A TRANSFORMATION METHOD TO CONSTRUCT FAMILY OF EXACTLY SOLVABLE POTENTIALS IN QUANTUM MECHANICS

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A transformation method is applied to the second order ordinary differential equation satisfied by orthogonal polynomials to construct a family of exactly solvable quantum systems in any arbitrary dimensional space. Using the properties of orthogonal polynomials, the method transforms polynomial differential equation to D-dimensional radial Schrodinger equation which facilitates construction of exactly solvable quantum systems. The method is also applied using associated Laguerre and hypergeometric polynomials. The quantum systems generated from other polynomials are also briefly highlighted.

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1. Introduction

The Schrodinger equation plays a pivotal role in modern physics as its solution gives complete information of any given non-relativistic quantum system. Along the years, many authors have tried to obtain the exact solution of the Schrodinger equation for potentials of physical interest [1–12]. This is because, despite the intrinsic interest of the exactly solvable systems, these solutions can be used to get better approximated solutions for potentials which are physically interesting. To enhance the set of exactly solvable potentials, we follow a simple and compact transformation method [10, 13–16] which comprises of a co-ordinate transformation supplemented by a functional transformation. By applying this method, we transform the second order ordinary differential equation satisfied by special functions to standard Schrodinger equation in arbitrary D-dimensional Euclidean space and thus try to construct as many exactly solvable potentials as possible. The method is efficient in generating both power and non-power law type spherically symmetric potentials.

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The article is organized as follows. In Sec. 2, the detailed formalism of the theory is given. In Sec. 3, the application of the method using associated Laguerre polynomial is discussed. The solvable potentials obtained from hypergeometric, associated Legendre and Jacobi polynomials are also tabulated. The conclusions are discussed in Sec. 4.

2. Formalism

We consider a second order differential equation satisfied by a special function $Q(r)$

$$Q''(r) + M(r)Q'(r) + J(r)Q(r) = 0\,, \tag{1}$$

where a prime denotes differentiation with respect to its argument. $Q(r)$ will later be identified as one of the orthogonal polynomials.

The transformation method comprises of the following two steps

$$r \;\to\; g(r)\,, \tag{2}$$

$$\psi(r) \;=\; f^{-1}(r)Q(g(r))\,. \tag{3}$$

We implement the above prescription to equation (1) and obtain

$$\psi''(r) + \left(\frac{d}{dr}\ln\frac{f^2(r)\exp(\int M(g)dg)}{g'(r)}\right)\psi'(r) + \left(\frac{f''(r)}{f(r)} - \frac{g''(r)}{g'(r)}\frac{f'(r)}{f(r)} + g'(r)M(g)\frac{f'(r)}{f(r)} + g'^2 J(g)\right)\psi(r) = 0\,. \tag{4}$$

The radial Schrodinger equation in D-dimensional Euclidean space is ($\hbar = 1 = 2m$)

$$\psi''(r) + \frac{(D-1)}{r}\psi'(r) + \left(E_n - V(r) - \frac{\ell(\ell+D-2)}{r^2}\right)\psi(r) = 0\,. \tag{5}$$

Consistency of equations (4) and (5) demands that

$$\frac{d}{dr}\ln\frac{f^2(r)\exp(\int M(g)dg)}{g'(r)} = \frac{(D-1)}{r} \tag{6}$$

which fixes the form of $f(r)$ as

$$f(r) = N r^{\frac{(D-1)}{2}} g'^{\frac{1}{2}} \left(\exp\left(-\int M(g)dg\right)\right)^{\frac{1}{2}}\,, \tag{7}$$

where N is the integration constant and plays the role of the normalization constant of the wavefunctions.

Using (6) and (7) in equation (4) yields

$$\frac{\psi''(r)}{\psi(r)} + \frac{(D-1)}{r}\frac{\psi'(r)}{\psi(r)} = -\frac{1}{2}\{g,r\} + \frac{g'^2(r)}{4} \times \left[M^2(g) + 2M'(g) - 4J(g)\right] - \frac{(D-1)(D-3)}{4r^2}, \tag{8}$$

where the Schwartzian derivative symbol $\{g,r\}$ [17] is defined as

$$\{g,r\} = \frac{g'''(r)}{g'(r)} - \frac{3}{2}\frac{g''^2(r)}{g'^2(r)}.$$

From equations (3) and (7), the expression for normalizable wavefunction is

$$\psi(r) = N r^{-\frac{(D-1)}{2}} g'^{-\frac{1}{2}} \left(\exp\left(\int M(g)dg\right)\right)^{\frac{1}{2}} Q(g(r)). \tag{9}$$

The radial wavefunction $\psi(r) = \frac{u(r)}{r}$ has to satisfy the boundary condition $u(r) = 0$, in order to rule out singular solutions [18].

Expression (8) can be cast in the standard Schrodinger equation form (equation (5)) if we can write

$$-(E_n - V(r)) = -\frac{1}{2}\{g,r\} + \frac{g'^2(r)}{4} \times \left[M^2(g) + 2M'(g) - 4J(g)\right] - \frac{(D-1)(D-3)}{4r^2}. \tag{10}$$

Once we choose a particular orthogonal polynomial, $Q(g)$, to construct an exact solution of the Schrodinger equation, the characteristic functions of the polynomial $M(g)$, $J(g)$ get specified. We have to choose one or more than one terms containing the function $g(r)$ in expression (10) and put it equal to a constant to get the energy eigenvalues E_n. The procedure is worked out in detail for Laguerre and hypergeometric polynomials in the next section.

It is interesting to note that when the generated potential is purely non-power law, the potential given by expression (10) has a term $\frac{(D-1)(D-3)}{4r^2}$ which behaves as constant background attractive inverse square potential in any arbitrary dimension except for dimensions 1 and 3. For power law cases, this background potential and the potential coming from the Schwartzian derivative unite to give the correct centrifugal barrier potential in arbitrary dimensions.

3. Application of the transformation method

3.1. Construction of exactly solvable potentials from associated Laguerre polynomial

Identifying

$$Q(g(r)) = L_n^{\alpha}(g) \tag{11}$$

as the associated Laguerre polynomial, its characteristic functions $M(g)$ and $J(g)$ give

$$M(g) = \frac{\alpha + 1 - g}{g}\,, \tag{12}$$

$$J(g) = \frac{n}{g}\,. \tag{13}$$

Using equations (11), (12) and (13) in equation (8) yields

$$\frac{\psi''(r)}{\psi(r)} + \frac{(D-1)}{r}\frac{\psi'(r)}{\psi(r)} = \frac{1}{4}(\alpha^2 - 1)\frac{g'^2}{g^2} - \frac{1}{2}(2n + \alpha + 1) \times\frac{g'^2}{g} + \frac{g'^2}{4} - \frac{1}{2}\frac{g'''}{g'} + \frac{3}{4}\frac{g''^2}{g'^2} - \frac{(D-1)(D-3)}{4r^2}\,, \tag{14}$$

and equation (9) yields

$$\psi(r) = N r^{-\frac{(D-1)}{2}} g'^{-\frac{1}{2}} g^{\frac{\alpha+1}{2}} \exp\left(-\frac{g}{2}\right) L_n^{\alpha}(g)\,. \tag{15}$$

To convert equation (14) into a standard stationary state Schrodinger equation, we make one or more terms of the right-hand side of equation (14) a constant quantity. This enables us to get the energy eigenvalues E_n, the functional form of $g(r)$ and subsequently potential $V(r)$ and wavefunction $\psi(r)$.

(i) As a first case, let us choose

$$\frac{g'^2}{g^2} = c_1^2\,, \tag{16}$$

where c_1^2 is a real positive constant independent of r. Equation (16) gives the functional form of $g(r)$ as

$$g(r) = A_1 \exp(-c_1 r)\,, \tag{17}$$

where A_1 is an integration constant and for normalizability condition we consider here only the negative sign in the exponential. Using the value of $g(r)$ in equation (14) yields

$$E_n = -\frac{c_1^2 \alpha^2}{4}\,, \tag{18}$$

$$V(r) = A_1 c_1^2 \exp(-c_1 r)\left(\frac{A_1}{4}\exp(-c_1 r) - \frac{2n+\alpha+1}{2}\right) - \frac{(D-1)(D-3)}{4r^2}, \tag{19}$$

and from equation (15), we obtain

$$\psi(r) = N r^{-\frac{(D-1)}{2}} \exp\left(-\frac{c_1 \alpha r}{2}\right) \exp\left(-\frac{A_1 \exp(-c_1 r)}{2}\right) L_n^{\alpha-1}(\exp(-c_1 r)). \tag{20}$$

To express energy eigenvalues in terms of the quantum number n, we choose

$$\frac{2n+\alpha+1}{2} = \beta$$

a constant independent of n, which gives $\alpha = 2\beta - 2n - 1$ and $\beta \geq n + \frac{1}{2}$. This yields energy eigenvalues, potential and energy eigenfunction as $(A_1 = 1)$

$$E_n = -\frac{c_1^2}{4}(2\beta - 2n - 1)^2, \tag{21}$$

$$V(r) = c_1^2 \exp(-c_1 r)\left(\tfrac{1}{4}\exp(-c_1 r) - \beta\right) - \frac{(D-1)(D-3)}{4r^2}, \tag{22}$$

and

$$\psi(r) = N r^{-\frac{(D-1)}{2}} \exp\left(-(2\beta - 2n - 1)\frac{c_1 r}{2}\right) \exp\left(-\frac{\exp(-c_1 r)}{2}\right) \times L_n^{2\beta-2n-2}(\exp(-c_1 r)). \tag{23}$$

The potential given by expression (22) is non-power law and as our formalism suggests, it has an inverse square potential term in spaces where the dimensionality is other than 1 and 3.

(ii) Continuing the procedure to construct exactly solvable quantum system, we consider second term $\frac{g'^2}{g}$ of expression (14) to be constant independent of r, *i.e.*,

$$\frac{g'^2}{g} = c_2^2, \tag{24}$$

we get the functional form of $g(r)$ as

$$g(r) = \frac{c_2^2}{4} r^2. \tag{25}$$

Equations (14) and (25) yield

$$
\begin{aligned}
E_n &= \frac{1}{2}(2n_r + \alpha + 1)c_2^2\,, && (26)\\
V(r) &= \frac{c_2^4}{16}r^2 + \left(\alpha^2 - \frac{1}{4} - \frac{(D-1)(D-3)}{4}\right)\frac{1}{r^2}\,, && (27)
\end{aligned}
$$

and

$$\psi(r) = N r^{\alpha+1-\frac{D}{2}} \exp\left(-\frac{c_2^2}{8}r^2\right) L_{n_r}^{\alpha}\left(\frac{c_2^2}{4}r^2\right) . \tag{28}$$

To get the correct centrifugal barrier term in D-dimensional Euclidean space, we have to identify the coefficient of $\frac{1}{r^2}$ in expression (27) to be $\ell(\ell+D-2)$, which fixes the value of α as

$$\alpha = \ell + \frac{D-2}{2} . \tag{29}$$

Identifying

$$\frac{c_2^2}{2} = \omega\,, \tag{30}$$

expressions (26), (29) and (30) yield energy eigenvalues as

$$E_n = \omega\left(n + \frac{D}{2}\right)\,, \tag{31}$$

where the principal quantum number n is $(2n_r + \ell)$.

From equations (26) and (27), we get the potential and eigenfunction as

$$V(r) = \tfrac{1}{4}\omega^2 r^2 + \tfrac{\ell(\ell+D-2)}{r^2} \tag{32}$$

and

$$\psi(r) = N r^{\ell} \exp\left(-\frac{\omega r^2}{4}\right) L_{\frac{1}{2}(n-\ell)}^{\ell+\frac{D-2}{2}}\left(\frac{\omega r^2}{2}\right) \tag{33}$$

respectively.

(iii) Proceeding in a similar way, let

$$g'^2(r) = c_3^2\,. \tag{34}$$

This gives

$$g(r) = c_3 r \,. \tag{35}$$

As required for the normalizability of the wavefunction, we have taken the positive sign. Equations (14) and (35) yield

$$E_n = -\frac{c_3^2}{4} \,, \tag{36}$$

$$V(r) = -\frac{c_3}{2}(2n_r + \alpha + 1)\frac{1}{r} + \frac{1}{4r^2}\left(\alpha^2 - 1 - (D-1)(D-3)\right) \,, \tag{37}$$

and

$$\psi(r) = N r^{\frac{1}{2}(\alpha - D + 2)} \exp\left(-\frac{c_3}{2} r\right) L_n^{\alpha+1}(c_3 r) \,. \tag{38}$$

To get the correct form of centrifugal barrier term in D-dimensional Euclidean space, we have to identify the coefficient of $\frac{1}{r^2}$ in expression (37) to be $\ell(\ell + D - 2)$, which fixes the value of α as

$$\alpha + 1 = 2\ell + D - 1 \,. \tag{39}$$

Further, to get the energy eigenvalues in terms of the quantum number n, we identify the coefficient of $\frac{1}{r}$ in equation (37) to be e^2, which in atomic unit is 2, *i.e.*,

$$c_3 = \frac{4}{2n_r + \alpha + 1} \,. \tag{40}$$

Equations (36), (39) and (40) yield the energy eigenvalues

$$E_n = -\frac{1}{n^2} \,, \tag{41}$$

where the principal quantum number, n for the D-dimensional case is $n = n_r + \ell + \frac{D-1}{2}$, and reduces to the usual, $n = n_r + \ell + 1$ when $D = 3$. Now, the potential becomes

$$V(r) = -\frac{2}{r} + \frac{\ell(\ell + D - 2)}{r^2} \,, \tag{42}$$

and the wavefunction

$$\psi(r) = N r^{\ell} \exp\left(-\frac{r}{n}\right) L_n^{2\ell + D - 1}\left(\frac{2r}{n}\right) \,. \tag{43}$$

TABLE I

Summary of the quantum systems obtained from associated the Laguerre polynomial $L_n^\alpha(g)$.

$g(r)$	E_n	$V(r)$	$\psi(r)$
$\exp(-c_1 r)$	$-\frac{c_1^2}{4}(2\beta-2n-1)^2$	$c_1^2\exp(-c_1 r)\left(\frac{1}{4}\exp(-c_1 r)-\beta\right) -\frac{(D-1)(D-3)}{4r^2}$,	$Nr^{-\frac{(D-1)}{2}}\exp\left(-(2\beta-2n-1)\frac{c_1 r}{2}\right) \exp\left(-\frac{\exp(-c_1 r)}{2}\right)L_n^{2\beta-2n-2}(\exp(-c_1 r))$
		where $\frac{2n+\alpha+1}{2}=\beta$	
$\frac{c_2^2}{4}r^2$	$\omega(n+\frac{D}{2})$,	$\frac{1}{4}\omega^2 r^2+\frac{\ell(\ell+D-2)}{r^2}$	$Nr^\ell\exp\left(-\frac{\omega r^2}{4}\right)L_{\frac{1}{2}(n-\ell)}^{\ell+\frac{D-2}{2}}\left(\frac{\omega r^2}{2}\right)$,
	where $\frac{c_2^2}{2}=\omega$		where $\alpha=\ell+\frac{D-2}{2}$
$c_3 r$	$-\frac{1}{n^2}$,	$-\frac{2}{r}+\frac{\ell(\ell+D-2)}{r^2}$,	$Nr^\ell\exp(-\frac{r}{n})L_n^{2\ell+D-1}(\frac{2r}{n})$,
	where $n=n_r+\ell+\frac{D-1}{2}$	where $c_3=\frac{4}{2n_r+\alpha+1}$	where $\alpha+1=2\ell+D-1$

(iv) As a fourth choice, let

$$\frac{g''^2}{g'^2} = c_4^2\,. \tag{44}$$

We get the functional form of $g(r)$ similar to that of the first choice which gives us a similar type of quantum system as obtained in that case.

The summary of the constructed exactly solvable systems is given in Table I.

3.2. Construction of exactly solvable potentials from hypergeometric function

Identifying

$$Q(g(r)) = {}_2F_1(\alpha,\beta,\gamma;g) \tag{45}$$

as the hypergeometric function, its characteristic functions $M(g)$ and $J(g)$ give

$$M(g) = \frac{\gamma-(\alpha+\beta+1)g}{g(1-g)}\,, \tag{46}$$

$$J(g) = -\frac{\alpha\beta}{g(1-g)}\,. \tag{47}$$

Using equations (45),(46) and (47) in equation (8) yield

$$\frac{\psi''(r)}{\psi(r)} + \frac{(D-1)}{r}\frac{\psi'(r)}{\psi(r)} = \left(\frac{\gamma(\gamma-\alpha-\beta-1)}{2}+\alpha\beta\right)\frac{g'^2}{g} + \frac{\gamma(\gamma-2)}{4}\frac{g'^2}{g^2} + \left(\frac{\gamma(\gamma-\alpha-\beta-1)}{2}+\alpha\beta\right)\frac{g'^2}{(1-g)} + \left(\frac{(\alpha+\beta-\gamma)^2-1}{4}\right) \times\frac{g'^2}{(1-g)^2} - \frac{1}{2}\frac{g'''}{g'} + \frac{3}{4}\frac{g''^2}{g'^2} - \frac{(D-1)(D-3)}{4r^2}\,, \tag{48}$$

and equation (9) yield

$$\psi(r) = N r^{-\frac{(D-1)}{2}} g'^{-\frac{1}{2}} g^{\frac{\gamma}{2}} (1-g)^{\frac{\alpha+\beta-\gamma+1}{2}} {}_2F_1(\alpha,\beta,\gamma;g)\,. \tag{49}$$

To put equation (48) into the standard stationary state Schrodinger equation and to generate exactly solvable quantum systems, we follow the same procedure of equating different terms of the right-hand side of equation (48) to a constant. We summarize the different quantum systems thus obtained in Table II.

In a similar way, by identifying $Q(g)$ as another orthogonal polynomial and applying the above mentioned procedure, different exactly solvable potentials can be obtained. We have listed a few of them in Table III.

TABLE II

Summary of the quantum systems obtained from the hypergeometric function ${}_2F_1(\alpha, \beta, \gamma; g)$.

Relation	$g(r)$	E_n	$V(r)$	$\psi(r)$
$\frac{g'^2}{g} = p_1^2$	$\frac{p_1^2}{4} r^2$	$-2\Big(-2n(\beta+1)+(\ell+\frac{D}{2})(\ell+\frac{D}{2}+n-\beta-2)\Big)$	$\frac{r^2}{(1-r^2)}\Big(-4n(\beta+1)+(2\ell+D)(2\ell+D+2n-2\beta-4)+\frac{(-n+\beta-\ell-\frac{D}{2}+1)^2-1}{4(1-r^2)}\Big)+\frac{\ell(\ell+D-2)}{r^2}$, where $c=\ell+\frac{D}{2}$; taking $p_1=2$	$r^\ell(1-r^2)^{\frac{-n+\beta-\ell-\frac{D}{2}+2}{2}}\, {}_2F_1(-n,\beta+1,\ell+\frac{D}{2};r^2)$
$\frac{g'^2}{g^2} = p_2^2$	$A_2\exp(-p_2r)$	$-p_2^2\left(\frac{\beta_1^2-(n+1)^2}{2(n+1)}\right)^2$, where $(n+1)(n-1+\gamma)=\beta_1$	$-\frac{\beta_1^2A_2p_2^2\exp(-p_2r)}{1-A_2\exp(-p_2r)}-\frac{(D-1)(D-3)}{4r^2}$ with condition $\alpha+\beta-\gamma+1=\pm1$	$r^{-\frac{D-1}{2}}(A_2\exp(-p_2r))^{\frac{\beta_1^2-(n+1)^2}{2(n+1)}}(1-A_2\exp(-p_2r))\, {}_2F_1(-n,\frac{\beta_1^2+n+1}{n+1},\frac{\beta_1^2-n^2-n}{n+1};A_2\exp(-p_2r))$
$\frac{g'^2}{(1-g)^2} = p_3^2$	$1-A_3\exp(-p_3r)$	$-p_3^2\left(\frac{\gamma_1^2+(n+1)^2}{2(n+1)}\right)^2$, where $-\beta(n+1)+n+1=\gamma_1^2$	$\frac{p_3^2A_3\gamma_1^2\exp(-p_3r)}{(1-A_3\exp(-p_3r))}-\frac{(D-1)(D-3)}{4r^2}$ with $\gamma=1$	$r^{-\frac{(D-1)}{2}}(1-A_3\exp(-p_3r))\exp\left(-p_3\frac{\gamma_1^2+(n+1)^2}{2(n+1)}r\right)\, {}_2F_1(-n,\frac{n-(\gamma_1^2-1)}{n+1},2;(1-\exp(-p_3r)))$
$\frac{g''^2}{g'^2} = p_4^2$	$A_4\exp(-p_4r)$	$-p_4^2\left(\frac{\delta^2-(n+1)^2}{2(n+1)}\right)$ where $(n+1)(n-1+\gamma)=\delta$	$-\frac{\delta^2A_4p_4^2\exp(-p_4r)}{1-A_4\exp(-p_4r)}-\frac{(D-1)(D-3)}{4r^2}$ with condition $\alpha+\beta-\gamma+1=\pm1$	$r^{-\frac{D-1}{2}}(A_4\exp(-p_4r))^{\frac{\delta^2-(n+1)^2}{2(n+1)}}(1-A_4\exp(-p_4r))\, {}_2F_1\left(-n,\frac{\delta^2+n+1}{n+1},\frac{\delta^2-n^2-n}{n+1};A_4\exp(-p_4r)\right)$

In each case, we have taken $\alpha=-n$ to make ${}_2F_1(\alpha,\beta,\gamma;g)$ a polynomial.

TABLE III

Quantum systems obtained from other orthogonal polynomials.

Relation	$g(r)$	E_n	$V(r)$	$\psi(r)$
$\frac{g'^2}{1-g^2} = p_1^2$	$\sin p_1 r$	$-p_1^2(m^2 - n(n+1) - \frac{1}{2})$	$C_m^2 \tan^2 p_1 r - \frac{(D-1)(D-3)}{4r^2}$, where $p_1^2(m^2 - \frac{1}{4}) = C_m^2$	$Nr^{-\frac{D-1}{2}} \cos^{\frac{1}{2}} p_1 r P_n^m(\sin p_1 r)$
$\frac{g'^2}{(1-g^2)^2} = p_2^2$	$\tanh p_2 r$	$-(m-1)^2 p_2^2$	$-n(n-1)p_2^2 \operatorname{sech}^2 p_2 r - \frac{(D-1)(D-3)}{4r^2}$	$Nr^{-\frac{D-1}{2}} P_{n-1}^{m-1}(\tanh p_2 r)$
$\frac{g'^2}{1-g^2} = p_3^2$	$\sin p_3 r$	$\frac{1}{4}((\alpha-\beta)^2 - 4n(n+\alpha+\beta+1)$ $-2(\alpha+\beta)-2)p_3^2$	$c_1^2 \tan^2 p_3 r + c_2^2 \sec p_3 r \tan p_3 r$ $-\frac{(D-1)(D-3)}{4r^2}$ with $\frac{(\alpha^2-\beta^2)}{2} p_3^2 = c_1^2$; $\frac{(\alpha^2+\beta^2)}{2} p_3^2 = c_2^2$	$Nr^{-\frac{D-1}{2}} (\cos p_3 r)^{\frac{\alpha+\beta+1}{2}}$ $\left(\frac{1+\sin p_3 r}{1-\sin p_3 r}\right)^{\frac{\beta-\alpha}{4}} P_n^{(\alpha,\beta)}(\sin p_3 r)$
$\frac{g'^2}{(1-g^2)^2} = p_4^2$	$\tanh p_4 r$	$\frac{1}{4}((\alpha-\beta)^2 - 4n(n+\alpha+\beta+1)$ $-2(\alpha+\beta))p_4^2$	$c_3^2 \tanh^2 p_4 r + c_4^2 \tanh p_4 r$ $-\frac{(D-1)(D-3)}{4r^2}$ with $\frac{1}{4}((2n+\alpha+\beta+1)^2 - 1)p_4^2 = c_3^2$; $\frac{1}{2}(\alpha^2-\beta^2)p_4^2 = c_4^2$	$Nr^{-\frac{D-1}{2}} (\operatorname{sech} p_4 r)^{\frac{\alpha+\beta}{2}}$ $\exp\left(-\frac{(\alpha-\beta)}{2} p_4 r\right) P_n^{(\alpha,\beta)}(\tanh p_4 r)$

4. Conclusions

In this article, we have presented a simple transformation method of construction of exactly solvable potentials using the properties of orthogonal polynomials in the regime of non-relativistic quantum mechanics. The method is applied to construct spherically symmetric exactly solvable potentials in arbitrary D-dimensional Euclidean space. The number of possible exactly solvable potentials that can be constructed using a particular orthogonal polynomial depends on the number of $g(r)$ dependent terms on the right-hand side of equation (8), the mode of extraction of energy eigenvalues as discussed in the formalism and the normalizability of the eigenfunctions. We have listed exactly solvable potentials constructed from associated Laguerre, hypergeometric, associated Legendre and Jacobi polynomials. The method can, however, be applied to other orthogonal polynomials too. The constructed potentials are mostly non-power law with an inverse square potential $(D-1)(D-3)r^{-2}$ which vanishes for $D = 1$ and $D = 3$. For power law potential, this term along with the Schwartzian derivative give the correct form of centrifugal barrier term in D-dimensions (*e.g.*, equations (29), (32)). It is notable that we have explicitly kept the various constants such as integration constants, scale factors and characteristic constants in our expressions, which allows flexibility to the constructed exactly solvable potentials at the time of possible applications.

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