

MEASUREMENT OF THE CKM ANGLE γ AT THE LHCb*

AGNIESZKA DZIURDA

on behalf of the LHCb Collaboration

The H. Niewodniczański Institute of Nuclear Physics, Polish Academy of Sciences
Radzikowskiego 152, 31-342 Kraków, Poland

(Received April 16, 2013)

The document reports the first combined measurements of the CKM angle γ from LHCb, performed with b -hadron decays dominated by $b \rightarrow u$ and $b \rightarrow c$ tree-level amplitudes. Precision is achieved by averaging results from $B^\pm \rightarrow Dh^\pm$ ($h = K, \pi$), where $D \rightarrow h^+h^-$, $D \rightarrow K^+\pi^-$, $D \rightarrow K^+\pi^-\pi^+\pi^-$, and $D \rightarrow K_S^0 h^+h^-$. The measurements used data corresponding to an integrated luminosity of 1 fb^{-1} collected in 2011 by LHCb. Using only $B^\pm \rightarrow DK^\pm$ decays results, a best-fit value of $\gamma = (71.1^{+16.6}_{-15.7})^\circ$ is obtained. For the first time, information from $B^\pm \rightarrow D\pi^\pm$ decays is included in the combination. When these results are included, the best-fit value becomes $\gamma = 85.1^\circ$ and we set confidence limits of $\gamma \in [61.8; 67.8]^\circ$ or $[77.9; 92.4]^\circ$ at 68% C.L. and $\gamma \in [43.8; 101.5]^\circ$ at 95% C.L.

DOI:10.5506/APhysPolB.44.1429

PACS numbers: 13.25.Hw, 14.40.Nd

1. Introduction

In the Standard Model (SM), CP violation in weak interactions is described by a single, irreducible phase in the 3×3 Cabibbo–Kobayashi–Maskawa (CKM) mixing matrix [1]. The unitarity of the CKM matrix implies a set of relations among its elements V_{ij} , in particular the condition

$$V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0, \quad (1)$$

which can be presented in the complex plane as a Unitarity Triangle (UT). Overconstraining the unitarity triangle from precise measurements of all its sides and angles is, therefore, a test of the SM.

* Presented at the Cracow Epiphany Conference on the Physics After the First Phase of the LHC, Kraków, Poland, January 7–9, 2013.

The angle $\gamma = \arg(-V_{ud}V_{ub}^*/V_{cd}V_{cb}^*)$ is the least well measured of the CKM angles. The world average value $\gamma = (76 \pm 10)^\circ$ and $\gamma = (66 \pm 12)^\circ$ are found, respectively by UTFit [2] and CKMFitter [3] collaborations. This precision can be significantly improved with large datasets collected by LHCb, and measurements of γ is an important goal of the LHCb physics program.

The CKM angle γ can be determined using purely tree-level decays of the beauty hadrons which are expected to be insensitive to physics beyond the SM. In the case of the tree-level process, the CKM angle γ can be obtained by exploiting the interference between $b \rightarrow u$ and $b \rightarrow c$ transitions in decays of b -hadrons with a charm meson in the final state, such as $B^- \rightarrow Dh^-$, where h^- is a kaon or a pion. The decays with a kaon as a bachelor are more sensitive for γ measurement. In $B^- \rightarrow DK^-$ decays, the colour-favoured $B^- \rightarrow D^0K^-$ and the colour-suppressed $B^- \rightarrow \bar{D}^0K^-$ transitions interfere when the D^0 and \bar{D}^0 decay to a common final state. The two interfering amplitudes differ by a factor $r_B e^{i(\delta_B \pm \gamma)}$, where r_B is the magnitude of the ratio of the amplitudes $A(B^- \rightarrow \bar{D}^0K^-)$ and $A(B^- \rightarrow D^0K^-)$, and δ_B is their relative strong phase.

The relationship between γ and the physical observables depends on the D final state. Three separate methods of extracting the CKM angle γ can be considered: the GLW method [4, 5], where decays to CP eigenstates ($D \rightarrow K^+K^-$ and $D \rightarrow \pi^+\pi^-$) are used, the ADS method [6], where decays to flavour-specific eigenstates ($D \rightarrow K^+\pi^-$ and $D \rightarrow K^+\pi^-\pi^+\pi^-$) are used, and the GGSZ method [9], where self-conjugate three-body final states ($D \rightarrow K_S^0\pi^+\pi^-$ and $D \rightarrow K_S^0K^+K^-$) are used. All of them have been studied by the LHCb using data sample corresponding to an integrated luminosity of 1 fb^{-1} collected in 2011.

2. GLW method

In this method, the D meson is selected in the CP eigenstates ($f_{\text{CP}\pm}$) $D \rightarrow K^-K^+$ or $D \rightarrow \pi^-\pi^+$. The CKM angle γ can then be determined using triangle relation between amplitudes: $A(B^- \rightarrow D^0K^-)$, $A(B^- \rightarrow \bar{D}^0K^-)$ and $A(B^- \rightarrow D_{\text{CP}+}K^-)$, where $D_{\text{CP}} = (D^0 + \bar{D}^0)/\sqrt{2}$. The experimental disadvantage of this method comes from the fact that $B^- \rightarrow \bar{D}^0K^-$ is relatively smaller than $B^- \rightarrow D^0K^-$. The usual GLW observables follow

$$R_{\text{CP}+} = 2 \frac{\Gamma(B^- \rightarrow D_{\text{CP}+}K^-) + \Gamma(B^+ \rightarrow D_{\text{CP}+}K^+)}{\Gamma(B^- \rightarrow D^0K^-) + \Gamma(B^+ \rightarrow \bar{D}^0K^+)}, \quad (2)$$

$$A_{\text{CP}+} = \frac{\Gamma(B^- \rightarrow D_{\text{CP}+}K^-) - \Gamma(B^+ \rightarrow D_{\text{CP}+}K^+)}{\Gamma(B^- \rightarrow D_{\text{CP}+}K^-) + \Gamma(B^+ \rightarrow D_{\text{CP}+}K^+)} \quad (3)$$

and they are connected to the three unknowns γ , r_B and δ_B through

$$R_{\text{CP}+} = 1 + r_B^2 + 2r_B \cos(\delta_B) \cos(\gamma), \quad (4)$$

$$A_{\text{CP}+} = \frac{2r_B \sin(\delta_B) \sin(\gamma)}{R_{\text{CP}+}}. \quad (5)$$

For the $B^- \rightarrow D\pi^-$ decays, analogous relations can be obtained by replacing hadronic parameters r_B and δ_B by $D\pi$ versions r_B^π and δ_B^π . The expected CP asymmetries in the $B^- \rightarrow D\pi^-$ decays are smaller than the corresponding ones in the $B^- \rightarrow DK^-$, since the value of r_B , which controls the size of the CP interference, is naively $\propto 20$ times smaller, but the large yields in the $B^- \rightarrow D\pi^-$ modes help constrain the mass shape in the fit.

The fit results for $D \rightarrow \pi^-\pi^+$ and $D \rightarrow K^-K^+$ are shown in Figs. 1 and 2, respectively. The first evidence of non-zero CP asymmetry, A_{CP} , between decays: $B^- \rightarrow DK^-$ and $B^+ \rightarrow DK^+$ with

$$\begin{aligned} D \rightarrow K^+K^-, \quad A_{\text{CP}}^{(KK)} &= 0.148 \pm 0.037 \pm 0.010, \\ D \rightarrow \pi^+\pi^-, \quad A_{\text{CP}}^{(\pi\pi)} &= 0.135 \pm 0.066 \pm 0.010 \end{aligned}$$

is observed. In average,

$$\begin{aligned} A_{\text{CP}+} &= 0.145 \pm 0.032 \pm 0.010, \\ R_{\text{CP}+} &= 1.007 \pm 0.038 \pm 0.012 \end{aligned}$$

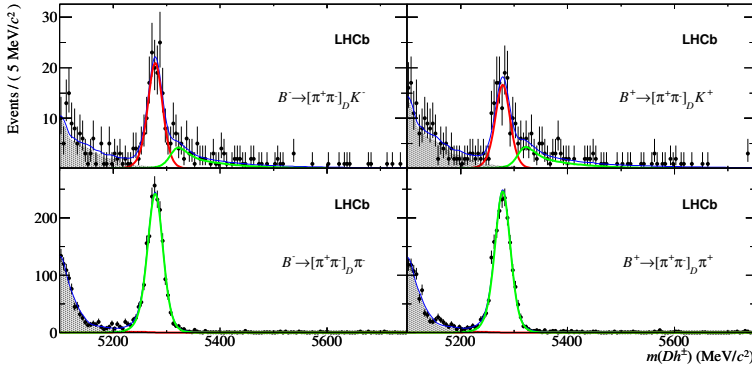


Fig. 1. Invariant mass distributions of selected $B^\pm \rightarrow [\pi^+\pi^-]_D h^\pm$ candidates. The left plots are B^- candidates, B^+ are on the right. In the top plots, the B candidates are reconstructed with a kaon mass hypothesis, in the bottom row and they are reconstructed with a pion mass hypothesis. The dark curve represents the $B^\pm \rightarrow DK^\pm$ events, the light curve $B^\pm \rightarrow D\pi^\pm$. The shaded contribution are partially reconstructed events and the total PDF includes the combinatorial component.

are obtained. In all mentioned results, the first uncertainty is statistical, the second one is systematic. A combined 4.5σ significance for CP violation in these modes is found. No significant asymmetry in the corresponding $B^- \rightarrow D\pi^-$ modes is observed. Further results on GLW are given in Ref. [7].

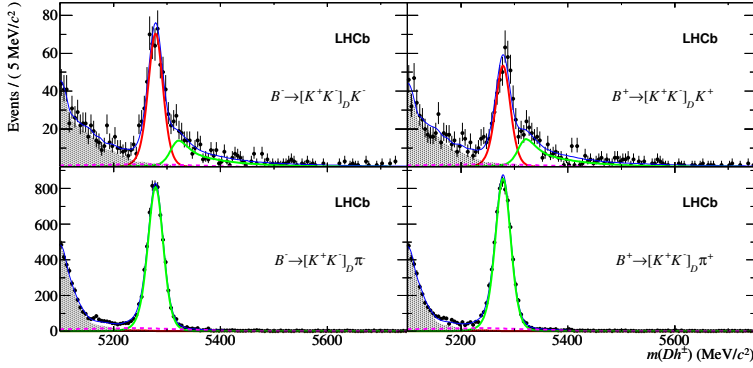


Fig. 2. Invariant mass distributions of selected $B^\pm \rightarrow [K^+K^-]_D h^\pm$ candidates. See the caption of Fig. 1 for a full description. The contribution from $\Lambda_b \rightarrow \Lambda_c^\pm h^\mp$ decays is indicated by the dashed line.

3. ADS method

In the ADS method, the interfering amplitudes have comparable magnitudes and hence large interference effects can occur. In particular, B decay rate is the result of the interference of the colour allowed $B^- \rightarrow D^0 K^-$ decay followed by the doubly Cabibbo suppressed $D^0 \rightarrow \pi^- K^+$ decay, and the colour suppressed $B^- \rightarrow D^0 K^-$ decay followed by the Cabibbo allowed $D^0 \rightarrow \pi^+ K^-$ decay. The same method can be used for $D^0 \rightarrow \pi^- K^+ \pi^- \pi^+$ and $D^0 \rightarrow \pi^+ K^- \pi^- \pi^+$ finale states. The experimental observables are

$$R_{\text{ADS}} = \frac{\Gamma(B^- \rightarrow [f]_D K^-) + \Gamma(B^+ \rightarrow [\bar{f}]_D K^+)}{\Gamma(B^- \rightarrow [\bar{f}]_D K^-) + \Gamma(B^+ \rightarrow [f]_D K^+)}, \quad (6)$$

$$A_{\text{ADS}} = \frac{\Gamma(B^- \rightarrow [f]_D K^-) - \Gamma(B^+ \rightarrow [\bar{f}]_D K^+)}{\Gamma(B^- \rightarrow [f]_D K^-) + \Gamma(B^+ \rightarrow [\bar{f}]_D K^+)}. \quad (7)$$

The observables R_{ADS} and A_{ADS} are related to the physics parameters by the following equations

$$R_{\text{ADS}} = r_B^2 + r_D^2 + 2r_B r_D C_f \cos(\delta_B + \delta_D) \cos(\gamma), \quad (8)$$

$$A_{\text{ADS}} = \frac{2r_B r_D C_f \sin(\delta_B + \delta_D) \sin(\gamma)}{R_{\text{ADS}}}, \quad (9)$$

where $[f]_D$ indicates the final state f from D^0 or \bar{D}^0 meson, r_D is the ratio of the magnitudes of D decay amplitudes $r_D = |A(D^0 \rightarrow f)|/|A(\bar{D}^0 \rightarrow f)|$, δ_D is the difference of the strong phases and C_f is a coherence factor equal to 1 for $D^0 \rightarrow \pi^\pm K^\pm$ and between 0 and 1 for $D^0 \rightarrow \pi^\pm K^\mp \pi^- \pi^+$. This method can be also adapted to $B^- \rightarrow D\pi^-$ decays.

The LHCb has published measurements of CP violation in both 2-body $D^0 \rightarrow \pi^\pm K^\pm$ [7] and 4-body $D^0 \rightarrow \pi^\pm K^\mp \pi^- \pi^+$ [8] final states. The D hadronic parameters in these analyses are taken separately for each mode from CLEO measurements [12]. The invariant mass distribution of both the suppressed $B^- \rightarrow DK^-$ and $B^- \rightarrow D\pi^+$ modes, separated by B charge, are shown in Figs. 3 and 4, respectively, for $D^0 \rightarrow \pi^\pm K^\pm$ and $D^0 \rightarrow \pi^\pm K^\mp \pi^- \pi^+$ final states.

As an important result of the first analysis is the first observation of the rare ADS mode, $B^- \rightarrow (K^- \pi^+)_D K^+$, with more than 10σ significance. There is an evidence for large CP asymmetry in the $B^- \rightarrow DK^-$ mode,

$$A_{\text{ADS}(K)}^{K\pi} = -0.52 \pm 0.15 \pm 0.02,$$

and a hint of asymmetry in the $B^- \rightarrow D\pi^-$,

$$A_{\text{ADS}(\pi)}^{K\pi} = 0.143 \pm 0.062 \pm 0.011.$$

The partial rates

$$R_{\text{ADS}(K)}^{K\pi} = 0.0152 \pm 0.0020 \pm 0.0004,$$

$$R_{\text{ADS}(\pi)}^{K\pi} = 0.00410 \pm 0.00025 \pm 0.00005$$

are obtained, respectively. In all measured results, the first uncertainty is statistical, the second one is systematic.

Moreover, the suppressed 4-body ADS modes in both the $B^- \rightarrow DK^-$ and $B^- \rightarrow D\pi^-$ decays have not previously been observed. In addition to the first observation of these modes at LHCb, with a significance which exceeds 5σ and 10σ , respectively, there is a hint of CP asymmetry,

$$A_{\text{ADS}(K)}^{K3\pi} = -0.42 \pm 0.22$$

in the $B^- \rightarrow DK^-$. The asymmetry in $B^- \rightarrow D\pi^-$ mode is measured to be

$$A_{\text{ADS}(\pi)}^{K3\pi} = +0.13 \pm 0.10.$$

The partial rates

$$R_{\text{ADS}(K)}^{K3\pi} = 0.0124 \pm 0.0027,$$

$$R_{\text{ADS}(\pi)}^{K3\pi} = 0.00369 \pm 0.00036$$

are found. Here uncertainties contain both statistical and systematic components.

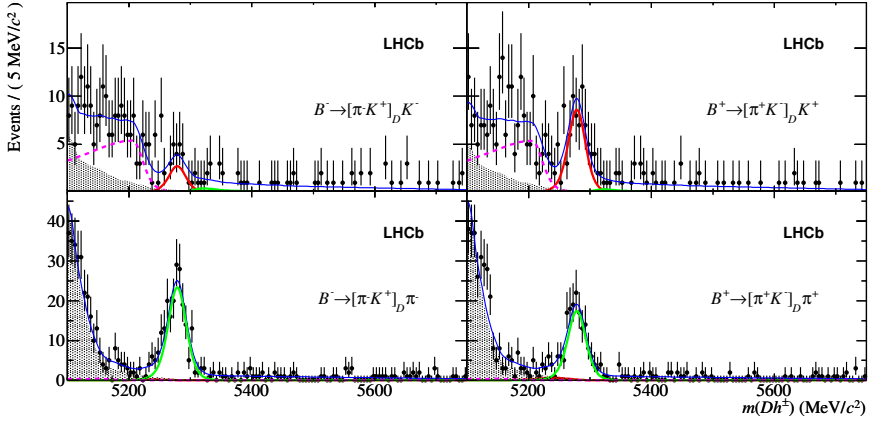


Fig. 3. Invariant mass distributions of selected $B^\pm \rightarrow [\pi^\pm K^\mp]_D h^\pm$ candidates. See the caption of Fig. 1 for a full description. The dashed line here represents the partially reconstructed, but Cabibbo favoured, $B_s^0 \rightarrow \bar{D}^0 K^- \pi^+$ and $\bar{B}_s^0 \rightarrow D^0 K^+ \pi^-$ decays, where the pions are lost.

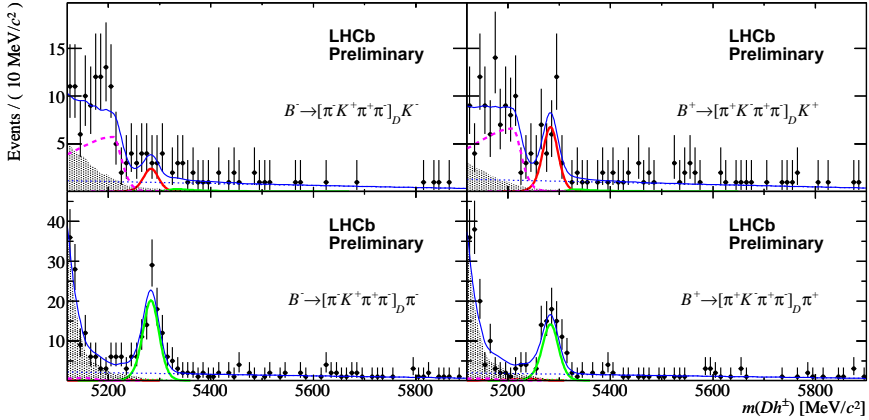


Fig. 4. Invariant mass distributions of selected $B^\pm \rightarrow [\pi^\pm K^\mp \pi^- \pi^+]_D h^\pm$ candidates. See the caption of Fig. 1 for a full description. The dashed line here represents the partially reconstructed, but Cabibbo favoured, $B_s^0 \rightarrow \bar{D}^0 K^- \pi^+$ and $\bar{B}_s^0 \rightarrow D^0 K^+ \pi^-$ decays, where the pions are lost.

4. GGSZ

In the GGSZ analysis, the neutral D mesons are selected in three-body self-conjugate final states such as $D \rightarrow K_S^0 h^+ h^-$. The sensitivity to γ comes from the different interference pattern for $B^+ \rightarrow DK^+$ and $B^- \rightarrow DK^-$ decays, in the $D \rightarrow K_S^0 h^+ h^-$ Dalitz plot. The amplitude of the decay $B^- \rightarrow DK^-$, $D \rightarrow K_S^0 h^+ h^-$ can be written as the superposition of the

$B^- \rightarrow D^0 K^-$ and $B^- \rightarrow \bar{D}^0 K^-$ contributions

$$A_B(m_+^2; m_-^2) = A + r_B e^{i(\delta_B - \gamma)} \bar{A}, \quad (10)$$

where m_+^2 and m_-^2 are the invariant masses squared of the $K_S^0 h^+$ and $K_S^0 h^-$ combinations, respectively, that define the position of the decay in the Dalitz plot, $A = A(m_+^2, m_-^2)$ is the $D^0 \rightarrow K_S^0 h^+ h^-$ amplitude, and $\bar{A} = \bar{A}(m_+^2, m_-^2)$ the $\bar{D}^0 \rightarrow K_S^0 h^+ h^-$ amplitude. The conjugate amplitudes are related by $A(m_+^2, m_-^2) = \bar{A}(m_-^2, m_+^2)$. The Dalitz plots are binned into $2N$ (from $-N$ to N excluding zero) regions symmetric under the exchange $m_+^2 \leftrightarrow m_-^2$. For positive bins, $m_+^2 > m_-^2$ is satisfied. A strong-phase difference $\delta_D(m_+^2, m_-^2) = \arg \bar{A} - \arg A$ between the \bar{D}^0 and D^0 decay is obtained at each point in the Dalitz plot. A model-independent approach is taken in the first LHCb analysis with this method [10], which uses the CLEO [11] measurements of δ_D in bins of the Dalitz plot. The chosen binning schemes are shown in Fig. 5.

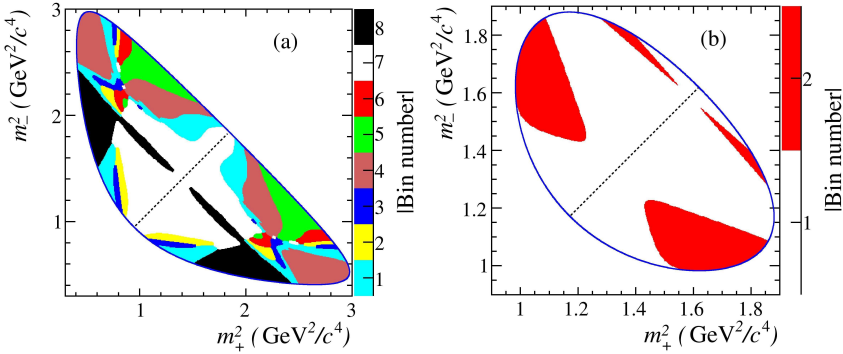


Fig. 5. Binning choices for (left) $D \rightarrow K_S^0 \pi^+ \pi^-$ and (right) $D \rightarrow K_S^0 K^+ K^-$. The diagonal line separates the positive and negative bins.

The population of each positive (negative) bin in the Dalitz plot arising from B^+ decays is $N_{\pm i}^+$ ($N_{\pm i}^-$), and that from B^- decays is $N_{\pm i}^-$ ($N_{\pm i}^+$)

$$N_{\pm i}^+ = h_{B^+} \left[K_{\mp i} + (x_+^2 + y_+^2) K_{\pm i} + 2\sqrt{K_i K_{-i}}(x_{+c\pm i} \mp y_{+s\pm i}) \right], \quad (11)$$

$$N_{\pm i}^- = h_{B^-} \left[K_{\pm i} + (x_-^2 + y_-^2) K_{\mp i} + 2\sqrt{K_i K_{-i}}(x_{-c\pm i} \pm y_{-s\pm i}) \right], \quad (12)$$

where K_i is the number of events in bin of flavour-tagged D decays, c_i (s_i) is $\cos(\sin)$ of strong phase difference in each bin (taken from CLEO-c). The index $\pm i$ varies over the number of bins, $\pm i = \pm 8(\pm 2)$ for $D \rightarrow K_S^0 \pi^+ \pi^-$ ($D \rightarrow K_S^0 K^+ K^-$). The sensitivity to γ enters through the Cartesian parameters

$$x_{\pm} = r_B \cos(\delta_B \pm \gamma) \quad \text{and} \quad y_{\pm} = r_B \sin(\delta_B \pm \gamma). \quad (13)$$

LHCb has performed the measurement using $B^- \rightarrow DK^-$ to the $D \rightarrow K_S^0 \pi^+ \pi^-$ and $D \rightarrow K_S^0 K^+ K^-$ final states. The measured Dalitz plots are shown in Figs. 6 and 7, respectively. In this analysis, the $B^- \rightarrow D\pi^-$ modes are firstly, used constrain the $B^- \rightarrow DK^-$ mass shape in the fit and secondly, to determine the variation in the reconstruction efficiency over the Dalitz plot. The assumption of no CP violation in these decays is made and a systematic uncertainty is assigned. The following results are obtained

$$\begin{aligned} x_- &= (0.0 \pm 4.3 \pm 1.5 \pm 0.6) \times 10^{-2}, \\ x_+ &= (-10.3 \pm 4.5 \pm 1.8 \pm 1.4) \times 10^{-2}, \\ y_- &= (2.7 \pm 5.2 \pm 0.8 \pm 2.3) \times 10^{-2}, \\ y_+ &= (-0.9 \pm 3.7 \pm 0.8 \pm 3.0) \times 10^{-2}, \end{aligned}$$

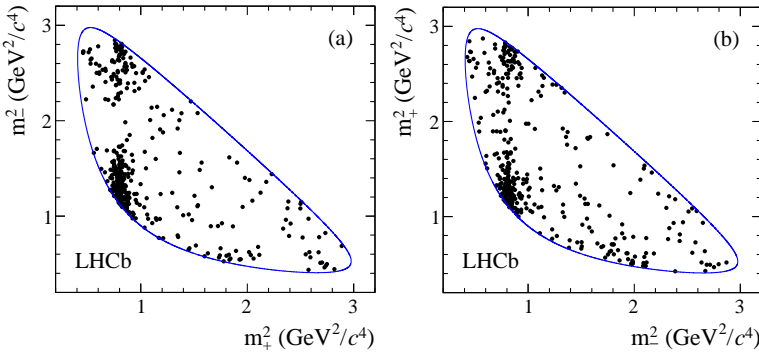


Fig. 6. Dalitz plots of $B^- \rightarrow DK^-$ candidates in the signal region for $D \rightarrow K_S^0 \pi^+ \pi^-$ divided between (left) B^+ and (right) B^- . The boundaries of the kinematically-allowed regions are also shown.

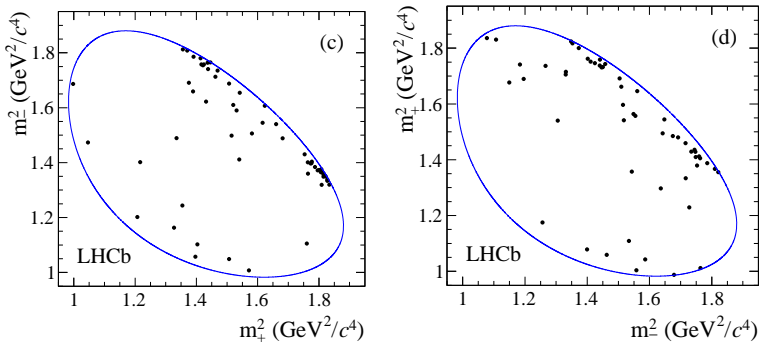


Fig. 7. Dalitz plots of $B^- \rightarrow DK^-$ candidates in the signal region for $D \rightarrow K_S^0 K^+ K^-$ decays, divided between (left) B^+ and (right) B^- . The boundaries of the kinematically-allowed regions are also shown.

where the first uncertainty is statistical, the second is systematic and the third arises from the experimental knowledge of the (c_i, s_i) parameters. The best fit values for x_{\pm} and y_{\pm} with 1σ , 2σ , 3σ contour of statistical uncertainty are shown in Fig. 8. A non-zero value of the angle between the two vectors joining the origin in the xy plane and the best fit values is a signature of CP violation.

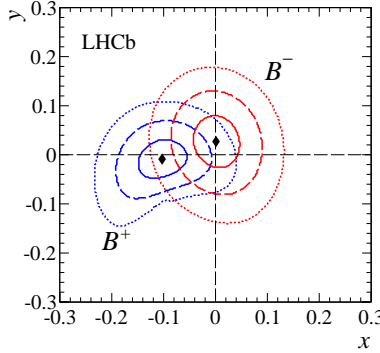


Fig. 8. One (solid), two (dashed) and three (dotted) standard deviation confidence levels for (x_+, y_+) (dark) and (x_-, y_-) (light) as measured in $B^- \rightarrow DK^-$ decays (statistical only). The points represent the best fit central values.

5. The LHCb γ average

All mentioned above measurements are used in the first γ combination performed by LHCb. The strategy is to maximize the total likelihood built from the product of the probability density functions (PDFs) f_i of input experimental observables A_i

$$\mathcal{L}(\vec{\alpha}) = \prod_i f_i \left(\vec{A}_i^{\text{obs}} \middle| \vec{A}_i(\vec{\alpha}_i) \right), \quad (14)$$

where $\vec{\alpha}_i$ is a set of parameters, the \vec{A}_i^{obs} are the measured central values of the observables and $\vec{A}_i(\vec{\alpha}_i)$ are the truth relations. The additional inputs for $D \rightarrow K^+\pi^-$ and $D \rightarrow K^+\pi^-\pi^+\pi^-$ are taken from CLEO [12]. The recent evidence for a difference in the CP asymmetries in $D \rightarrow K^+K^-$ and $D \rightarrow \pi^+\pi^-$, $\Delta a_{\text{CP}}^{\text{dir}} = (-0.656 \pm 0.154) \times 10^{-2}$ [13], is taken into account in the combination, however it has only marginal effects on the final results.

The combination of all the LHCb results from $B^- \rightarrow DK^-$ decays gives at 68% C.L.

$$\begin{aligned} \gamma &= (71.1^{+16.6}_{-15.7})^\circ, \\ r_B &= 0.092 \pm 0.008, \\ \delta_B &= (112.0^{+12.6}_{-15.5})^\circ. \end{aligned}$$

The approximately Gaussian behaviour for all observables is shown in Fig. 9. The results are in good agreement with the averages recently published by BaBar [14] and Belle [15].

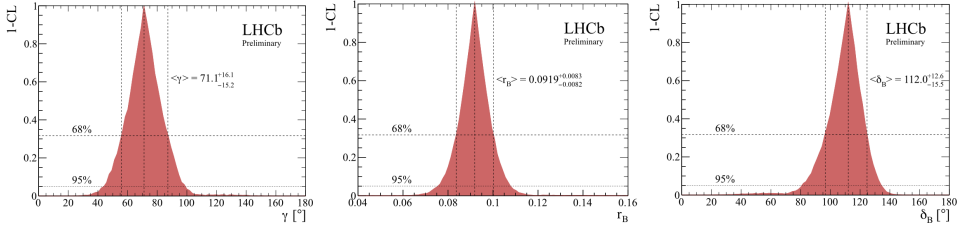


Fig. 9. Left: 1-C.L. curve for γ for the combination of the DK parts of the two-body GLW/ADS, of the GGSZ measurement, and of the $D \rightarrow K^+\pi^-\pi^+\pi^-$ ADS measurement. Middle: 1-C.L. curve for the r_B . Right: 1-C.L. curve for δ_B .

In addition, LHCb has performed for the first time γ average from both $B^- \rightarrow DK^-$ and $B^- \rightarrow D\pi^-$ modes. A double-peak structure is observed in γ 1-C.L. curve as is shown in Fig. 10. The confidence limits are set

$$\begin{aligned} \gamma &\in [61.8, 67.8]^\circ \quad \text{or} \quad [77.9, 92.4]^\circ \quad \text{at} \quad 68\% \text{ C.L.}, \\ \gamma &\in [43.8, 101.5]^\circ \quad \text{at} \quad 95\% \text{ C.L.}, \end{aligned}$$

where all values are modulo 180° . The best fit value is shifted to 85° , however the 95% confidence level is essentially unchanged. The impact of $B^- \rightarrow D\pi^-$ channel in the combination is probably larger than naively expected. It corresponds to rather high values of $r_B(D\pi^-)$ ($\in [0.010, 0.024]$ at 68% C.L.). Further information about γ combination performed by the LHCb can be found in Ref. [16].

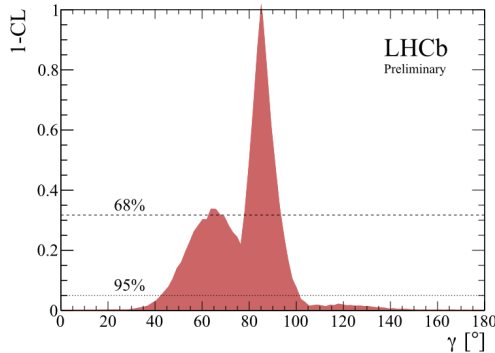


Fig. 10. 1-C.L. curve for the full DK and $D\pi$ combination.

6. Conclusion

Using 1 fb^{-1} of data collected in 2011 a combined measurement of the CKM angle γ is performed by LHCb. The obtained results are in good agreement and have comparable uncertainties with the world best values and results from other experiments. An additional 2 fb^{-1} has been collected in 2012 by LHCb which will significantly improve the result in the near future. Moreover, understanding and eventually fully exploiting $B^- \rightarrow D\pi^-$ decays will be investigated with the analysis of the 2012 dataset.

REFERENCES

- [1] M. Kobayashi, T. Maskawa, *Prog. Theor. Phys.* **49**, 652 (1973).
- [2] D. Derkach [UTfit Collaboration], [arXiv:1301.3300](https://arxiv.org/abs/1301.3300) (2013), <http://utfit.org/UTfit/>
- [3] J. Charles *et al.* [CKMfitter Group], *Eur. Phys. J. C* **41**, 1 (2005); <http://ckmfitter.in2p3.fr/>
- [4] M. Gronau, D. London, *Phys. Lett.* **B253**, 483 (1991).
- [5] M. Gronau, D. Wyler, *Phys. Lett.* **B265**, 172 (1991).
- [6] D. Atwood, I. Dunietz, A. Soni, *Phys. Rev. Lett.* **78**, 3257 (1997).
- [7] R. Aaij *et al.* [LHCb Collaboration], *Phys. Lett.* **B712**, 203 (2012) [*Erratum ibid.*, **B713**, 351 (2012)].
- [8] R. Aaij *et al.* [LHCb Collaboration], LHCb-CONF-2012-030 (2012).
- [9] A. Giri, Y. Grossman, A. Soffer, J. Zupan, *Phys. Rev.* **D68**, 054018 (2003).
- [10] R. Aaij *et al.* [LHCb Collaboration], *Phys. Lett.* **B718**, 43 (2012).
- [11] J. Libby *et al.* [CLEO Collaboration], *Phys. Rev.* **D82**, 112006 (2010).
- [12] N. Lowrey *et al.* [CLEO Collaboration], *Phys. Rev.* **D80**, 031105 (2009).
- [13] Y. Amhis *et al.* [Heavy Flavor Averaging Group], [arXiv:1207.1158](https://arxiv.org/abs/1207.1158) [hep-ex].
- [14] J. Lees *et al.* [BABAR Collaboration], *Phys. Rev.* **D87**, 052015 (2013) [[arXiv:1301.1029](https://arxiv.org/abs/1301.1029) [hep-ex]].
- [15] K. Trabelsi [Belle Collaboration], [arXiv:1301.2033](https://arxiv.org/abs/1301.2033) [hep-ex].
- [16] R. Aaij *et al.* [LHCb Collaboration], LHCb-CONF-2012-032, 2012.