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RADIATIVE $S \to V\gamma$ VERTEX IN QCD SUM RULES

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We calculate the coupling constants $g_{SV\gamma}$, where S and V denote scalar, $a_0(980)$ and $f_0(980)$, mesons and vector, ρ and ω, mesons in QCD sum rules. A comparison of our estimates on the coupling constants with the results existing in the literature is also presented.

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1. Introduction

It is important to study the light scalar mesons $f_0(980)$ and $a_0(980)$ in hadron physics. For isoscalar $f_0(980)$ and isovector $a_0(980)$, the nature and the quark substructure have been a subject of discussion. Besides the questions of their nature and their properties, the roles of scalar mesons in the hadronic processes are interesting and popular subject. The radiative decay $V \to PP'\gamma$ with $V = \phi, \omega, \rho$ and $P = \pi^0, \eta, K^0$ have been studied to derive relevant information on the properties of the scalar mesons, $f_0(980)$, $a_0(980)$ and σ. Especially, the decays $\phi \to \pi^0\pi^0\gamma$, $\phi \to \pi^0\eta\gamma$, $\phi \to K^0\overline{K}^0\gamma$ and $\rho \to \pi^0\pi^0\gamma$ are valuable processes for these scalar mesons, while the decays $\omega \to \pi^0\pi^0\gamma$, $\omega \to \pi^0\eta\gamma$ and $\rho \to \pi^0\eta\gamma$ cannot be considered because their scalar meson contributions are too small [1]. Nevertheless, in order to recognize to the scalar mesons, $f_0(980)$ and $a_0(980)$, we can also investigate the radiative decays involving the scalar mesons, such as, $f_0 \to \omega\gamma$, $f_0 \to \rho\gamma$, $a_0 \to \omega\gamma$ and $a_0 \to \rho\gamma$ in different theoretical methods. Although an experimental analysis of the reactions $f_0 \to \omega\gamma, \rho\gamma$ and $a_0 \to \omega\gamma, \rho\gamma$ has been analyzed at WASA at COSY [2], we have not more definite experimental data for these decays recently. However, an experimental data are important that they will include the source of information to the solution of the scalar meson

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puzzle. On the theoretical side, we can find different predictions for these decays in [3–6]. Let us back to our main problem which is analyzed the transition constants of the $S \to V\gamma$ decays.

The coupling constants $g_{SV\gamma}$ were already studied in the literature [7–11]. For the coupling constants $g_{a_0\omega\gamma}$ and $g_{a_0\rho\gamma}$, the results given in Ref. [7] were $g_{a_0\omega\gamma} = 0.75 \pm 0.20$ and $g_{a_0\rho\gamma} = 2.00 \pm 0.50$, and also $g_{a_0\omega\gamma} = 0.45 \pm 0.10$ and $g_{a_0\rho\gamma} = 1.30 \pm 0.30$. In addition, the coupling constant $g_{f_0\rho\gamma}$ was analyzed and found the values in different intervals as $1.99 \pm 0.56 \leq\mid g_{f_0\rho\gamma} \mid\leq 3.86 \pm 0.97$ and $1.97 \pm 0.57 \leq\mid g_{f_0\rho\gamma} \mid\leq 1.87 \pm 0.54$ [10]. And, the coupling constant $g_{a_0\rho\gamma}$ was calculated as $0.82 \pm 0.34 \leq\mid g_{a_0\rho\gamma} \mid\leq 0.85 \pm 0.36$ and $1.97 \pm 0.57 \leq\mid g_{a_0\rho\gamma} \mid\leq 0.85 \pm 0.36$ in [11]. Here, differences in the results came from values of some parameters including the methods.

The aim of this work is to calculate the coupling constants $g_{SV\gamma}$ in QCD sum rules which was suggested by Shifman, Vainstain and Zakharov [12]. This method has been one of the most effective tools to study low energy properties of hadrons. Furthermore, we also investigate the $S \to V\gamma$ decays with $V = \rho$ and ω and $S = f_0$ and a_0 using the obtained values of the coupling constants.

2. Formalism

In order to calculate the radiative $S \to V\gamma$ transition constants in the framework of the QCD sum rules, we start with the following correlator

$$\prod\nolimits_{\alpha\beta}(p,p') = \int d^4x\, d^4y e^{ip.y} e^{-ip'x} \left\langle 0 \left| T\left\{ J^{\gamma}_{\alpha}(0) J^{V}_{\beta}(x) J_S(y) \right\} \right| 0 \right\rangle . \quad (1)$$

Here, J^V_β and J_S are the quark currents that create the vector mesons, $V = \omega$ and ρ and scalar mesons $S = a_0$ and f_0, respectively

$$\begin{aligned} J_{a_0} &= (1/2)\left(\overline{u}u - \overline{d}d\right) , \\ J_{f_0} &= (1/\sqrt{2})\left(\overline{u}u + \overline{d}d\right) , \\ J^{\omega}_{\alpha} &= (1/2)\left(\overline{u}\gamma_\alpha u + \overline{d}\gamma_\alpha d\right) , \\ J^{\rho}_{\alpha} &= (1/2)\left(\overline{u}\gamma_\alpha u \;-\; \overline{d}\gamma_\alpha d\right) , \end{aligned} \quad (2)$$

and $J^{\gamma}_{\alpha} = (e_u \overline{u}\gamma_\alpha u + e_d \overline{d}\gamma_\alpha d)$ is the electromagnetic current.

In order to calculate the coupling constants $g_{SV\gamma}$, we use the relevant Feynman diagrams shown in Fig. 1. The diagram in Fig. 1 (d) gives a new contribution which was not considered in the similar calculation of the coupling constant so far. Hence, we use the definition of the term as

$$\begin{aligned} &\int d^4z\, e^{iqz} \left\langle 0 \left| T\left\{ J^{\mu}_{\rm el.}(z) \overline{q}(x) \sigma_{\alpha\beta} q(0) \right\} \right| 0 \right\rangle \\ &= i e_q \langle \overline{q}q \rangle (g_{\mu\alpha} q_\beta - g_{\mu\beta} q_\alpha)\chi + O(x) . \end{aligned} \quad (3)$$

We obtain also the following expression from Eq. (1)

$$\prod_{\mu\alpha} = -i\chi N_c e_q \langle \overline{q}q\rangle \frac{1}{p^2}(p_\alpha q_\mu - g_{\mu\alpha} p\, q)\,, \tag{4}$$

where χ is the magnetic susceptibility of the quark condensate. When applying double Borel transformations, the term in Eq. (4) goes to zero. Therefore, to find the contribution which is from Fig. 1 (d), we follow the references [13, 14]. The expression in Eq. (3) is converted into an expression on the Light Cone as in [13, 14]

$$\begin{aligned} &\int d^4z\; e^{iqz}\;\; \langle 0|T\{J^\mu_{\text{el.}}(z)\overline{q}(x)\sigma_{\alpha\beta}q(0)\}|0\rangle \\ &= ie_q\langle\overline{q}q\rangle \int du e^{iuqx}\{(g_{\mu\alpha}q_\beta - g_{\beta\mu}q_\alpha)[\chi\varphi_\gamma(u) + \ldots]\}\,, \end{aligned} \tag{5}$$

where e_q is the quark charge, the function $\varphi_\gamma(u)$ is the leading twist-2 photon wave function. Hence, using this form of equation, we obtain the contribution part including the magnetic susceptibility as follows

$$A_\chi = ie_q\chi \langle\overline{q}q\rangle\, \varphi_\gamma(u_0)M^2 \left(1 - e^{-s_0/M^2}\right)\,, \tag{6}$$

where the term $(1 - e^{-\frac{s_0}{M^2}})$ is the function used to subtract the continuum, s_0 is the continuum threshold, and

$$u_0 = \frac{M_2^2}{M_1^2 + M_2^2}\,, \qquad M^2 = \frac{M_1^2 M_2^2}{M_1^2 + M_2^2}\,. \tag{7}$$

Here, M_1^2 and M_2^2 are the Borel parameters in the vector mesons V, ρ and ω, and scalar mesons, a_0 and f_0, respectively. We will set $M_1^2 = M_2^2 = 2M^2$ because the masses of the vector mesons and the scalar mesons which are analysed in this paper, are very close to each other. Thus, we obviously get that $U_0 = 1/2$.

We now add the ideas discussed above to calculate the coupling constant $g_{SV\gamma}$. To begin with, consider the phenomenological part of the sum rule. We introduce the overlap amplitude of the vector mesons and the scalar mesons

$$\begin{aligned} \left\langle V\left(p'\right)\,\middle|\, J^V_\beta \,\middle|\, 0\right\rangle \;&=\; \lambda_V U_\beta\,, \\ \langle 0\,|J_S|\,S(p)\rangle \;&=\; \lambda_S\,, \end{aligned} \tag{8}$$

where U_β is the polarization vector of the vector meson. The physical part of the correlation function is given by

$$\prod_{\alpha\beta}(p,p') = \frac{\langle 0\,|J_S|\,S(p)\rangle \left\langle S(p)|J^\alpha_\gamma|V(p')\right\rangle \left\langle V(p')|J^V_\beta|0\right\rangle}{\left(p^2 - m_S^2\right)\left(p'^2 - m_V^2\right)} + \ldots \tag{9}$$

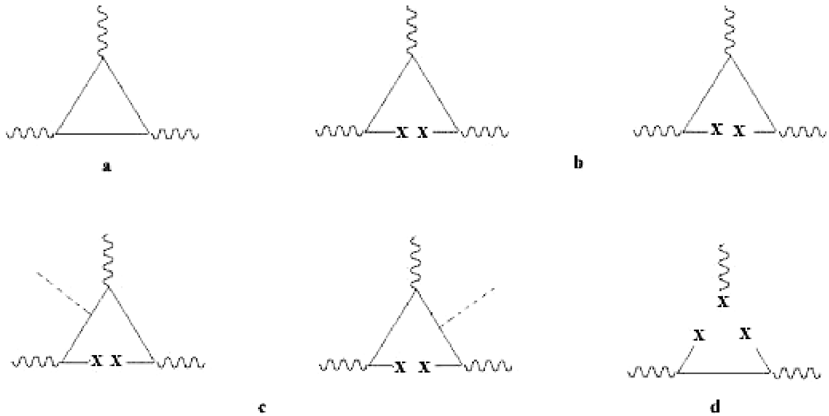

Fig. 1. Feynman diagrams for the $SV\gamma$ vertex.

The dots denote the contributions from the higher states and the continuum. Substituting Eq. (8) into Eq. (9), we arrive at the expression for the phenomenological part of the sum rules as

$$\prod_{\alpha\beta}^{\text{phen.}} = -i\frac{e}{M_V} g_{SV\gamma} \frac{\lambda_S \lambda_V U_\beta}{\left(p^2 - m_S^2\right)\left(p'^2 - m_S^2\right)} \left(p\, q U_\alpha - U\, q p_\alpha\right) \tag{10}$$

with $p' = p+q$. The $g_{SV\gamma}$ coupling constants are defined through the relation

$$\left\langle S(p)\left| J_\alpha^\gamma \right| V(p')\right\rangle = -i\frac{e}{M_V} g_{SV\gamma} K\left(q^2\right)\left(p\, q U_\alpha - U\, q p_\alpha\right)\,, \tag{11}$$

where $K(q^2)$ is the form factor with $K(0) = 1$.

Let us now turn to the theoretical part. Thus, we consider the perturbative contribution and the power corrections from operators of different dimensions to the correlator. First, we consider the perturbative part. Its contribution is given by one diagram of the lowest-order bare loop in Fig. 1 (a). Next, the power corrections from the operators which are proportional to vacuum condensate are considered. We restrict ourselves to the operators of dimensions $d = 3$ and $d = 5$ only since we work in the limit of $m_q = 0$ and in this limit only operators of these dimensions make contributions. The contribution of the power corrections from operators of different dimensions are given by one diagram of the dimension $d = 3$ in Fig. 1 (b) and three diagrams of the dimension $d = 5$ in Fig. 1 (c). The explicit calculation of the expression corresponding to Fig. 3 (left) gives zero in the limit $m_q = 0$.

The expression of the coupling constants $g_{f_0 V\gamma}$ and $g_{a_0 V\gamma}$ in QCD sum rules are obtained by equating the theoretical and the phenomenological part of the correlator then, the double Borel transformations of the variables $Q^2 = -p^2$ and $Q'^2 = -p'^2$ on both sides. Finally, we obtain the following expressions for the coupling constants, $g_{SV\gamma}$,

$$g_{f_0 V\gamma} e^{-\frac{m_{f_0}^2}{M_1^2}} e^{-\frac{m_V^2}{M_2^2}} = \frac{1}{2\sqrt{2}}\{g\}\left[A_\chi + 3 + m_0^2\left[\frac{3}{8M_1^2} + \frac{5}{8M_2^2}\right]\right], \quad (12)$$

$$g_{a_0 V\gamma} e^{-\frac{m_{a_0}^2}{M_1^2}} e^{-\frac{m_V^2}{M_2^2}} = \{g\}\left[A_\chi + \frac{3}{4} + m_0^2\left[\frac{3}{32M_1^2} + \frac{5}{32M_2^2}\right]\right]. \quad (13)$$

where $\{g\} = \frac{m_V}{\lambda_S \lambda_V}(e_u \mp e_d)\langle \bar{q}q\rangle$ is the term including overlap amplitude and quark charges, and $[A_\chi = -\varphi_\gamma(u_0)\chi M^2(1 - e^{-\frac{s_0}{M^2}})]$ is the term coming from the definition of the magnetic susceptibility.

In order to analyze the obtained sum rule, we consider two models, Model I and Model II. The contributions coming from the Feynman diagrams shown in Fig. 1 (a), (b) and (c) are considered in Model I, and Model II is constructed by adding the contribution coming from the Feynman diagram in Fig. 1 (d) to Model I.

3. Results

Using the following numerical values for the various parameters, we obtain the values of the coupling constants. For the parameters the following values are taken: $m_0^2 = (0.8 \pm 0.02)$ GeV^2, $\langle \bar{u}u\rangle = \langle \bar{d}d\rangle = (-0.014 \pm 0.002)$ GeV^3 [15], $m_{f_0} = 0.98$ GeV, $m_{a_0} = 0.98$ GeV, $m_\omega = 0.78$ GeV and $m_\rho = 0.77$ GeV. We use the overlap amplitude for the scalar mesons, f_0 and a_0, and the vector mesons ω and ρ as $\lambda_{f_0} = (0.18 \pm 0.015)$ GeV^2 [16], $\lambda_{a_0} = (0.21 \pm 0.05)$ GeV^2 [7], $\lambda_\rho = (0.17 \pm 0.01)$ GeV^2, $\lambda_\omega = (0.16 \pm 0.01)$ GeV^2 [17]. Furthermore, we analyze the dependence of the sum rule for the coupling constants, $g_{SV\gamma}$, on independent variations of the continuum threshold S_0 and the Borel parameters M_1^2 and M_2^2. For the vector channel, the limits $M_1^2 = 1.2$ GeV^2 and 1.0 GeV$^2 \leq M_2^2 \leq 1.4$ GeV^2 determine the allowed interval [18]. In Fig. 2 and Fig. 3, we present the coupling constants $g_{f_0\omega\gamma}$, $g_{f_0\rho\gamma}$, $g_{a_0\omega\gamma}$ and $g_{a_0\rho\gamma}$ as a function of the Borel parameter M_2^2 for different values of M_1^2. From the graphs in Fig. 2 and Fig. 3, we see that the variations of the coupling constants $g_{SV\gamma}$ as a function of the Borel parameters M_1^2 for different values of M_2^2 are quite steady. Hence, the determined values for the coupling constants, $g_{SV\gamma}$, in Model I are

$$\begin{aligned} g_{f_0\omega\gamma} &= 0.81 \pm 0.14\,, \qquad g_{a_0\omega\gamma} = 1.48 \pm 0.36\,, \\ g_{f_0\rho\gamma} &= 2.23 \pm 0.40\,, \qquad g_{a_0\rho\gamma} = 0.45 \pm 0.14\,, \end{aligned} \quad (14)$$

where errors come from the values of the overlap amplitudes λ_{f_0}, λ_{a_0}, λ_ω and λ_ρ and the values of vacuum condensates and the Borel parameters.

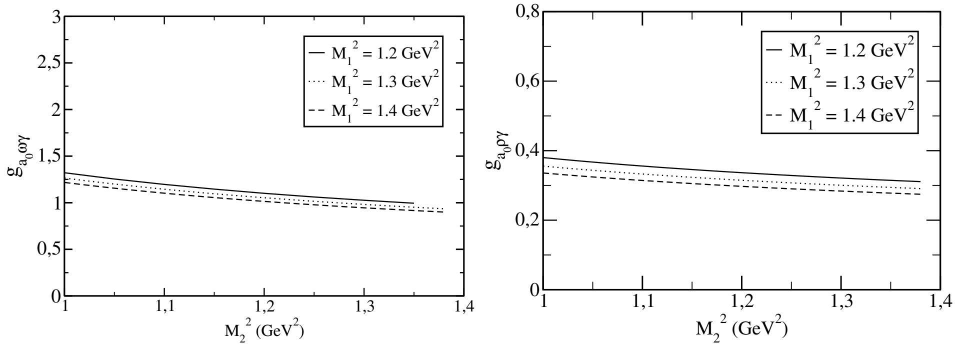

Fig. 2. The coupling constants $g_{a_0\omega\gamma}$ and $g_{a_0\rho\gamma}$ as a function of the Borel parameter M_2^2 for different values of M_1^2.

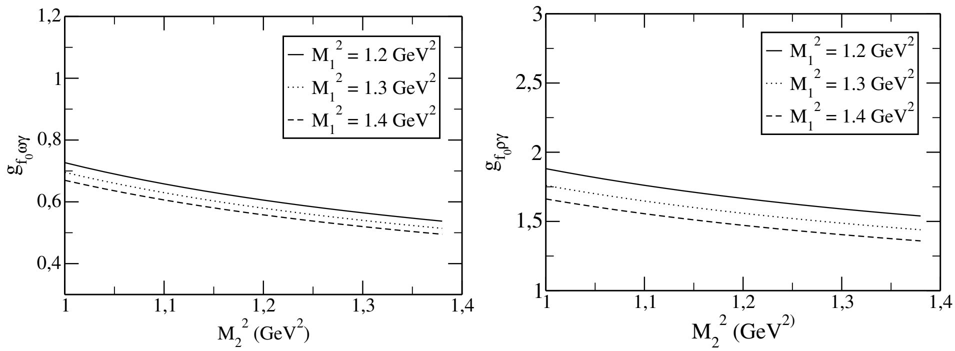

Fig. 3. The coupling constants $g_{f_0\omega\gamma}$ and $g_{f_0\rho\gamma}$ as a function of the Borel parameter M_2^2 for different values of M_1^2.

Model II includes the term which has a parameter called the magnetic susceptibility of the quark condensate, χ, and for which we use the value $\chi = (-3.15 \pm 0.10)$ GeV^2 [19]. In Fig. 4 and Fig. 5, we present the dependence of the coupling constants, $g_{SV\gamma}$, on the Borel parameters $M_1^2 = 1$ GeV^2 at the values of the continuum threshold $s_0 = 2.0$, 2.2 and 2.4 GeV^2. These graphs show that the sum rule is quite stable with the variations of M^2. Then, we obtain the coupling constants $g_{SV\gamma}$ in Model II as

$$\begin{aligned} g_{f_0\omega\gamma} &= 1.31 \pm 0.25\,, \qquad g_{a_0\omega\gamma} = 2.39 \pm 0.54\,, \\ g_{f_0\rho\gamma} &= 3.61 \pm 0.72\,, \qquad g_{a_0\rho\gamma} = 0.72 \pm 0.21\,, \end{aligned} \tag{15}$$

where, in addition to previous uncertainty, errors come from the variation of the continuum threshold and the magnetic susceptibility of the quark condensate. These values are different than the previous values of the coupling constants, $g_{SV\gamma}$. These differences can arise from the term including the parameter χ whose contribution is dominant in Model II.

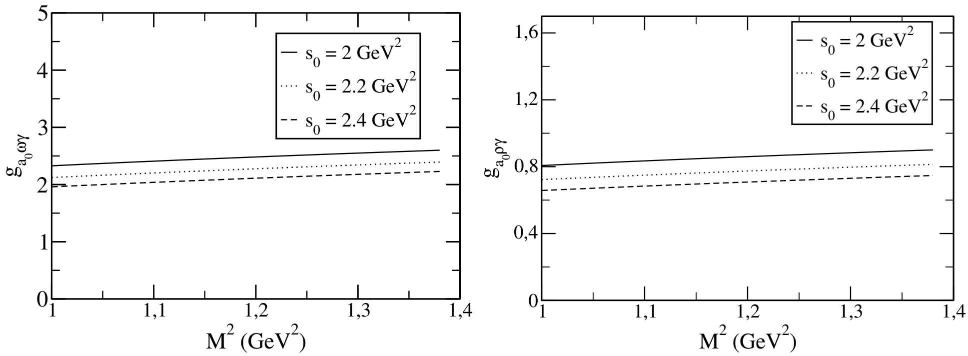

Fig. 4. The dependence of the coupling constants $g_{a_0\omega\gamma}$ and $g_{a_0\rho\gamma}$ on the Borel parameter M^2 for the values of the continuum threshold parameter $s_0 = 2$ GeV^2, $s_0 = 2.2$ GeV^2 and $s_0 = 2.4$ GeV^2.

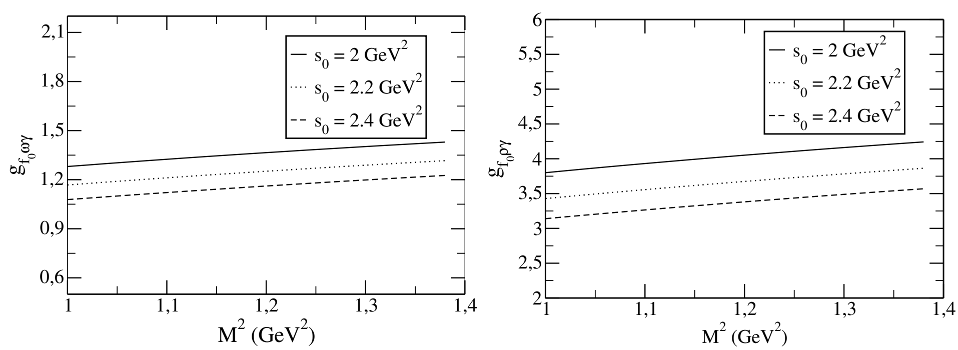

Fig. 5. The dependence of the coupling constants $g_{f_0\omega\gamma}$ and $g_{f_0\rho\gamma}$ on the Borel parameter M^2 for the values of the continuum threshold parameter $s_0 = 2$ GeV^2, $s_0 = 2.2$ GeV^2 and $s_0 = 2.4$ GeV^2.

In Tables I and II, we present the other obtained values for the coupling constants, $g_{SV\gamma}$, and what we have obtained in this work. The results shown in these tables indicate that the values obtained in the present work are compatible with the other studies in the literature.

TABLE I

The obtained values of the coupling constants $g_{f_0V\gamma}$ in Model I and Model II, and the values in the literature.

Ref.	$g_{f_0\omega\gamma}$
[9]	(0.68 ∓ 0.02)–(1.25 ∓ 0.02)
[9]	(0.78 ∓ 0.02)–(1.30 ∓ 0.02)
[9]	(0.69 ∓ 0.02)–(0.71 ∓ 0.02)
Model I	0.81 ∓ 0.14
Model II	1.31 ∓ 0.25
Ref.	$g_{f_0\rho\gamma}$
[10]	(1.99 ∓ 0.56)–(3.86 ∓ 0.97)
[10]	(1.97 ∓ 0.57)–(1.87 ∓ 0.54)
[11]	(1.75 ∓ 0.53)–(1.12 ∓ 0.34)
Model I	2.23 ∓ 0.40
Model II	3.61 ∓ 0.72

TABLE II

The obtained values of the coupling constants $g_{a_0\mathrm{V}\gamma}$ in Model I and Model II. Comparison of different predictions and present work.

Ref.	$g_{a_0\omega\gamma}$
[7]	0.75 ∓ 0.20
[7]	0.45 ∓ 0.10
[8]	2.57 ∓ 0.21
Model I	1.48 ∓ 0.36
Model II	2.39 ∓ 0.54
Ref.	$g_{a_0\rho\gamma}$
[7]	2.0 ∓ 0.50
[7]	1.30 ∓ 0.30
[11]	(0.96 ∓ 0.44)–(0.85 ∓ 0.38)
[10]	(0.82 ∓ 0.34)–(0.85 ∓ 0.36)
[10]	(1.97 ∓ 0.57)–(0.85 ∓ 0.36)
Model I	0.45 ∓ 0.14
Model II	0.72 ∓ 0.21

In Tables I and II, the values of the coupling constants, $g_{f_0\omega\gamma}$ [9], $g_{f_0\rho\gamma}$ [11], $g_{a_0\omega\gamma}$ [7] and $g_{a_0\rho\gamma}$ [7, 11] were obtained with the Feynman diagrams which we used in Model I. We can see from the tables that the results of the coupling constants are different from our obtained values. These differences come from the different sign in the equation obtained for the coupling constants, and also, from the different values of the some input parameters which are used in calculation. We can use the obtained results of the coupling constants for analyzing the radiative decays of scalar mesons into vector mesons, $S \to V\gamma$. Thus, we get some values of the decay widths of the decays $S \to V\gamma$, to compare with other analysis. Anyway, the radiative decays involving the scalar mesons were studied in [3, 5, 6, 20–22]. Hence, we can study a vector meson dominance model for the decay amplitudes of the types $S \to V\gamma$, where S and V denote the scalar and vector mesons considering the tree-level diagrams for the process. In order to compute the amplitude for $S \to V\gamma$ decay, we describe the $SV\gamma$-vertex by the effective Lagrangian as [23]

$$L_{SV\gamma} = \frac{e}{M_v} g_{SV\gamma} \partial^{\alpha} V^{\beta} (\partial_{\alpha} A_{\beta} - \partial_{\beta} A_{\alpha}) S \,, \tag{16}$$

where V denotes the vector meson, ω and ρ, field, A is a photon field, S is a scalar meson, a_0 and f_0, field. From this Lagrangian, we can derive the corresponding partial width as follows

$$\Gamma(S \to V\gamma) = \frac{\alpha}{24} g^2_{SV\gamma} \frac{\left(M_S^2 - M_V^2\right)^3}{M_S^5} \,, \tag{17}$$

where $g_{SV\gamma}$ are the coupling constants for the decays $S \to V\gamma$. Using this expression, we would like to find the decay width of the $S \to V\gamma$ decay, for which we use the values of the coupling constants, $g_{SV\gamma}$ obtained in the present work. Therefore, we have the following results for Model I

$$\begin{aligned} \Gamma(f_0 \to \omega\gamma) &= 10.07 \pm 4 \text{ keV} \,, \\ \Gamma(f_0 \to \rho\gamma) &= 86.70 \pm 14 \text{ keV} \,, \\ \Gamma(a_0 \to \omega\gamma) &= 33.65 \pm 6.3 \text{ keV} \,, \\ \Gamma(a_0 \to \rho\gamma) &= 3.53 \pm 1.0 \text{ keV} \,, \end{aligned} \tag{18}$$

and for Model II

$$\begin{aligned} \Gamma(f_0 \to \omega\gamma) &= 25.18 \pm 5.4 \text{ keV} \,, \\ \Gamma(f_0 \to \rho\gamma) &= 217.63 \pm 30 \text{ keV} \,, \\ \Gamma(a_0 \to \omega\gamma) &= 83.82 \pm 13 \text{ keV} \,, \\ \Gamma(a_0 \to \rho\gamma) &= 8.65 \pm 3.17 \text{ keV} \,. \end{aligned} \tag{19}$$

There is a wide hierarchy between $\Gamma(f_0 \to \omega\gamma)$, and $\Gamma(f_0 \to \rho\gamma)$ and between $\Gamma(a_0 \to \omega\gamma)$ and $\Gamma(a_0 \to \rho\gamma)$. Let us compare our predictions on partial widths of the $S \to V\gamma$ decays with the existing theoretical results and experimental values in the literature. Hence, an experimental measurement of these resonances would be very valuable to analyze these decays and the theoretical models are improved. Experimental data about these processes have not been found yet, therefore, we list all theoretical results in Table III. We can see that there is a large dispersion among the different theoretical models, and to solve the differences, studying of these radiative decays is very important and interesting to the theoretical models.

TABLE III

Comparison of the QCD sum rule prediction for the decay width of $S \to V\gamma$ decays with other approaches. The unit is in keV.

Ref.	$f_0 \to \omega\gamma$	$f_0 \to \rho\gamma$	$a_0 \to \omega\gamma$	$a_0 \to \rho\gamma$
[3] [I]	126 ± 20	19 ± 5	641 ± 87	3 ± 1
[3] [II]	88 ± 17	3.3 ± 2	641 ± 87	3 ± 1
[5]	$14.1^{+21.5}_{-7.8}$	$24.1^{+21.5}_{-11.6}$	$11.6^{+5.2}_{-0.8}$	$12.4^{+5.5}_{-0.8}$
[6]	59.67	81.43	114.7	8.47
[20]	3.4	3.4	3.4	3.4
[21] [Est. I]	59.4	50.8	34.9	36.4
[21] [Est. II]	31.7	3.8	44.6	46.6
[21] [Est. III]	63.4	66.4	44.6	46.6
[22]	4.3 ± 1.3	4.2 ± 1.1	31 ± 13	11 ± 4
This work Model I	10.07 ± 4	86 ± 14	33.65 ± 6.3	3.53 ± 1
This work Model II	25.96 ± 5.4	198 ± 30	86.29 ± 13	9.03 ± 3.17

The scalar mesons, $f_0(980)$ and $a_0(980)$, have an experimental decay width as $\Gamma_{f_0} = (40 \pm 100)$ MeV and $\Gamma_{a_0} = (50 \pm 100)$ MeV, respectively [24]. Using the experimental values of the decay width of the f_0 and a_0 mesons we can determine some values for the branching ratio of the decay $S \to VP$. Calculated values of the branching ratios of the decays are shown in Tables IV and V.

In conclusion, we analyzed the coupling constants $g_{SV\gamma}$ of the $S \to V\gamma$ transition by employing the QCD sum rules and we propose that it should be add a term including magnetic susceptibility, when these coupling constants are studied. The obtained values for the coupling constants $g_{SV\gamma}$ are given in Table I and Table II with the relative errors. Our results are generally in agreement with the other predictions. The obtained values will not be compared with the experimental data at present. We expect that it could be done in the near future.

TABLE IV

The branching ratios of the decays $S \to V\gamma$ for the values of the decay widths $\Gamma(f_0 \to \omega\gamma) = 10.7$ keV $\Gamma(f_0 \to \rho\gamma) = 86.70$ keV, $\Gamma(a_0 \to \omega\gamma) = 33.65$ keV and $\Gamma(a_0 \to \rho\gamma) = 3.53$ keV in Model I.

$\Gamma_{f_0}^{\rm tot}$ [MeV]	$\mathrm{Br}(f_0 \to \omega\gamma)$	$\Gamma_{f_0}^{\rm tot}$ [MeV]	$\mathrm{Br}(f_0 \to \rho\gamma)$
40	2.51×10^{-4}	40	2.16×10^{-3}
50	2.01×10^{-4}	50	1.73×10^{-3}
60	1.67×10^{-4}	60	1.44×10^{-3}
70	1.43×10^{-4}	70	1.23×10^{-3}
80	1.25×10^{-4}	80	1.08×10^{-3}
90	1.11×10^{-4}	90	9.63×10^{-4}
100	1.00×10^{-4}	100	8.67×10^{-4}
$\Gamma_{a_0}^{\rm tot}$ [MeV]	$\mathrm{Br}(a_0 \to \omega\gamma)$	$\Gamma_{a_0}^{\rm tot}$ [MeV]	$\mathrm{Br}(a_0 \to \rho\gamma)$
50	6.73×10^{-4}	50	7.06×10^{-5}
60	5.60×10^{-4}	60	5.88×10^{-5}
70	4.80×10^{-4}	70	5.04×10^{-5}
80	4.20×10^{-4}	80	4.41×10^{-5}
90	3.73×10^{-4}	90	3.92×10^{-5}
100	3.36×10^{-4}	100	3.53×10^{-5}

TABLE V

The branching ratios of the decays $S \to V\gamma$ for the values of the decay widths $\Gamma(f_0 \to \omega\gamma) = 25.18$ keV $\Gamma(f_0 \to \rho\gamma) = 217$ keV, $\Gamma(a_0 \to \omega\gamma) = 83.82$ keV and $\Gamma(a_0 \to \rho\gamma) = 8.65$ keV in Model II.

$\Gamma_{f_0}^{\rm tot}$ [MeV]	$\mathrm{Br}(f_0 \to \omega\gamma)$	$\Gamma_{f_0}^{\rm tot}$ [MeV]	$\mathrm{Br}(f_0 \to \rho\gamma)$
40	6.29×10^{-4}	40	4.75×10^{-3}
50	5.03×10^{-4}	50	3.96×10^{-3}
60	4.19×10^{-4}	60	3.30×10^{-3}
70	3.59×10^{-4}	70	2.82×10^{-3}
80	3.14×10^{-4}	80	2.47×10^{-3}
90	2.79×10^{-4}	90	2.20×10^{-3}
100	2.51×10^{-4}	100	1.98×10^{-3}
$\Gamma_{a_0}^{\rm tot}$ [MeV]	$\mathrm{Br}(a_0 \to \omega\gamma)$	$\Gamma_{a_0}^{\rm tot}$ [MeV]	$\mathrm{Br}(a_0 \to \rho\gamma)$
50	1.67×10^{-3}	50	1.80×10^{-4}
60	1.39×10^{-3}	60	1.50×10^{-4}
70	1.19×10^{-3}	70	1.29×10^{-4}
80	1.04×10^{-3}	80	1.12×10^{-4}
90	9.31×10^{-4}	90	1.00×10^{-4}
100	8.38×10^{-4}	100	9.03×10^{-5}

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REFERENCES

[1] R. Escribano, *Phys. Rev.* **D74**, 114020 (2006).

[2] M. Buscher, *AIP Conf. Proc.* **1030**, 40 (2008) [`arXiv:0804.2452 [hep-ph]`].

[3] D. Black, M. Harada, J. Schechter, `arXiv:hep-ph/0306065`.

[4] M. Harada, `arXiv:hep-ph/0408189`.

[5] R. Escribano, P. Masjuan, J. Nadal, *Phys. Lett.* **B670**, 27 (2008) [`arXiv:0806.3007 [hep-ph]`].

[6] M.K. Volkov, Yu.M. Bystritskiy, E.A. Kuraev, `arXiv:0811.3773 [hep-ph]`.

[7] A. Gokalp, O. Yilmaz, *Eur. Phys. J.* **C22**, 323 (2001).

[8] C. Aydin, A.H. Yilmaz, *Mod. Phys. Lett.* **A21**, 1297 (2006).

[9] C. Aydin, A.H. Yilmaz, *Acta Phys. Pol. B* **37**, 1769 (2006).

[10] C. Aydin *et al.*, *Eur. Phys. J.* **C48**, 111 (2006).

[11] C. Aydin *et al.*, *Phys. Rev.* **D73**, 074020 (2006).

[12] M.A. Shifman, A.I. Vainstain, V.I. Zakharov, *Nucl. Phys.* **B147**, 385 (1979).

[13] B.L. Ioffe, A.V. Smilga, *Nucl. Phys.* **B232**, 109 (1984).

[14] I.I. Balitsky, V.M. Braun, *Nucl. Phys.* **B311**, 541 (1988).

[15] V.M. Balyaev, B.L. Ioffe, *Zh. Eksp. Teor. Fiz.* **83**, 876 (1982).

[16] F. De Fazio, M.R. Pennington, *Phys. Lett.* **B520**, 78 (2001).

[17] Shi-lin Zhu, W.-Y.P. Hwang, Ze-Sen Yang, *Phys. Lett.* **B420**, 8 (1998).

[18] V.L. Eletsky, B.L. Ioffe, Ya.I. Kogan, *Phys. Lett.* **B122**, 423 (1983).

[19] P. Ball, V.M. Braun, N. Kivel, *Nucl. Phys.* **B649**, 263 (2003).

[20] Yu. Kalashnikova *et al.*, *Phys. Rev.* **C73**, 045203 (2006) [`arXiv:nucl-th/0512028`].

[21] S. Ivashyn, A.Y. Korchin, *Eur. Phys. J.* **C54**, 89 (2008) [`arXiv:0707.2700 [hep-ph]`].

[22] H. Nagahiro, L. Roca, E. Oset, *Eur. Phys. J.* **A36**, 73 (2008) [`arXiv:0802.0455 [hep-ph]`].

[23] A.I. Titov, T.-S.H. Lee, H. Toki, O. Streltsova, *Phys. Rev.* **C60**, 035205 (1999).

[24] J. Beringer *et al.* [Particle Data Group], *Phys. Rev.* **D86**, 010001 (2012).