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THE ANNIHILATION CROSS SECTION OF THE DARK MATTER WHICH IS DRIVEN BY SCALAR UNPARTICLE

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We analyze the annihilation cross section of the Dark Matter which interacts with the Standard Model sector over the scalar unparticle propagator. We observe that the annihilation cross section of the dark matter pair is sensitive to the dark matter mass and the scaling dimension of scalar unparticle. We estimate a range for the dark matter mass and the scaling dimension of scalar unparticle by using the current dark matter abundance.

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1. Introduction

The visible matter is considerably less than the amount of matter required in the Universe, and 23% of the present Universe [1–5] is contributed by the Dark Matter (DM) that is not detectable by the emitted radiation. Although the nature of the DM is at present not known, the weakly interacting massive particles (WIMPs) [1] are among the promising candidates of the DM and they are expected in the mass range of 10 GeV–a few TeV. WIMPs do not decay into the Standard Model (SM) particles since they are stable, however, they disappear by pair annihilation (see, for example, [6, 7]). One needs a theoretical framework beyond the SM in order to explain the nature of the DM and its stability that can be ensured by an appropriate discrete symmetry in various models (for details see, for example, [8] and references therein). From the experimental point of view, the indirect detection of the DM candidate is based on the current relic density which can be explained by thermal freeze-out of their pair annihilation. By using the current DM abundance by the WMAP Collaboration [5], one gets the appropriate range

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for the annihilation cross section and obtains a valuable information about the nature of the DM. In the present work, we take an additional scalar SM singlet DM field $\phi_{\rm S}$ (see [9–15]) and assume that it interacts with the SM sector over the scalar unparticle propagator. Unparticles [16, 17] arise from the interaction of the SM and the ultraviolet sector with a non-trivial infrared fixed point at high energy level. They are massless and they have non-integral scaling dimension $d_{\rm U}$. The unparticle sector weakly interacts with the SM on and the interactions of unparticles with the SM fields in the low energy level is defined by the effective Lagrangian

$$\mathcal{L}_{\rm eff} = \frac{\eta}{\Lambda_{\rm U}^{d_{\rm U}+d_{\rm SM}-n}}\, O_{\rm SM}\, O_{\rm U} \tag{1}$$

with the unparticle operator $O_{\rm U}$, the energy scale $\Lambda_{\rm U}$, the space-time dimension n and the parameter η which carries information about the energy scale of ultraviolet sector, the low energy sector and the matching coefficient [16–18]. In order to formulate the DM annihilation, we start with the effective Lagrangian which obeys the Z_2 symmetry $\phi_{\rm S} \to -\phi_{\rm S}$

$$\mathcal{L}_{\rm S} = \frac{1}{2}\,\partial_\mu\,\phi_{\rm S}\,\partial^\mu\,\phi_{\rm S} - \frac{\lambda}{4}\,\phi_{\rm S}^4 - \frac{1}{2}\,m_{\rm S}^2\,\phi_{\rm S}^2 - \frac{\lambda_0}{\Lambda_{\rm U}^{d_{\rm U}-2}}\phi_{\rm S}^2\,O_{\rm U}\,, \tag{2}$$

where λ_0[1] is the effective coupling which leads to tree level DM–DM-scalar unparticle interaction. Here, the DM scalar $\phi_{\rm S}$ has no vacuum expectation value and the Z_2 symmetry guarantees the stability of $\phi_{\rm S}$ which appears as pairs and it cannot decay into the SM particles. On the other hand, they are expected to annihilate into the SM particles with the annihilating cross section which obeys the observed DM abundance. The annihilation process is driven by the exchange particle(s) and, here, we assume that the scalar unparticle propagator is responsible for this annihilation. The scalar unparticle propagator is obtained by using the scale invariance [17, 19]

$$\int d^4x\, e^{ipx}\ \langle 0|T\,(O_{\rm U}(x)\,O_{\rm U}(0))|0\rangle$$
$$= i\frac{A_{d_{\rm U}}}{2\,\pi}\int\limits_0^\infty ds\,\frac{s^{d_{\rm U}-2}}{p^2-s+i\epsilon} = i\,\frac{A_{d_{\rm U}}}{2\,\sin\,(d_{\rm U}\pi)}\,\left(-p^2-i\epsilon\right)^{d_{\rm U}-2}\,, \tag{3}$$

where $A_{d_{\rm U}} = \frac{16\,\pi^{5/2}}{(2\,\pi)^{2\,d_{\rm U}}}\,\frac{\Gamma(d_{\rm U}+\frac{1}{2})}{\Gamma(d_{\rm U}-1)\,\Gamma(2\,d_{\rm U})}$ and the function $\frac{1}{(-p^2-i\epsilon)^{2-d_{\rm U}}}$ becomes $\frac{1}{(-p^2-i\epsilon)^{2-d_{\rm U}}} \to \frac{e^{-i\,d_{\rm U}\,\pi}}{(p^2)^{2-d_{\rm U}}}$ for $p^2>0$ with a non-trivial phase which appears as a result of non-integral scaling dimension.

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[1] Notice that we consider λ_0 as universal coupling (see, for example, [19]), *i.e.*, we take $\eta=\lambda_0$ and $n=4$ in Eq. (1).

The total averaging annihilation rate of the DM can be obtained by the process $\phi_S\, \phi_S \to U \to X_{SM}$,

$$\langle \sigma\, v_r \rangle \;=\; \frac{4\,\lambda_0^2}{m_S\, \Lambda_U^{2\,(d_U-2)}} \left(\frac{A_{d_U}}{2\,\sin\, d_U\, \pi} \left(\frac{1}{4\, m_S^2} \right)^{2-d_U} \right)^2 \Gamma \left(\tilde{U} \to X_{SM} \right) , \quad (4)$$

where $\Gamma(\tilde{U} \to X_{SM}) = \sum_i \Gamma(\tilde{U} \to X_{i\,SM})$ with virtual unparticle $\tilde{U}$ having mass $m_U = 2\, m_S$ (see [20, 21]) and $v_r = \frac{2\, p_{CM}}{m_S}$ is the average relative speed of two DM scalars (see, for example, [15]). At this stage, we present the functions $\Gamma(\tilde{U} \to X_{i\,SM})$ which appear in the annihilation cross section arising from possible annihilations that are valid for the DM mass range we choose (see Section 2): In this range, the annihilations to the fermion–antifermion pairs[2], photon pair, gluon pair and WW^*, ZZ^* can exist. For the fermion–antifermion output, we have

$$\Gamma \left(\tilde{U} \to f\, \bar{f} \right) = \sum_f \frac{N_f \left(c_U^{ff} \right)^2}{8\, \pi\, m_U^2} \left(m_U^2 - 4\, m_f^2 \right)^{\frac{3}{2}} , \qquad (5)$$

where $N_f = 1\,(3)$ for leptons (quarks) and $c_U^{ff} = \frac{\lambda_0}{\Lambda_U^{d_U-1}}$. The one for the photon–photon (gluon–gluon) pair reads

$$\Gamma \left(\tilde{U} \to V\, V \right) = \frac{\beta\, m_U^3}{64\, \pi} \left| c_U^{VV} \right|^2 , \qquad (6)$$

where $c_U^{VV} = \frac{4\, i\, \lambda_0}{\Lambda_U^{d_U}}$ and $\beta = 1\,(2)$ for $V = \gamma\,(g)$. Finally, for WW^* and ZZ^* output, we get[3]

$$\Gamma \left(\tilde{U} \to W(Z) W(Z)^* \right) = \sum_{ij} \Gamma_{ij} \left(\tilde{U} \to W(Z) W(Z)^* \right) , \qquad (7)$$

with

$$\Gamma_{ij} \left(\tilde{U} \to W(Z) W(Z)^* \right) =$$
$$\frac{(2\,\pi)^4}{2\, m_U} \int \delta \left[P - \sum_{i=1}^{3} p_i \right] \prod_{i=1}^{3} \frac{d^3 p_i}{(2\,\pi)^3\, 2\, E_i}\, N_f \left| M_{ij}^{W(Z)} \right|^2 , \qquad (8)$$

[2] The annihilations into top–antitop quark pair and top quark–antineutrino do not exist.

[3] Notice that, in this expression, we ignore the mass of neutrinos.

where p_i (p_j, p_3) is the outgoing charged lepton or down quark (incoming neutrino or up quark, outgoing W boson) four momentum for $\Gamma_{ij}(\tilde{U} \to WW^*)$, the outgoing lepton or quark (antilepton or antiquark, outgoing Z boson) four momentum for $\Gamma_{ij}(\tilde{U} \to ZZ^*)$. In Eq. (8), $|M^W_{ij}|^2$ reads

$$\begin{aligned}
\left|M^W_{ij}\right|^2 = & \frac{16\, g^2\, c^2_{\mathrm{U}W}\, |V_{ij}|^2\, \left((p_i+p_j)\cdot p_3\right)^2}{m^6_W\, \left(m^2_W-(p_i+p_j)^2\right)^2} \\
& \times \left\{2\, m^2_i\, m^2_j\, m^2_W\, \left(m^2_i+m^2_j-2\, m^2_W\right)+2\, \left(m^2_i+m^2_j\right)\, m^2_W\, (p_i\cdot p_j)^2\right. \\
& -\, 2\, \left((m_i-m_W)\,(m_i+m_W)\, p_i\cdot p_3+m^2_i\, p_j\cdot p_3\right) \\
& \times \left(m^2_j\, p_i\cdot p_3+(m_j-m_W)\,(m_j+m_W)\, p_j\cdot p_3\right) \\
& +\, p_i\cdot p_j\, \left(\left(m^4_i+6\, m^2_i\, m^2_j+m^4_j\right)\, m^2_W-\left(m^2_i+m^2_j\right)\right. \\
& \left.\left. \times \left(2\, m^4_W+\left((p_i+p_j)\cdot p_3\right)^2\right)-m^6_W\right)\right\}\, . \qquad (9)
\end{aligned}$$

Here, $c_{\mathrm{U}W}=\frac{\lambda_0}{\Lambda^{d_{\mathrm{U}}}_{\mathrm{U}}}$, V_{ij} is the CKM matrix element for up–down quark pairs and $V_{ij}=1$ for neutrino-charged lepton. Finally, $|M^Z_{ij}|^2$ is

$$\begin{aligned}
\left|M^Z_{ij}\right|^2 = & \frac{32\, g^2\, c_{\mathrm{U}Z}\, \left((p_i+p_j)\cdot p_3\right)^2}{c^2_W\, m^6_Z\, \left(m^2_Z-(p_i+p_j)^2\right)^2} \\
& \times \left\{\, \left(c^2_{\mathrm{L}}+c^2_{\mathrm{R}}\right)\, \left(2\, m^2_i\, m^2_j\, \left(m^2_Z\, \left(m^2_i+m^2_j-2\, m^2_Z\right)-\left((p_i+p_j)\cdot p_3\right)^2\right)\right.\right. \\
& +\, p_i\cdot p_j\, \left(m^2_Z\, \left(m^4_i+m^4_j-m^4_Z-2\, m^2_j\, \left(m^2_Z-p_i\cdot p_j\right)\right.\right. \\
& \left.\left. +\, 2\, m^2_i\, \left(3\, m^2_j-m^2_Z+p_i\cdot p_j\right)\right)-\left(m^2_i+m^2_j\right)\, \left((p_i+p_j)\cdot p_3\right)^2\right) \\
& +\, 2\, m^2_Z\left(m^2_j\, p_i\cdot p_3\, (p_i+p_j)\cdot p_3\right. \\
& \left.\left. +\, p_j\cdot p_3\, \left(-m^2_Z\, p_i\cdot p_3+m^2_i\, (p_i+p_j)\cdot p_3\right)\right)\right) \\
& -\, 2\, c_{\mathrm{L}}\, c_{\mathrm{R}}\, m_i\, m_j\, \left(m^2_Z\, \left(m^4_i+m^4_j+3\, m^4_Z-2\, m^2_j\, \left(m^2_Z-2\, p_i\cdot p_j\right)\right.\right. \\
& \left. -\, 4\, m^2_Z\, p_i\cdot p_j+4\, (p_i\cdot p_j)^2+2\, m^2_i\, \left(m^2_j-m^2_Z+2\, p_i\cdot p_j\right)\right) \\
& \left.\left. -\, m^2_i+m^2_j-2\, m^2_Z+2\, p_i\cdot p_j)\, \left((p_i+p_j)\cdot p_3\right)^2\,\right)\right\}, \qquad (10)
\end{aligned}$$

where $c_{UZ} = \frac{\lambda_0}{\Lambda_U^{d_U}}$, $c_L = \frac{-1}{2} + s_W^2 \left(\frac{1}{2}\right)$ for charged lepton (neutrino), $c_L = \frac{-1}{2} + \frac{s_W^2}{3} \left(\frac{1}{2} - \frac{2\, s_W^2}{3}\right)$ for down quark (up quark), $c_R = s_W^2\, (0)$ for charged lepton (neutrino) and $c_R = \frac{s_W^2}{3} \left(-\frac{2\, s_W^2}{3}\right)$ for down quark (up quark).

Now, we are ready to analyze annihilation cross section $\langle \sigma\, v_r \rangle$ and, by using the expression for the relic abundance,

$$\Omega\, h^2 = \frac{x_f\, 10^{-11}\, \mathrm{GeV}^{-2}}{\langle \sigma\, v_r \rangle}\,, \tag{11}$$

with $x_f = 25$ [15, 22–24], we get the range $2.21 \times 10^{-9}\, \mathrm{GeV}^{-2} \leq \langle \sigma\, v_r \rangle \leq 2.44 \times 10^{-9}\, \mathrm{GeV}^{-2}$. Here, we respect the upper and the lower bounds of the present relic abundance [5]

$$\Omega\, h^2 = 0.1109 \pm 0.0056\,. \tag{12}$$

2. Discussion

In the present work, we analyze the annihilation cross section of the DM which interacts with the SM sector over the scalar unparticle propagator. The DM–DM-unparticle coupling λ_0 plays an important role in the annihilation process and we study its numerical value by respecting the estimated upper and lower bounds of the annihilation cross section of the DM, namely, $2.21 \times 10^{-9}\, \mathrm{GeV}^{-2} \leq \langle \sigma\, v_r \rangle \leq 2.44 \times 10^{-9}\, \mathrm{GeV}^{-2}$. Furthermore, the scaling dimension of scalar unparticle, the energy scale Λ_U and the DM mass m_S are among the free parameters of this scenario. For the scaling dimension d_U, we choose the well known range of $1 < d_U < 2$ (see [17, 25]). We consider the DM mass m_S in the interval $10\, \mathrm{GeV} \leq m_S \leq 70\, \mathrm{GeV}$ and we take the energy scale $\Lambda_U = 10$ TeV.

In Fig. 1, we plot the DM mass m_S dependence of the coupling λ_0 for the annihilation cross section $\langle \sigma\, v_r \rangle$ and different values of d_U. Here, the lower, intermediate, upper solid (long dashed; dashed) lines represent λ_0 for $d_U = 1.1$; 1.3; 1.5 and $\langle \sigma\, v_r \rangle_{AV}$ ($\langle \sigma\, v_r \rangle_{Max}$; $\langle \sigma\, v_r \rangle_{Min}$). We observe that the coupling λ_0 is sensitive to m_S and this sensitivity increases with the increasing values of the scaling dimension d_U. For small values of d_U and m_S, λ_0 is more restricted and their increasing values result in λ_0 lying in a broader range. In order to get the present experimental result of $\langle \sigma\, v_r \rangle$, λ_0 must be of the order of magnitude of 0.01 for $1.1 < d_U < 1.3$ and it must reach 0.1 for $d_U = 1.5$ for the DM mass values $m_S > 40\, \mathrm{GeV}$. For completeness, we present the scaling dimension d_U dependence of the coupling λ_0 for the annihilation cross section $\langle \sigma\, v_r \rangle$ and different values of m_S in Fig. 2. Here, the lower, intermediate, upper solid (long dashed; dashed) line represents λ_0 for $m_S = 30$; 50; $70\, \mathrm{GeV}$ and $\langle \sigma\, v_r \rangle_{AV}$ ($\langle \sigma\, v_r \rangle_{Max}$; $\langle \sigma\, v_r \rangle_{Min}$). This figure also shows the strong sensitivity of the coupling λ_0 to the scaling dimension d_U. The coupling reaches the numerical values of the order of 1.0 for the upper bounds of d_U, namely for $d_U \sim 1.9$.

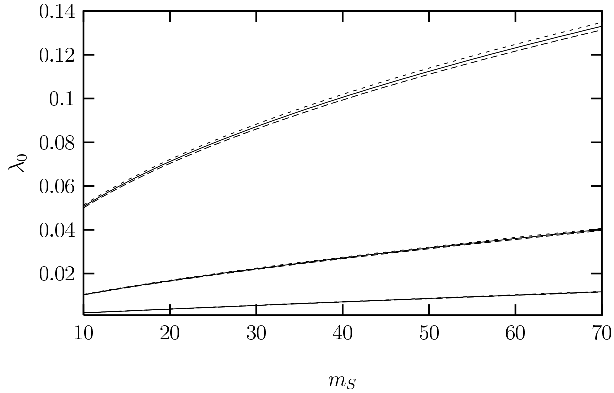

Fig. 1. λ_0 as a function of m_S. Here, the lower, intermediate, upper solid (long dashed; dashed) line represents λ_0 for $d_S = 1.1;\ 1.3;\ 1.5$ and $\langle \sigma\, v_r \rangle_{AV}$ ($\langle \sigma\, v_r \rangle_{Max}$; $\langle \sigma\, v_r \rangle_{Min}$).

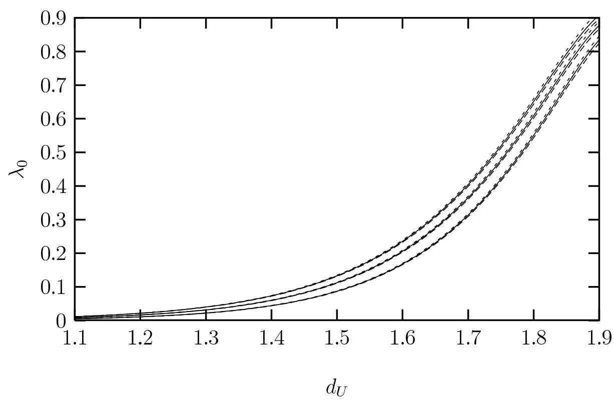

Fig. 2. λ_0 as a function of d_U. Here, the lower, intermediate, upper solid (long dashed; dashed) line represents λ_0 for $m_S = 30;\ 50;\ 70$ GeV and $\langle \sigma\, v_r \rangle_{AV}$ ($\langle \sigma\, v_r \rangle_{Max}$; $\langle \sigma\, v_r \rangle_{Min}$).

Figures 3 and 4 represent m_S and d_U dependence of the annihilation cross section $\langle \sigma\, v_r \rangle$ and, in both figures, the straight solid lines show the estimated upper and lower bounds.

Figure 3 is devoted to the annihilation cross section $\langle \sigma\, v_r \rangle$ with respect to m_S for different values of d_U and λ_0. Here, the solid (long dashed; dashed) line represents $\langle \sigma\, v_r \rangle$ for $d_U = 1.1$ and $\lambda_0 = 0.01$ ($d_U = 1.2$ and $\lambda_0 = 0.01$;

$d_U = 1.5$ and $\lambda_0 = 0.1$). We observe that the $\langle \sigma\, v_r \rangle$ is obtained in the estimated range for $d_U = 1.1$ and $\lambda_0 = 0.01$ in the case of $m_S \sim 60\,\mathrm{GeV}$. For $d_U = 1.2$ and $\lambda_0 = 0.01$, the DM mass should be light, namely $m_S \sim 25\,\mathrm{GeV}$, to get $\langle \sigma\, v_r \rangle$ in the estimated range. For $d_U = 1.5$ and $\lambda_0 = 0.1$, $\langle \sigma\, v_r \rangle$ lies in the estimated range for $m_S \sim 40\,\mathrm{GeV}$. We see that, for a fixed coupling λ_0 (for $\lambda_0 = 0.01$ see this figure), the increase in the scaling dimension d_U results in the decrease in the mass m_S so that $\langle \sigma\, v_r \rangle$ lies in the estimated range.

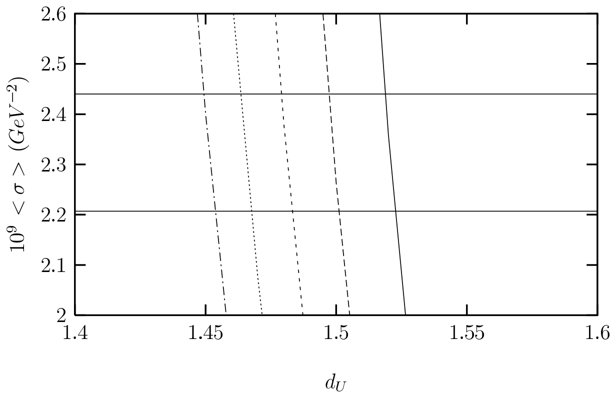

Fig. 3. The annihilation cross section $\langle \sigma\, v_r \rangle$ as a function of m_S. Here, the solid (long dashed; dashed) line represents $\langle \sigma\, v_r \rangle$ for $d_U = 1.1$ and $\lambda_0 = 0.01$ ($d_U = 1.2$ and $\lambda_0 = 0.01$; $d_U = 1.5$ and $\lambda_0 = 0.1$).

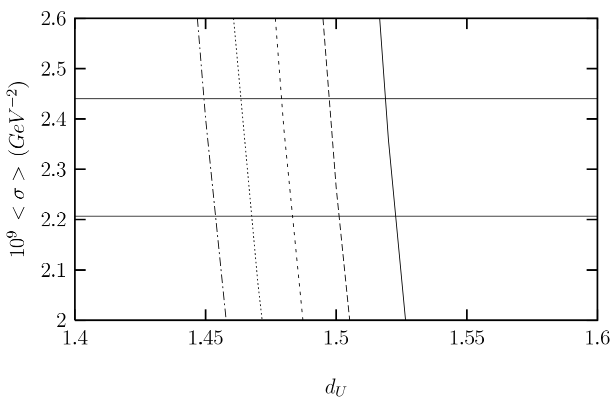

Fig. 4. The annihilation cross section $\langle \sigma\, v_r \rangle$ as a function of d_U. Here, the solid (long dashed; dashed; dotted; dot-dashed) line represents $\langle \sigma\, v_r \rangle$ for $m_S = 30\,(40; 50; 60; 70)\,\mathrm{GeV}$.

Figure 4 represents the annihilation cross section $\langle \sigma\, v_{\rm r} \rangle$ with respect to $d_{\rm U}$ for $\lambda_0 = 0.1$ and different values of $m_{\rm S}$. Here, the solid (long dashed; dashed; dotted; dot-dashed) line represents $\langle \sigma\, v_{\rm r} \rangle$ for $m_{\rm S} = 30\,(40; 50; 60; 70)$ GeV. This figure shows that $\langle \sigma\, v_{\rm r} \rangle$ lies in the estimated range when $m_{\rm S}$ respects $30\,{\rm GeV} < m_{\rm S} < 70\,{\rm GeV}$ and $d_{\rm U} \sim 1.5$.

3. Summary

The annihilation cross section $\langle \sigma\, v_{\rm r} \rangle$ is sensitive to the DM–DM-unparticle coupling λ_0, the DM mass $m_{\rm S}$ and the scaling dimension $d_{\rm U}$. We observe that the coupling λ_0 is strongly restricted for the small values of $d_{\rm U}$ and $m_{\rm S}$. The experimental result of $\langle \sigma\, v_{\rm r} \rangle$ is obtained if λ_0 is of the order of magnitude of 0.01 (0.1) for $1.1 < d_{\rm U} < 1.3$ ($d_{\rm U} \sim 1.5$) in the case of $m_{\rm S} > 40\,{\rm GeV}$. For $d_{\rm U} \sim 1.9$, λ_0 reaches the numerical values of the order of 1.0.

With the forthcoming, more accurate experimental measurements one will provide a considerable information about the mechanism driving the possible annihilation process of the DM and the role of unparticle physics on this process.

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