AUTOMATION IN FIXED-ORDER CALCULATIONS*

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An overview is presented of the developments in fixed-order calculations for hard scattering cross sections over the last one and a half decade.

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1. Introduction

After a few years of operation, the experiments at the Large Hadron Collider have gathered a large amount of data, the analysis of which has already led to the achievement of one of its primary goals: the discovery of a candidate for the Higgs boson. The main challenge in such analyses is the separation of backgrounds from signals. The former is defined as "known" physics, and in the case of searches for the Higgs boson refers to the part of the physical processes that can be described by the Standard Model without involvement of the Higgs boson. The latter is the contribution that does involve the Higgs boson.

Separating signals from backgrounds often requires precise theoretical predictions of the latter. For example, in processes when the Higgs boson is produced together with a top and an anti-top quark, and the Higgs boson decays into an bottom–anti-bottom quark pair, the experimental efficiency of identifying the jets coming from the bottom quarks and the necessary phase space cuts to reduce other multi-jet backgrounds lead to a substantial smearing of what would be a sharp Higgs resonance peak in the distribution of the invariant mass of the bottom quark jets. Theoretical knowledge of the backgrounds then becomes crucial for a successful analysis.

The example above immediately illustrates one of the complications in the calculation of such backgrounds: they often involve processes with many jets and/or particles in the final state. Including the decay of the top and

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anti-top quarks in the example, this multiplicity is eight. Part of the necessary calculations concern the so-called *hard scattering process*, and involve the application of perturbative quantum field theory, and quantum chromo dynamics (QCD) in particular. In such calculations, the number of final-state particles and jets, or rather *partons*, has a dramatic effect on the computational complexity. The need for high precision and, consequently, the higher orders in perturbation theory further increase this complexity. These circumstances have, over the course of the last one and a half decade, led to an effort towards automation of such calculation, both with respect to the design of suitable computation algorithms and explicit computer programs.

2. Factorization in hard scattering processes

The typical quantities to be calculated for hadron scattering processes are cross sections and their differentiations with respect to kinematical variables, like transverse momentum components and rapidities of final-state jets and/or particles. The calculations largely follow the structure of the naïve factorization formula

$$d\sigma(P_a,P_b\to\{P_i\}) = \int \prod_{j=a,b,1,2,\ldots} dx_j\, f_j(x_j)\, d\hat{\sigma}\left(x_aP_a, x_bP_b \to \left\{x_i^{-1}P_i\right\}\right) , \tag{1}$$

where P_a, P_b are the momenta of the scattering hadrons and $\{P_i\}$ represents the set of momenta of the observed final-state jets. The integrand is factorized into universal parton density functions (PDFs) describing the low-energy physics of the initial-state hadrons, universal fragmentation functions describing the low-energy physics of the final-state jets, and a process-dependent partonic cross section $\hat{\sigma}$. The latter is calculated within perturbative QCD, which will be the subject of the following sections. Complications arise because of infra-red singularities appearing in those calculations, but in [1], Eq. (1) has been established rigorously, valid to all orders in QCD, within the so-called *collinear factorization*. It requires the introduction of a factorization scale, which can be interpreted as the threshold between the aforementioned low and high-energy physics, on which both the universal functions and the partonic cross section depend. The full cross section cannot depend on this unphysical scale, and one of the reasons to include higher perturbative orders in a calculation is to reduce this scale dependence. The scale dependence of the PDFs and fragmentation functions can also be calculated with perturbative QCD, via evolution equations. In practice, the role of the fragmentation functions is mostly played by parton-shower programs including hadronization simulation. These are also used to describe initial-state radiation, taking over part of the role of the PDFs.

The following sections will deal with the leading-order (LO) and next-to-leading order (NLO) calculation of partonic cross sections.

3. Numerical evaluation of amplitudes

The calculation of partonic cross sections involves the calculation of phase space integrals. At leading order in perturbation theory, they have the following form

$$\hat{\sigma}_{a,b\to n} = \int d\Phi(p_a, p_b \to \{p\}_n) \left|\mathcal{M}_{a,b\to n}\right|^2 \mathcal{O}(p_a, p_b, \{p\}_n)\,, \tag{2}$$

where p_a, p_b are the momenta of the initial-state partons, and $\{p\}_n$ represents the set of momenta of the final-state partons and/or particles. The integral over the phase space of these final-state momenta is denoted as the integral over $\Phi(p_a, p_b \to \{p\}_n)$, and $\mathcal{O}(p_a, p_b, \{p\}_n)$ represents an infra-red safe observable, which at LO means that it must vanish whenever any parton becomes arbitrarily soft, or any pair of partons becomes arbitrarily collinear, since for such phase space configurations the tree-level squared scattering amplitude $|\mathcal{M}_{a,b\to n}|^2$ becomes singular. In practice, the observable includes phase space cuts, cutting out those singular regions. The number of necessary phase space restrictions grows at least as the square of the number of final-state partons, making it impossible to perform the integral analytically, and essentially leaving the Monte Carlo method as the only option.

The essence of the Monte Carlo method is that one only needs to be able to evaluate the integrand, in this case the squared amplitude, numerically and that, in principle, no analytic knowledge, other than square integrability, is required. In practice, some knowledge of the peak structure is still preferred in order to perform importance sampling to increase the rate of convergence of the Monte Carlo integration process. This is the effort to reduce the number of integrand evaluations, necessary to reach a given accuracy, as much as possible. The remaining task is then to perform the evaluation of the integrand at each phase space point as efficiently as possible.

The squared amplitude itself is a sum over the helicities of the partons and particles. Instead of performing this sum analytically, which in the case of many final-state particles leads to huge expressions, it can be performed explicitly, by evaluating the amplitude as function of both momenta and helicities, for all configurations of helicities, and adding up the squares of the absolute values of the obtained numbers. One can even perform this sum within the Monte Carlo set up, by choosing random helicity configurations. The task is then to numerically evaluate the amplitude as function of momenta and helicity configurations as efficiently as possible.

4. Numerical Dyson–Schwinger recursion

Telling from the Feynman rules in QCD, and the triple-gluon vertex in particular, one might expect large expressions for scattering amplitudes involving many gluons. It turns out, however, that helicity amplitudes can be written in a compact and elegant form. For a tree-level n-gluon amplitude as function of momenta, helicity, and color, we have

$$\mathcal{M}(\{p,\lambda,a\}_n) = i\mathrm{g}^{n-2} \sum_{\text{perm.}} \mathrm{Tr}\big(T^{a_1}T^{a_2}\cdots T^{a_n}\big)\mathcal{A}\left(p_1^{\lambda_1}, p_2^{\lambda_2}, \ldots, p_n^{\lambda_n}\right), \quad (3)$$

where $T^a, a = 1, 2, \ldots, 8$ are the generators of the color symmetry group SU(3), and the sum is over all permutations of the enumerated variables, excluding cyclic permutations. The *dual amplitudes* $\mathcal{A}$ contain only planar Feynman graphs, and depending on the helicity configuration, very compact expressions exist [2], for example

$$\mathcal{A}\left(p_1^-, p_2^+, \ldots, p_{j-1}^+, p_j^-, p_{j+1}^+, \ldots, p_n^+\right) = \frac{\langle p_1\, p_j\rangle^4}{\langle p_1\, p_2\rangle\langle p_2\, p_3\rangle\cdots\langle p_{n-1}\, p_n\rangle\langle p_n\, p_1\rangle}.$$

Each bracket in the expression above represents the contraction of the Weyl spinors associated with the momenta inside the bracket, and eventually represents just two complex multiplications.

The correctness of the expression above can be proven with the help of the Berends–Giele recursive relations [3]

$$\begin{aligned} A^\mu_{i,j} \;=\; & \frac{-i}{p_{i,j}^2}\left[\sum_{k=i}^{j-1} V^\mu_{\nu\rho}\left(p_{i,k}, p_{k+1,j}\right)\, A^\nu_{i,k} A^\rho_{k+1,j}\right. \\ & \left. + \sum_{k=i}^{j-2}\sum_{l=k+1}^{j-1} W^\mu_{\nu\rho\sigma}\, A^\nu_{i,k} A^\rho_{k+1,l} A^\sigma_{l+1,j}\right], \end{aligned} \quad (4)$$

where we denote $p_{i,j} = p_i + p_{i+1} + \cdots + p_j$, and

$$\begin{aligned} V^\mu_{\nu\rho}(p,q) \;&=\; \frac{i}{\sqrt{2}}\left[(p-q)^\mu g_{\nu\rho} + 2g^\mu_\rho\, q_\nu - 2g^\mu_\nu\, p_\rho\right], \\ W^\mu_{\nu\rho\sigma} \;&=\; \frac{i}{2}\left[2g^\mu_\rho g_{\nu\sigma} - g^\mu_\nu g_{\rho\sigma} - g^\mu_\sigma g_{\rho\nu}\right]. \end{aligned} \quad (5)$$

The *off-shell current* $A^\mu_{i,j}$ is the sum of all planar tree-level sub-graphs with external on-shell gluons $i, i+1, \ldots, j$ and one off-shell gluon the momentum $p_{i,j}$ of which is the sum of the momenta of the on-shell gluons. The dual amplitude for n gluons is then given by $i\varepsilon_\mu(p_n)p_n^2 A^\mu_{1,n-1}$, where $\varepsilon_\mu(p_n)$ is

the polarization vector of gluon number n, and p_n^2 formally cancels the superfluous propagator denominator in $A^\mu_{1,n-1}$. The starting points $A^\mu_{i,i}$ of the recursion are the polarization vectors of the on-shell external gluons.

The correctness of the recursive relation can easily be derived from the structure of planar tree-level multi-gluon amplitudes. The vertices of Eq. (5) are the QCD 3- and 4-gluon vertices stripped from their color content, and restricted to the application of planar graphs. As mentioned, the recursive relation can be used algebraically, to prove compact expressions for multi-gluon helicity amplitudes, but it can also be used to evaluate them directly. Starting from explicit numerical values of the momenta, explicit numerical values of the polarization vectors can be constructed, and the recursion can be performed numerically, to end up with a numerical value of the amplitude. This computation is very efficient, its complexity grows polynomially with the number of gluons, and is not restricted to certain helicity configurations, but can be applied for arbitrary polarization vectors. Considering the fact that the only expressions that go into such a computation are those for the vertices, one can understand that the recursive relation embodies a computational algorithm for tree-level amplitudes prescribed directly by the QCD Lagrangian.

One may ask whether this reasoning works for arbitrary Lagrangians, and generalizes to not-necessarily planar graphs, and the answer is positive, as was first pointed out in [4]. The generalization to not-necessarily planar graphs reduces the factorial computational complexity, implicit in Eq. (3) due to the sum over permutations, to exponential. Soon after, the calculations of tree-level amplitudes with essentially arbitrary numbers of external gluons appeared [5, 6]. The purely numerical nature of the algorithms allows for arbitrary representations of the external polarization vectors and internal color structure, allowing again for inventive Monte Carlo approaches to perform the necessary helicity and color sums.

Since the algorithm is essentially prescribed by the Lagrangian itself of the field theory under consideration, and can be recognized as a recursive perturbative solution of the Dyson–Schwinger equations of motion, it is reasonably easy to automate for arbitrary scattering processes, and several automatic programs based on this approach were developed [7]. Besides the use of the Dyson–Schwinger approach, also the conventional approach using Feynman graphs was applied to construct automatic programs for amplitude calculation, some of them employing sophisticated computational techniques to deal with the factorial growth of complexity inherent in any approach based on Feynman graphs [8].

It has to be stressed that many of these programs actually calculate cross sections, and also automate the phase space integration.

5. Merging with parton showers

LO calculations like described above need phase space cuts for the final-state partons to avoid the singular behavior of the tree-level matrix element. Summarizing such cuts into a single energy scale, as a minimum energy or transversal momentum of the partons, or a minimum invariant mass for any pair of partons, the calculated value of the cross section will depend dramatically on this scale. The calculation is not valid for small values of the scale, and in those regions of phase space all orders in the perturbative series need to be included. Of course, this is not possible exactly, but other approximations can be applied, taking into account only leading contributions in the logarithms of ratios of relevant energy scales. Parton shower programs do exactly this, within a Monte Carlo environment [9]. These on their own, on the other hand, cannot describe processes with several hard jets very well, suggesting to combine parton showers with the LO calculations.

This technique is referred to as *merging* of LO programs with parton shower programs. The idea is to let the LO program generate an event with a number of final-state parton momenta, and to let the parton shower program "dress up" the event further. The challenge is to avoid double-counting, or double-populating certain regions of phase space.

Parton shower programs create multi-parton phase space points by consecutive branching of momenta such that the final phase space point is directly generated following the correct distribution, within the approximations inherent in the parton shower. This means that every event generated by a parton shower has a well-defined tree structure. Events from a LO program are weighted, or the result from an un-weighting procedure, and do not have an inherent tree structure. An important part of the merging procedure, therefore, involves the repeated application of jet algorithms on LO-generated events to assign the necessary tree structure to the events. Hence jet algorithms are another important ingredient in the merging process. A more detailed description of various methods and approaches, and comparisons of their applications to several combinations of LO programs and parton shower programs can be found in [10].

6. NLO calculations

The result of LO calculations also dramatically depends on the value of the renormalization/factorization scale entering via the strong coupling constant and the factorization procedure. This dependence can be reduced by including higher orders in the perturbative series. It turns out that this also influences the shape of several differntial distributions, *i.e.* that these cannot be completely accurately calculated at LO. The NLO contribution

to the partonic cross section can formally be written as

$$\hat{\sigma}^{\mathrm{NLO}}_{a,b\to n} = \int d\Phi_n \, 2\mathrm{Re}\Big\{\mathcal{M}^{(0)}_{a,b\to n}\mathcal{M}^{(1)}_{a,b\to n}\Big\}\,\mathcal{O}^{\mathrm{LO}}_n + \int d\Phi_{n+1}\left|\mathcal{M}^{(0)}_{a,b\to n+1}\right|^2 \mathcal{O}^{\mathrm{NLO}}_{n+1}\,. \tag{6}$$

The notation for the phase space integration elements has been condensed a bit compared to Eq. (2). Notice that it involves one more final-state parton in the second term. This term represents the real-radiation contribution, and involves a tree-level amplitude with one more final-state parton. The super-script explicitly indicates the number or loops in the amplitude, which is zero. This number is non-zero in the first term, representing the virtual contribution, and involves the one-loop amplitude.

6.1. Real-radiation contribution

The notation for the observables has been changed too. The one in the real-radiation term takes one more parton as argument. The super-script indicates that, contrary to the one in the virtual contribution, this observable *does* allow for one parton to become arbitrarily soft, or one pair of partons to become arbitrarily collinear. This clearly makes the integral divergent, which embodies a first complication in NLO calculations. The divergencies formally cancel partly against IR divergencies coming from the one-loop amplitude and are partly removed within the factorization prescription. But somehow, the divergencies must be extracted before the numerical integral can be performed. The divergencies in the one-loop amplitude also come from an extra integration, the one-loop integral, but this is an integral over a full, unrestricted four-dimensional space and can be performed analytically within dimensional regularization.

The problem can be solved with the introduction of subtraction terms, which have the same divergent behavior as the real-radiation matrix element, but are constructed such that the extra parton can be integrated out analytically. The real-radiation term is replaced with

$$\int d\Phi_{n+1}\left[\left|\mathcal{M}^{(0)}_{a,b\to n+1}\right|^2 \mathcal{O}^{\mathrm{NLO}}_{n+1} - \sum_i \mathcal{D}^{(i)}_{n+1}\theta^{(i)}_{n+1}\mathcal{O}^{\mathrm{LO}}_n \circ T^{(i)}_{n\leftarrow n+1}\right]. \tag{7}$$

$\mathcal{D}^{(i)}_{n+1}$ is a function matching the divergent behavior for a few singular phase space regions, and $\theta^{(i)}_{n+1}$ may be restricting phase space to those regions. The mapping $T^{(i)}_{n\leftarrow n+1}$ projects the $(n+1)$ particle phase space to an n-particle phase space, and is necessary because the observable takes only n momenta as arguments. The subtraction terms must be constructed including this

observable, because the divergencies of the integrated terms

$$\int d\Phi_1 \left(\mathcal{D}^{(i)}_{n+1}\theta^{(i)}_{n+1}\right) \circ T^{(i)}_{n\to n+1}\, \mathcal{O}^{\mathrm{LO}}_n \tag{8}$$

must cancel against those in the virtual contribution. Notice that the mapping appearing in these terms is the inverse of the ones in the real-radiation term, in the sense that they construct an $(n+1)$-particle phase space point from an n-particle phase space point and the three integration variables represented by Φ_1.

Explicitly elaborated approaches to this method that are most used are Catani–Seymour dipole subtraction [11] and FKS subtraction [12]. In the former, the functions $\theta^{(i)}_{n+1}$ are, in the original approach, identical to 1. The method is quite universal, but has the disadvantage that the singularities are spread over many subtraction terms, and their number grows as the cube of the number of partons.

In the FKS method, the functions $\theta^{(i)}_{n+1}$ are a partition of 1. One can, therefore, take the summation over subtraction terms outside the phase space integral, and end up with a well-defined integral for each subtraction term separately. For each of these, the phase space integration can be factorized into an n-particle integration, and the three-dimensional integral in which the cancellation happens. As a consequence, the mappings $T^{(i)}_{n\leftarrow n+1}$ are not needed. The number of terms grows only as the square of the number of partons.

6.2. Virtual contribution

Another complication in NLO calculations is the one-loop amplitude. It was the final bottleneck, and its resolution is regularly referred to as the "NLO revolution". One-loop amplitudes are more complicated than tree-level amplitudes because they contain, for a given number of external legs, many more graphs, and because the one-loop integral has to be performed. It was known already how to perform calculations formally, but the combination of the two issues can lead to huge expressions that will essentially be impossible to handle if not treated carefully.

In general, a one-loop amplitude can be written as a sum of terms, each of which has the form

$$I = \int d^\omega q\, \frac{N(q)}{D_1(q)D_2(q)\cdots D_l(q)}\,, \qquad D_j(q) = (q+p_j)^2 - m_j^2 + i0\,, \tag{9}$$

where each p_i is some combination of the external momenta, and $N(q)$ is a polynomial in q. The integral is defined within dimensional regularization.

Its calculation implies finding the coefficients in the Laurent expansion

$$I = \frac{I_{-2}}{(\omega-4)^2} + \frac{I_{-1}}{\omega-4} + I_0 + O(\omega-4)\,. \tag{10}$$

One approach is to perform the loop integration numerically. This requires the introduction of subtraction terms which cancel both the infra-red and ultra-violet behavior of the integrand, and whose integrals are easier to perform analytically than the original loop integral. Furthermore, the integration has to be deformed into complex space to avoid the poles in the denominators of Eq. (9), and this has to happen such that also all subtraction terms stay valid. This approach has been successfully applied to multi-jet production in e^+e^- collisions [13].

The alternative is to expand the one-loop amplitude in terms of universal integrated one-loop functions

$$I = \sum_j c_j f_j\,. \tag{11}$$

The functions f_j should be universal in the sense that they only depend on values of external momenta and possible parameters like particle masses, and no other ingredients coming with the particular scattering process under consideration. Having a method to evaluate these, the problem of calculating the one-loop amplitude is reduced to that of obtaining the coefficients c_j.

It can be shown that the minimal set of functions needed are the *scalar integrals*, given by Eq. (9) with $N(q) = 1$, and up to $l = 4$. One has to be careful in dimensional regularization, and depending on the particular approach, the functions for $l = 5$ are also needed. One may, however, choose to increase the *universal* set of one-loop functions, and traditionally the set of *tensor integrals* was used, given by Eq. (9) with $N(q)$ as a product of components of q, and l reaching up to the number of external momenta. The tensor integrals can be calculated recursively, with the scalar integrals as starting point of the recursion [14, 15]. The traditional approach aimed at finding expressions for the coefficients c_j in terms of external momenta and polarization vectors. This was done graph-by-graph, and would lead to huge expressions, becoming impossible to handle for more than 6 external particles. The solution to this problem was proposed in [16], and worked out further in [17, 18], and comes down to calculating the coefficients numerically via recursive relations.

In the alternative approach, the coefficients of the scalar functions are determined directly. Whereas tree-level amplitudes are only polynomial functions of the external momenta, one-loop amplitudes also contain logarithms of ratios of invariants of the external momenta, which eventually appear in

the scalar functions. The branch cuts of these logarithms can be related to cuts in Feynman graphs. This led to the idea to extract the coefficients for the scalar functions via the cuts in Feynman graphs. Originally restricted to two-particle cuts [19], the approach became more powerful with the applications of multi-particle cuts [20].

Inspired by the integrand-level tensor reduction of [15], the integrand-level equivalent of Eq. (11) was used to extract the coefficients in [21]. It can be written as an inversion problem, with the numerator $N(q)$ on the l.h.s. of the equation, and sums of products of the denominator factors on the r.h.s, which can be approached completely numerically. Any value of q leads to a valid equation, but choosing q such that a number of denominator factors vanish triangularizes the equation partly, and facilitates convenient solutions. This also establishes the connection with the previously mentioned method, since putting denominator factors to zero corresponds to cutting internal lines in Feynman graphs.

6.3. Calculations and automation

Using the techniques described above, in various combinations, an impressive list of NLO calculations for processes with four or more particles in the final state have been performed last years [22]. It constitutes the so-called (enlarged) *Les Houches wishlist* of background and signal processes [23]. Such calculations require a high level of automation, and some of the developed tools were made public. A complete and public tool for such calculations is Helac-NLO [24]. It goes beyond the possibilities in the number of final state particles of old tools like MCFM [25], NLOjet [26] and VBFNLO [27]. Many of the calculations cited before have been performed with BlackHat [28] in combination with Sherpa [29]. Also the MadGraph-framework has been completed to perform full NLO calculations [30–32]. Besides, independent tools have been developed to perform the real-radiation integral [33, 34], and to calculate one-loop amplitudes [35, 36]

7. Matching NLO calculations with parton showers

Earlier, we described the technique of merging LO calculations with parton showers, combining the exclusiveness of parton showers with a more accurate treatment of processes with many hard jets. Improving the overall normalization to NLO precision in a consistent manner is called *NLO matching*. Following the formalism of [37], we can write

$$\hat{\sigma}_n^{\mathrm{LO+NLO}} = \int d\Phi_n \bar{B}_n \, \mathcal{O}_n^{\mathrm{LO}} + \int d\Phi_{n+1} \sum_j A_{n+1}^{(j)} \Big[\mathcal{O}_{n+1}^{\mathrm{NLO}} - \mathcal{O}_n^{\mathrm{LO}} \circ T_{n \leftarrow n+1}^{(j)} \Big] + \int d\Phi_{n+1} \left[\left| \mathcal{M}_{n+1}^{(0)} \right|^2 - \sum_j A_{n+1}^{(j)} \right] \mathcal{O}_{n+1}^{\mathrm{NLO}} , \tag{12}$$

where

$$\bar{B}_n = \left| \mathcal{M}_n^{(0)} \right|^2 + 2\mathrm{Re}\Big\{ \mathcal{M}_{a,b\to n}^{(0)} \mathcal{M}_{a,b\to n}^{(1)} \Big\} + \sum_j \int d\Phi_1 \, \mathcal{D}_{n+1}^{(j)} \circ T_{n\to n+1}^{(j)} + \sum_j \int d\Phi_1 \Big[A_{n+1}^{(j)} \circ T_{n\to n+1}^{(j)} - \mathcal{D}_{n+1}^{(j)} \circ T_{n\to n+1}^{(j)} \Big] . \tag{13}$$

One can easily see that most terms formally cancel in Eq. (12), leaving only the sum of the r.h.s. of Eq. (2) and Eq. (6). Additional subtraction terms $A_{n+1}^{(j)}$ have been introduced, with, in principle, the only restriction that all integrals should be convergent. $\bar{B}_n$ contains everything that lives in the Born phase space, including integrated subtraction terms. NLO matching procedures are based on the observation that the first line of Eq. (12) can, up to higher orders in the coupling constant, be written as

$$\int d\Phi_n \bar{B}_n \left[\Delta^{(A)}(t_0) \mathcal{O}_n^{\mathrm{LO}} + \sum_j \int_{t_0} d\Phi_1 \frac{A_{n+1}^{(j)} \circ T_{n\to n+1}^{(j)}}{\left| \mathcal{M}_n^{(0)} \right|^2} \Delta^{(A)}(t) \, \theta\big(t(\Phi_1) - t_0\big) \, \mathcal{O}_{n+1}^{\mathrm{NLO}} \circ T_{n\to n+1}^{(j)} \right] , \tag{14}$$

where

$$\Delta^{(A)}(t) = \exp\left\{ - \sum_j \int d\Phi_1 \, \theta\big(t(\Phi_1) - t\big) \, \frac{A_{n+1}^{(j)} \circ T_{n\to n+1}^{(j)}}{\left| \mathcal{M}_n^{(0)} \right|^2} \right\} \tag{15}$$

is the Sudakov form factor associated with the subtraction terms $A_{n+1}^{(j)}$. Matching is performed by generating an event either according to the first or the second line of Eq. (12) (excluding the second term on the first line). If the second line is chosen, the event is kept as it is, and if the first term is chosen, the event is processed through a one-step Sudakov branching algorithm. This procedure gives the correct starting condition for the parton shower avoiding possible double-counting.

In [37] it is explained how different choices of the auxiliary subtraction terms $A^{(j)}_{n+1}$ correspond to the two main procedures worked out in literature, Mc@NLO [38] and Powheg [39, 40]. They have been applied to several explicit calculations recently [41]. Also, automation is under development [42].

8. Summary

We presented an overview of developments in fixed-order calculations for hard scattering cross sections. Starting from the automation of LO calculations, we proceeded to merging multi-leg matrix elements with parton showers. We presented the complications that had to be dealt with to move to NLO, and discussed matching NLO calculations with parton showers.

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