytical results for the Landau gauge, which corresponds to taking the limit $\lambda_R \rightarrow \infty$. As it is known, in this case the infrared singularities disappear [26, 46], so we expect $\beta \rightarrow 0$, since the fermion propagator should have an ordinary pole at physical mass and not a branch point. Therefore, before taking the limit, we put $\beta = \beta_0/\lambda_R$ with constant β_0 to be determined. With all other assumptions identical as above, we get similar results as before: $\tau/m = \zeta$, $Z_3 = 1 - \zeta/3$ and

$$1 = \frac{\zeta}{2} \left(2 + \frac{1}{\beta_0} \right) \implies \beta_0 = \frac{\zeta}{2(1 - \zeta)}, \quad (63)$$

and hence for $\zeta \ll 1$,

$$\beta = \frac{\zeta}{2\lambda_R(1 - \zeta)} \approx \frac{\zeta}{2\lambda_R} \xrightarrow{\lambda_R \rightarrow \infty} 0. \quad (64)$$

The result for the mass renormalization in this case is

$$\frac{\delta m}{m} = \frac{\zeta}{2} \left[1 - 2 \ln \left(\frac{\zeta}{2} \right) \right]. \quad (65)$$

The nonanalyticity in ζ observed in the last formula and in (62), which manifests itself through the presence of a logarithmic function of the coupling constant, is similar in nature to the one found by us in QED₄ [43], where it constituted a reflection of the possible ill behavior of the perturbation series [48].

The solutions of the set of nonlinear equations (54)–(57) involving gauge parameter λ_R can, in general, be gauge dependent. To clarify this point, we performed a couple of plots of the model parameters for different gauges, *i.e.* for $\lambda_R = 1, 3, 10, \infty$, the last case corresponding to the Landau gauge. In Fig. 6, the dependence of τ and β on λ_R is demonstrated. Figure 7 shows the same dependence of δm and Z_3 .

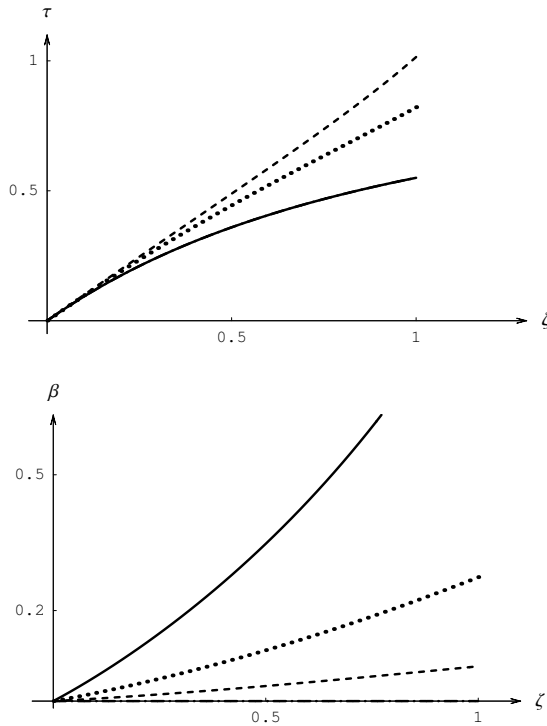


Fig. 6. The comparison of the behavior of photon mass τ (upper plot) and parameter β (lower plot) for different gauges: solid line — $\lambda_R = 1$, dotted line — $\lambda_R = 3$, dashed line — $\lambda_R = 10$, mixed line — $\lambda_R = \infty$ (Landau gauge). The latter is not visible on the upper plot, since it is overlapping with the solid line.

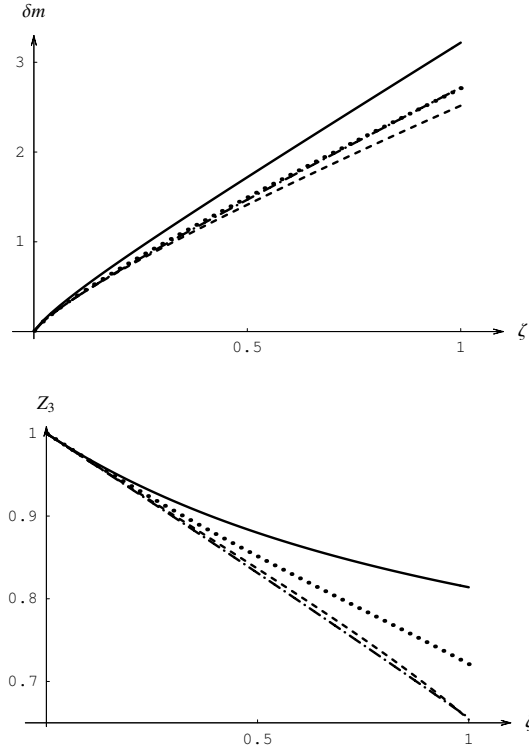


Fig. 7. The comparison of the behavior of fermion mass renormalization δm (upper plot) and charge renormalization constant Z_3 (lower plot) for different gauges: solid line — $\lambda_R = 1$, dotted line — $\lambda_R = 3$, dashed line — $\lambda_R = 10$, mixed line — $\lambda_R = \infty$ (Landau gauge). On the upper plot, the lines for $\lambda_R = 3$ and for Landau gauge follow close curves.

It may be easily seen that, apart from the parameter β , which is and may be obviously gauge dependent, all other values become practically gauge independent for weak coupling. To get the reliable results for strong coupling, we need to go beyond the first approximation consisting of the simple assumptions (8), (12) and (16). The same refers to other nonperturbative (but not exact) calculations.

At the end, we would like to note, that from the dependence of δm (Fig. 5 and Fig. 7) one sees, that for certain values of parameter ζ , we have $\delta m = m$ (i.e. $m_0 = 0$). Assuming value $\delta m/m = 1$ as fixed, equations (54)–(57) may be in turn solved for ζ , τ , β and Z_3 . In the case of the Landau gauge, the following values are obtained

$$\frac{\tau}{m} \approx \zeta \approx 0.34, \quad (66)$$

from which we get the generated masses (the Lagrangian in this case does not contain any masses) to be

$$m \approx 2.94 \times \frac{e^2}{4\pi}, \quad \tau = \frac{e^2}{4\pi}, \quad (67)$$

and the parity becomes broken. For two other parameters, we find

$$\beta_0 \approx 0.21 \quad (\text{naturally } \beta = 0), \quad Z_3 \approx 0.89. \quad (68)$$

A comment should be made here. The obtained effect of parity breaking (to some extent noted also in [49]) is not at variant with the Vafa–Witten theorem [50, 51] stating that there is no spontaneous parity breaking in theories with vectorlike fermions. The three-dimensional QED with two-component fermions is a very special theory, also in that aspect, that it evades the proof of this theorem. The crucial point in Vafa’s and Witten’s argument is the positivity of the boson measure (after integrating out the fermion degrees of freedom) which demands the positivity of the determinant of the Dirac operator. This positivity may be easily proved in four component version of the theory, because by the application of the matrix γ^5 , one sees that eigenvalues are always paired in such a way that the unwanted signs cancel. However, in spinor version of QED no such γ^5 matrix exists (there is no matrix anticommuting with matrices γ^0 , γ^1 and γ^2), so the above argument fails.

The results of [52], which support the Vafa–Witten conclusion, are not in conflict with the present work either. They were obtained with the assumptions $m \ll \alpha$ or $m \gg \alpha$, which correspond to $\zeta \gg 1$ or $\zeta \ll 1$, respectively, while in our case, we have $\zeta \approx 0.34$. One has also to stress that the conclusions of [52] are got in the limit of infinitely many flavors, while we have one flavor theory. The number of flavors, particularly if it is even, may be essential for the positivity of the determinant spoken above.

Appendix A

Spectral integrals

Below, we collect the set of integrals involving spectral density (14), that we used in Secs. 4 and 5. First, one can find that for $a^2 < m^2$, one has

$$\int dM \rho(M) \left(\frac{1}{(m^2 - a^2)^{1/2}} - \frac{1}{(M^2 - a^2)^{1/2}} \right) = \frac{m^{2\beta} \Gamma(\beta + 1/2)}{\sqrt{\pi} \Gamma(\beta + 1) (m^2 - a^2)^{\beta + 1/2}}, \quad (\text{A.1})$$

and similarly,

$$\int dM \rho(M) \left(\frac{1}{(m^2 - a^2)^{3/2}} - \frac{1}{(M^2 - a^2)^{3/2}} \right) = \frac{2m^{2\beta} \Gamma(\beta + 3/2)}{\sqrt{\pi} \Gamma(\beta + 1) (m^2 - a^2)^{\beta + 3/2}}, \quad (\text{A.2})$$

where Γ is the Euler function. From the former result by the appropriate substitution, we obtain

$$\begin{aligned} & \int dM \rho(M) \left(\frac{1}{(m^2 - k^2 x(1-x))^{1/2}} - (m \rightarrow M) \right) \\ &= \frac{\Gamma(\beta + 1/2)}{\sqrt{\pi} \Gamma(\beta + 1)} \frac{m^{2\beta}}{(m^2 - k^2 x(1-x))^{\beta + 1/2}}, \end{aligned} \quad (\text{A.3})$$

$$\begin{aligned} & \int dM \rho(M) \left(\frac{m}{(m^2 - k^2 x(1-x))^{1/2}} - (m \rightarrow M) \right) \\ &= \frac{\Gamma(\beta + 1/2)}{\sqrt{\pi} \Gamma(\beta + 1)} \frac{m^{2\beta + 1}}{(m^2 - k^2 x(1-x))^{\beta + 1/2}}, \end{aligned} \quad (\text{A.4})$$

where we additionally used the relation

$$\int dM \rho(M) M f(M^2) = m \int dM \rho(M) f(M^2). \quad (\text{A.5})$$

With a slight modification of (A.3) and (A.4), one can easily get the other set of integrals

$$\begin{aligned} & \int dM \rho(M) \left(\frac{1}{(m^2 x - k^2 x(1-x))^{1/2}} - (m \rightarrow M) \right) \\ &= \frac{\Gamma(\beta + 1/2)}{\sqrt{\pi} \Gamma(\beta + 1)} \frac{m^{2\beta} x^\beta}{(m^2 x - k^2 x(1-x))^{\beta + 1/2}}, \end{aligned} \quad (\text{A.6})$$

$$\begin{aligned} & \int dM \rho(M) \left(\frac{m}{(m^2 x - k^2 x(1-x))^{1/2}} - (m \rightarrow M) \right) \\ &= \frac{\Gamma(\beta + 1/2)}{\sqrt{\pi} \Gamma(\beta + 1)} \frac{m^{2\beta + 1} x^\beta}{(m^2 x - k^2 x(1-x))^{\beta + 1/2}}. \end{aligned} \quad (\text{A.7})$$

From (A.2), one finds

$$\begin{aligned} & \int dM \rho(M) \left(\frac{1}{(m^2 x - k^2 x(1-x))^{3/2}} - (m \rightarrow M) \right) \\ &= \frac{2\Gamma(\beta + 3/2)}{\sqrt{\pi}\Gamma(\beta + 1)} \frac{m^{2\beta} x^\beta}{(m^2 x - k^2 x(1-x))^{\beta+3/2}}, \end{aligned} \quad (\text{A.8})$$

$$\begin{aligned} & \int dM \rho(M) \left(\frac{m}{(m^2 x - k^2 x(1-x))^{3/2}} - (m \rightarrow M) \right) \\ &= \frac{2\Gamma(\beta + 3/2)}{\sqrt{\pi}\Gamma(\beta + 1)} \frac{m^{2\beta+1} x^\beta}{(m^2 x - k^2 x(1-x))^{\beta+3/2}}, \end{aligned} \quad (\text{A.9})$$

and also

$$\begin{aligned} & \int dM \rho(M) \left(\frac{1}{(m^2 x - k^2 x(1-x) + \tau^2(1-x))^{1/2}} - (m \rightarrow M) \right) \\ &= \frac{\Gamma(\beta + 1/2)}{\sqrt{\pi}\Gamma(\beta + 1)} \frac{m^{2\beta} x^\beta}{(m^2 x - k^2 x(1-x) + \tau^2(1-x))^{\beta+1/2}}. \end{aligned} \quad (\text{A.10})$$

The last integral we will need is lengthy and, therefore, we will not give it in its full complexity. Happily, in Sec. 5 only singular terms when $p^2 \rightarrow m^2$ are necessary

$$\begin{aligned} & \int dM \rho(M) \left(\frac{1}{m^2 - p^2} \frac{1}{(m^2 x - p^2 x(1-x) + \tau^2(1-x))^{1/2}} - (m \rightarrow M) \right) \\ & \approx \frac{m^{2\beta}}{(p^2 x^2 + \tau^2(1-x))^{1/2}} \frac{1}{(m^2 - p^2)^{\beta+1}}. \end{aligned} \quad (\text{A.11})$$

Appendix B

Parametric integrals

The parametric integrals needed for the vacuum polarization tensor in Sec. 4 are

$$\int_0^1 dx \frac{x(1-x)}{(1-x(1-x)k^2/m^2)^{\beta+1/2}} = \frac{1}{6} {}_2F_1(2, \beta + 1/2; 5/2; k^2/4m^2), \quad (\text{B.1})$$

$$\int_0^1 dx \frac{1}{(1-x(1-x)k^2/m^2)^{\beta+1/2}} = {}_2F_1(1, \beta + 1/2; 3/2; k^2/4m^2). \quad (\text{B.2})$$

In Sec. 5, there appear two other parametric integrals, we denoted by \mathcal{S}_1 and \mathcal{S}_2 (formulae (52) and (53)), which may, after elementary calculation, be expressed by simple functions

$$\begin{aligned}\mathcal{S}_1 &= m\mathcal{K}_1(m^2, \tau^2) + 2\tau\mathcal{K}_2(m^2, \tau^2) = \int_0^1 dx \frac{m(x+2) + 2\tau}{(m^2x^2 + \tau^2(1-x))^{1/2}} \\ &= 1 - \frac{\tau}{m} + \left(2 + \frac{2\tau}{m} + \frac{\tau^2}{2m^2}\right) \ln(2m/\tau + 1),\end{aligned}\tag{B.3}$$

$$\begin{aligned}\mathcal{S}_2 &= \mathcal{K}_1(m^2, \tau^2) + 2m^2 \frac{\partial \mathcal{K}_1(m^2, \tau^2)}{\partial m^2} + 4m\tau \frac{\partial \mathcal{K}_2(m^2, \tau^2)}{\partial m^2} \\ &= -\tau \int_0^1 dx \frac{(2m+\tau)x^2 + \tau(x-2)}{(m^2x^2 + \tau^2(1-x))^{1/2}} = \frac{1}{m} \left[2 + \frac{2\tau}{m} - \left(\frac{2\tau}{m} + \frac{\tau^2}{m^2} \right) \ln(2m/\tau + 1) \right],\end{aligned}\tag{B.4}$$

where \mathcal{K}_1 and \mathcal{K}_2 were defined in equations (48) and (49).

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