AN EMPIRICAL FORMULA FOR MASS SPECTRA OF QUARKS AND LEPTONS WHICH MIGHT BE COMPOSITE

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We present an overall empirical formula that, after specification of its free parameters, describes precisely the mass spectrum of charged leptons and is suggested to reproduce correctly also the mass spectra of neutrinos, and up and down quarks (together, twelve masses with twelve specified free parameters are presented). Imposing *a priori* some constraints on three parameters, we predict three of lepton and quark masses, especially the taon mass as $m_\tau = 1776.80$ MeV. We also present a more theoretical argumentation in favor of our mass formula. In Appendix, an option of composite quarks is briefly outlined, where elementary color-triplet quark-like fermions are bound with a color-triplet isoscalar scalar boson ($\mathbf{3}^* \times \mathbf{3}^* \to \mathbf{3}$ color coupling). This option plays here the role of a theoretical hint at imposing a constraint *a priori* on a free parameter in our mass formula for quarks. The possibility of leptons composed from the same preons as are quarks is considered in brief ($\mathbf{3}^* \times \mathbf{3} \to \mathbf{1}$ color coupling).

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Any triplet of particle masses, as these for leptons and quarks, can be phenomenologically parametrized in very different ways by making use of three free parameters. When some parameters are constrained *a priori*, one may get various mass predictions, correct or wrong.

In this note, we will consider the particular parametrization

$$m_N = \rho_N \, \mu \left[N^2 + \frac{\varepsilon - 1}{N^2} - \eta (N-1) \right] \tag{1}$$

in terms of three mass-dimensional parameters

$$\mu \,, \qquad \mu\varepsilon \,, \qquad \mu\eta \,, \tag{2}$$

where

$$N = 1,\ 3,\ 5 \tag{3}$$

is a quantum number numerating the masses

$$m_1\,, \qquad m_3\,, \qquad m_5\,, \tag{4}$$

while

$$\rho_1 = \frac{1}{29}\,, \qquad \rho_3 = \frac{4}{29}\,, \qquad \rho_5 = \frac{24}{29} \tag{5}$$

stand for generation-weighting factors satisfying the normalization condition $\sum_N \rho_N = 1$. Explicitly, the mass formula (1) reads

$$\begin{aligned} m_1 &= \frac{\mu}{29}\,\varepsilon\,, \\ m_3 &= \frac{\mu}{29}\,\frac{4}{9}(80+\varepsilon-18\eta)\,, \\ m_5 &= \frac{\mu}{29}\,\frac{24}{25}\,(624+\varepsilon-100\eta)\,. \end{aligned} \tag{6}$$

It is a transformation of parameters μ, ε, η into masses m_1, m_3, m_5. Its inverse transformation gets the form

$$\begin{aligned} \mu &= \frac{29}{12928}\left[75m_5 - 4(225m_3 - 82m_1)\right]\,, \\ \varepsilon &= \frac{29}{\mu}m_1 = \frac{12928m_1}{75m_5 - 4(225m_3-82m_1)}\,, \\ \eta &= \frac{8}{3}\,\frac{125m_5 - 6(351m_3 - 136m_1)}{75m_5 - 4(225m_3 - 82m_1)}\,, \end{aligned} \tag{7}$$

allowing to fit the free parameters μ, ε, η to experimental values of mases m_1, m_3, m_5. If this can be done with some of the parameters constrained *a priori*, we may obtain some predictions for the mass spectrum.

The reason why we consider here the particular formula (1) is its wonderful propriety of precisely reproducing the triplet of charged-lepton masses m_e, m_μ, m_τ , when we impose *a priori* the constraint

$$\eta = 0\,. \tag{8}$$

In fact, for $m_1 = m_e\,,\, m_3 = m_\mu\,,\, m_5 = m_\tau$, the third formula (7) with $\eta = \eta^{(e)} = 0$ gives the prediction [1, 2]

$$m_\tau = \frac{6}{125}\,(351m_\mu - 136m_e) = 1776.7964\ \text{MeV} = 1776.80\ \text{MeV} \tag{9}$$

versus the experimental value [3]

$$m_\tau = 1776.82 \pm 0.16 \text{ MeV} \tag{10}$$

(the same in both Refs. [3]), when the experimental figures $m_e = 0.5109989$ MeV and $m_\mu = 105.65837$ MeV [3] are used as the only input. The first two formulae (7) determine then the parameters

$$\mu^{(e)} = \frac{29(9m_\mu - 4m_e)}{320} = 85.9924 \text{ MeV}\,, \qquad \varepsilon^{(e)} = \frac{320 m_e}{9m_\mu - 4m_e} = 0.17229\,. \tag{11}$$

In three last paragraphs of this note before Appendix (*cf.*, in particular, Eq. (22)) we present a tentative justification of the mass formula (1) in terms of the notion of "intrinsic interactions".

Now, let us try to impose *a priori* on the formula (1) or (6) the constraint

$$\varepsilon \to 0 \tag{12}$$

and conjecture that it is the case for neutrinos: $m_1 = m_{\nu_1}$, $m_3 = m_{\nu_2}$, $m_5 = m_{\nu_3}$ with $\varepsilon = \varepsilon^{(\nu)} \to 0$. In this case, from the second formula (7), we predict

$$m_{\nu_1} \to 0 \tag{13}$$

and determine then that

$$\begin{aligned} m_{\nu_2} &= \sqrt{\Delta m^2_{21} + m^2_{\nu_1}} \to \sqrt{\Delta m^2_{21}} = 8.8 \times 10^{-3} \text{ eV}\,, \\ m_{\nu_3} &= \sqrt{\Delta m^2_{32} + m^2_{\nu_2}} \to \sqrt{\Delta m^2_{32} + \Delta m^2_{21}} = 5.0 \times 10^{-2} \text{ eV}\,, \end{aligned} \tag{14}$$

when we use the experimental estimates $\Delta m^2_{21} \equiv m^2_{\nu_2} - m^2_{\nu_1} = 7.7 \times 10^{-5}\,\text{eV}^2$ and $\Delta m^2_{32} \equiv m^2_{\nu_3} - m^2_{\nu_2} = 2.4 \times 10^{-3}\,\text{eV}^2$ (assuming that $\Delta m^2_{32} > 0$) [3]. From the formulae (7), we determine the parameters $\mu^{(\nu)} = -9.3 \times 10^{-3}$ eV and $\eta^{(\nu)} = 7.9$ as responsible for $m_{\nu_2} = 8.8 \times 10^{-3}$ eV and $m_{\nu_3} = 5.0 \times 10^{-2}$ eV (the minus sign at $\mu^{(\nu)}$ is consistent with the seesaw mechanism).

In the case of up and down quarks, the concept of mass loses its observable status getting rather an effective character because of their confinement in hadrons (*i.e.*, their nonasymptotic behavior). In this note, we will accept for our discussion the particular quark-mass experimental estimates given in the second Ref. [3]

$$\begin{aligned} m_{u,d} &= \begin{cases} 2.3^{+0.7}_{-0.5} \text{ MeV} \\ 4.8^{+0.7}_{-0.3} \text{ MeV} \end{cases}, \\ m_{c,s} &= \begin{cases} 1.275 \pm 0.025 \text{ GeV} \\ 95 \pm 5 \text{ MeV} \end{cases}, \\ m_{t,b} &= \begin{cases} 173.5 \pm 0.6 \;\; \pm 0,8 \text{ GeV} \\ 4.18 \pm 0.03 \text{ GeV} \end{cases} \end{aligned} \tag{15}$$

(of course, our results will depend on efficiency of this estimate). Then, let us try to fit to these effective masses (to their central values, for simplicity) the free parameters $\mu^{(u,d)}$, $\varepsilon^{(u,d)}$, $\eta^{(u,d)}$ through the inverse transformation (7), obtaining

$$\mu^{(u,d)} = \left\{ \begin{array}{ll} 26.6 & \mathrm{GeV} \\ 0.515 & \mathrm{GeV} \end{array} \right. , \qquad \varepsilon^{(u,d)} = \left\{ \begin{array}{l} 0,00251 \\ 0,270 \end{array} \right. , \qquad \eta^{(u,d)} = \left\{ \begin{array}{l} 4.27 \\ 3.78 \end{array} \right. . \tag{16}$$

We can see from Eqs. (16) that

$$\mu^{(u)}\varepsilon^{(u)} \mathbin{:} \mu^{(d)}\varepsilon^{(d)} = 2.3 \mathbin{:} 4.8 = 1 \mathbin{:} 2.1\,. \tag{17}$$

If, under some structural suggestions leading to the option of composite quarks (*cf.* Appendix), we tentatively conjecture that *a priori* Eq. (A.5) holds

$$\mu^{(u)}\varepsilon^{(u)} \mathbin{:} \mu^{(d)}\varepsilon^{(d)} = 1 \mathbin{:} 2.5 \tag{18}$$

then, we can get from the first Eq. (6) the prediction

$$m_u = 0.4 m_d = 1.9 \ \mathrm{MeV}\,, \tag{19}$$

when the experimental input $m_d = 4.8$ MeV is applied. In this case, the parameter $\mu^{(u)}\varepsilon^{(u)}$ $(= 0.4\mu^{(d)}\varepsilon^{(d)})$ can be considered as constrained *a priori*, while the remaining five independent quark parameters are free and can be determined from Eq. (6) with the use of experimental input of m_d, $m_{c,s}$, $m_{t,b}$ (*cf.* Eq. (16)). The large errors in the experimental estimates (15) of m_u and m_d allow for the tentative conjecture (18). This accepts the theoretical hint presented in Appendix.

Now, we would like to comment on the possible physical meaning of the quantum number $N = 1, 3, 5$ appearing in our overall empirical formula (1) for mass spectra of leptons and quarks. It is natural to presume that the quantum field of any fundamental fermion (lepton or quark) should carry an odd number of Dirac bispinor indices $\alpha_1, \alpha_2, \ldots, \alpha_N : \psi_{\alpha_1 \alpha_2 \ldots \alpha_N}(x)$. Among them, one bispinor index, say α_1, can be correlated with the Standard Model $\mathrm{SU(3)} \times \mathrm{SU_L(2)} \times \mathrm{U(1)}$ label (suppressed here) identifying the considered fermion with a particular lepton or quark. So, this index α_1 is distinguished from the remaining bispinor indices, $\alpha_2, \ldots, \alpha_N$ which are, in a natural way, expected to be undistinguishable from each other. Therefore, the bispinor indices $\alpha_2, \ldots, \alpha_N$ behave as physical objects obeying Fermi statistics along with the Pauli exclusion principle requiring them to be fully antisymmetrized. This implies that N can be equal to 1, 3, 5 only (since any α_i assumes four values 1, 2, 3, 4), and that the total spin of a fundamental fermion is reduced to spin $1/2$ connected with the distinguished bispinor index α_1. Hence, we can conclude that in Nature there are exactly three generations of leptons and quarks [4].

The fundamental fermions with $N = 1, 3, 5$ satisfy three Dirac equations that can differ by their mass terms. The gamma matrices in these Dirac equations with $N = 1, 3, 5$ (fulfilling the Dirac square-root condition $\sqrt{p^2} \rightarrow \mathit{\Gamma}_N^\mu p_\mu$) are [4]

$$\mathit{\Gamma}_N^\mu \equiv \frac{1}{\sqrt{N}} \left(\gamma_1^\mu + \gamma_2^\mu + \ldots + \gamma_N^\mu \right) = \gamma^\mu \otimes \underbrace{\mathbf{1} \otimes \ldots \otimes \mathbf{1}}_{N-1 \text{ times}} , \tag{20}$$

where

$$\left\{ \gamma_i^\mu, \gamma_j^\nu \right\} = 2g^{\mu\nu}\delta_{ij} \quad (i, j = 1, 2, \ldots, N) \tag{21}$$

form a Clifford algebra, while

$$\left\{ \mathit{\Gamma}_N^\mu, \mathit{\Gamma}_N^\nu \right\} = 2g^{\mu\nu} \underbrace{\mathbf{1} \otimes \mathbf{1} \otimes \ldots \otimes \mathbf{1}}_{N \text{ times}} \qquad \text{and} \qquad \{\gamma^\mu, \gamma^\nu\} = 2g^{\mu\nu}$$

define Dirac algebras, the second of them being the familiar one (γ^μ and $\mathbf{1}$ are conventional 4×4 Dirac matrices).

Note in addition that, in this model, the form (5) of fermion generation-weighting factors turns out to be justified [4]. This is due to the fact that only one component ψ_{α_1} of the field $\psi_{\alpha_1\alpha_2\ldots\alpha_N}(x)$ for $N = 1$, and its four components $\psi_{\alpha_1 12} = -\psi_{\alpha_1 21} = \psi_{\alpha_1 34} = -\psi_{\alpha_1 43}$ for $N = 3$ and also twenty four components being permutations of $\psi_{\alpha_1 1234}$ (equal to each other up to the sign) for $N = 5$ can be different from zero, when our "intrinsic Pauli principle" as well as special relativity and probabilistic interpretation of quantum theory [4] are invoked (then, for $N = 3$ twelve components $\psi_{\alpha_1 14} = -\psi_{\alpha_1 41}$, $\psi_{\alpha_1 32} = -\psi_{\alpha_1 23}$, $\psi_{\alpha_1 13} = -\psi_{\alpha_1 31}$, $\psi_{\alpha_1 24} = -\psi_{\alpha_1 42}$ and $\psi_{\alpha_1 11}$, $\psi_{\alpha_1 33}$, $\psi_{\alpha_1 22}$, $\psi_{\alpha_1 44}$ are zero). Here, the chiral representation is used, where $1 = (\uparrow, +1)$, $2 = (\downarrow, +1)$, $3 = (\uparrow, -1)$, $4 = (\downarrow, -1)$ with $\uparrow\downarrow$ being spin-1/2 projections and ± 1 eigenvalues of chirality γ^5.

In this way, we can construct an "intrinsically composite model" of leptons and quarks of three generations [4], where Dirac bispinor indices play the role of "intrinsic partons" of which all but one are undistinguishable and obey the "intrinsic Pauli principle", in contrast to one of them which is distinguished by "carrying" the Standard Model label of a lepton or quark. In the present note, we propose in this context a possible overall empirical formula for the mass spectra of leptons and quarks, where $N = 1, 3, 5$ is a quantum number.

Our intrinsically composite formalism outlined above might either have a fundamental character or be a dynamical S-wave approximation to a more conventional composite model for an odd number of spin-1/2 orbital partons bound mainly in S-states, where one of these partons is distinguished from the rest. In the first option, the conventional orbital partons appearing in the second option are replaced through an act of algebraic abstraction by new algebraic partons described by means of Dirac bispinor indices. Somewhat similarly, the fundamental notion of Dirac spin 1/2 has been arised from the notion of orbital angular momentum through an act of algebraic (group-theoretical) abstraction.

Thus, in our intrinsically composite model of leptons and quarks, there are involved one distinguished intrinsic parton corresponding to the Dirac bispinor index α_1 (correlated with the suppressed Standard Model label) and $N-1$ undistinguishable intrinsic partons related to the Dirac bispinor indices $\alpha_2, \ldots, \alpha_N$ ($N = 1,\, 3,\, 5$). All these partons "interact intrinsically", producing three mass terms in the formula (1). Of course, such "intrinsic interactions" are completely unknown, nevertheless we may try to make them out from the mass formula (1) in terms of the quantum number N and the generation-weighting factor ρ_N (determined in our model by Eq. (5)). In this way, we get an interpretation of the mass formula (1) and so — if it is accepted — a kind of theoretical justification of our spectral formula.

To this end, let us reasonably imagine that in the mass formula (1) we have

$$\begin{aligned} m_N &= (\text{intrinsic masses and two–body interactions of all partons } \alpha_1, \ldots, \alpha_N) \\ &+ (\text{correction to intrinsic mass of distinguished parton } \alpha_1) \\ &+ (\text{correction to intrinsic masses of undistinguishable partons } \alpha_2, \ldots, \alpha_N) \\ &= \rho_N \left(\sum_{i,j=1}^{N} a + b P_N^2 + \sum_{i=2}^{N} c \right) = \rho_N \left[a N^2 + b \frac{1}{N^2} + c(N-1) \right] , \end{aligned} \tag{22}$$

where all intrinsic partons $\alpha_1, \alpha_2, \ldots, \alpha_N$ are treated on an equal footing (*i.e.*, a and c are independent of $i, j = 1, 2, \ldots, N$ and $i = 2, \ldots, N$, respectively), while

$$P_N = \left[\frac{N!}{(N-1)!} \right]^{-1} \tag{23}$$

is the probability of finding the distinguished parton α_1 among all N partons $\alpha_1, \alpha_2, \ldots, \alpha_N$ of which $N-1$, $\alpha_2, \ldots, \alpha_N$, are undistinguishable (and P_N^2 labels a selfinteraction of α_1). Then, we can see that Eq. (22) gives us really the mass formula (1), if we use the notation

$$a = \mu\,, \qquad b = \mu(\varepsilon - 1)\,, \qquad c = -\mu\eta\,, \tag{24}$$

where $\eta \geq 0$ (when it is fitted to experimental masses). Note that charged leptons get here specific states corresponding to the limit $\eta \to 0$ (when η is fitted to their experimental masses). In fact, the input of experimental m_e and m_μ to the mass formula (1) is sufficient to predict precisely m_τ, if and only if $\eta \to 0$ (*cf.* Eqs. (8) and (9)). It means, in spirit of our interpretation (22), that the initial mass values of undistinguishable intrinsic partons "within" charged leptons require no significant correction, in contrast to those "within" neutrinos as well as "within" up and down quarks (of three generations).

Finally, a remark on the third term in the mass formula (22) is due. Here, we reject the option, where this third term should be

$$\rho_N \sum_{i=2}^{N} c\,(1-P_N)^2 = \rho_N\, c\, \frac{(N-1)^3}{N^2} \tag{25}$$

with $1-P_N = (N-1)/N$ denoting the possibility of finding one of the $N-1$ undistinguishable partons $\alpha_2, \ldots, \alpha_N$ among all N partons $\alpha_1, \alpha_2, \ldots, \alpha_N$. The reason for it is that the three-body coupling (25),

$$\rho_N\, c\, \frac{(N-1)^3}{N^2} = \rho_N \sum_{i,j,k=2}^{N} \frac{c}{N^2}\,, \tag{26}$$

is not so simple as the actual third term in Eq. (22) given by the one-body coupling $\rho_N\, c\,(N-1) = \rho_N \sum_{i=2}^{N} c$ correcting the component $2\rho_N\, a(N-1)$ in the first term of Eq. (22)

$$\rho_N\, a\, N^2 = \rho_N\, a\, \left[1 + 2(N-1) + (N-1)^2\right]\,. \tag{27}$$

Note that its component $\rho_N\, a\,(N-1)^2 = \rho_N \sum_{k,l=2}^{N} a$ is a two-body coupling.

Appendix A

Option of orbitally composite quarks and leptons[1]

[1] The resulting fundamental fermions (leptons and quarks) of three generations $N = 1, 3, 5$, carrying the distinguished Dirac bispinor index α_1 and the (suppressed) Standard Model label, may be either elementary (as usually expected, even if intrinsically composite) or, perhaps, orbitally composed from, say, two kinds of elementary color-triplet preons: Dirac preons and a scalar preon (*cf.* Appendix). In the second option, our construction, involving also undistinguishable Dirac bispinor indices $\alpha_2, \ldots, \alpha_N$, defines these Dirac preons in three generations $N = 1, 3, 5$, that become then intrinsically composite. Again, they may be either elementary or may describe a dynamical S-wave approximation to orbitally composite states of N spin-1/2 fermions carrying Dirac bispinor indices $\alpha_1, \alpha_2, \ldots, \alpha_N$ $(N = 1, 3, 5)$ and (suppressed) Standard Model label correlated with α_1. We vote for their intrinsic version not involving orbital compositness. Due to the binding color reduction $\mathbf{3}^* \times \mathbf{3}^* = \mathbf{3} + \mathbf{6}^* \to \mathbf{3}$ or $\mathbf{3}^* \times \mathbf{3} = \mathbf{1} + \mathbf{8} \to \mathbf{1}$, the color-triplet Dirac preons and the color-triplet scalar preon may compose the phenomenological quarks or leptons of three generations, respectively.

All leptons and quarks are usually expected to be elementary (even if they are intrinsically composite). Consider, however, a preliminary model, where all leptons, $\nu_N = \nu_1, \nu_2, \nu_3$ (the mass combinations of flavor ν_e, ν_μ, ν_τ) and $e_N = e^-, \mu^-, \tau^-$ $(N = 1, 3, 5)$ are to be discussed later, while all quarks, $u_N = u, c, t$ and $d_N = d, s, t$ $(N = 1, 3, 5)$ can be treated as orbitally composed from elementary color-triplet constituents, say, quark-like fermions U_N and D_N $(N = 1, 3, 5)$ with charges $Q_{U_N} = \frac{2}{3}$ and $Q_{D_N} = -\frac{1}{3}$, and a color-triplet isoscalar scalar boson S with charge $Q_S = -\frac{1}{3}$

$$u_N = (D_N^c\, S^c) \,, \qquad d_N = (-U_N^c\, S^c) \,. \tag{A.1}$$

They are orbitally composite color-triplet fermions due to the binding color reduction $\mathbf{3}^* \times \mathbf{3}^* = \mathbf{3} + \mathbf{6}^* \to \mathbf{3}$. In Eqs. (A.1), c denotes the charge-conjugation. Of course, the bound states u_N and d_N get the conventional charges

$$Q_{u_N} = \tfrac{1}{3} + \tfrac{1}{3} = \tfrac{2}{3} \,, \qquad Q_{d_N} = -\tfrac{2}{3} + \tfrac{1}{3} = -\tfrac{1}{3} \tag{A.2}$$

and conventional baryon-minus-lepton numbers

$$(B-L)_{u_N} = -\tfrac{1}{3} + \tfrac{2}{3} = \tfrac{1}{3} \,, \qquad (B-L)_{d_N} = -\tfrac{1}{3} + \tfrac{2}{3} = \tfrac{1}{3} \tag{A.3}$$

identified in this case with the baryon numbers $B_{u_N} = \frac{1}{3}$ and $B_{d_N} = \frac{1}{3}$ (then $L_{u_N} = 0$ and $L_{d_N} = 0$). Here, $Q = I^3 + \frac{B-L}{2}$, giving with the use of weak isospins $I^3_{U_N} = \frac{1}{2}$, $I^3_{D_N} = -\frac{1}{2}$, $I^3_S = 0$ the baryon-minus-lepton numbers $(B-L)_{U_N} = \frac{1}{3}$, $(B-L)_{D_N} = \frac{1}{3}$, $(B-L)_S = -\frac{2}{3}$ as are applied in Eqs. (A.3).

When assuming that the parameters $\mu^{(u)}\varepsilon^{(u)} = 29 m_u$ and $\mu^{(d)}\varepsilon^{(d)} = 29 m_d$ in the mass formula (1) or (6) are proportional to the electromagnetic selfenergies of bound states (A.1), and so proportional to

$$Q^2_{D_N} + Q^2_S = \left(\tfrac{1}{3}\right)^2 + \left(\tfrac{1}{3}\right)^2 = \tfrac{2}{9} \,, \qquad Q^2_{U_N} + Q^2_S = \left(\tfrac{2}{3}\right)^2 + \left(\tfrac{1}{3}\right)^2 = \tfrac{5}{9} \,, \tag{A.4}$$

we infer that

$$m_u : m_d = \mu^{(u)}\varepsilon^{(u)} : \mu^{(d)}\varepsilon^{(d)} = \tfrac{2}{9} : \tfrac{5}{9} = 1 : 2.5 \tag{A.5}$$

(here, electric radii of the involved constituents are presumed to be reasonably equal). Thus, from our preliminary model, the ratio $m_d/m_u = 2.5$ follows, turning out to be not far away from the experimental value 2.1 given in Eq. (17). The ratio 2.5 predicts $m_u = 1.9$ MeV, when the input $m_d = 4.8$ MeV is used.

Now, it is interesting to observe that with the use of the same elementary color-triplet constituents, the quark-like fermions U_N and D_N and the isoscalar scalar boson S, we can form orbitally composite color-singlet fermions due to the binding color reduction $\mathbf{3}^* \times \mathbf{3} = \mathbf{1} + \mathbf{8} \to \mathbf{1}$

$$(D_N^c\, S)\ , \qquad (-U_N^c\, S)\ , \tag{A.6}$$

alternative to the previous $\mathbf{3}^* \times \mathbf{3}^* = \mathbf{3} + \mathbf{6}^* \to \mathbf{3}$. These bound states gain the lepton-like charges Q

$$\tfrac{1}{3} - \tfrac{1}{3} = 0\,, \qquad -\tfrac{2}{3} - \tfrac{1}{3} = -1\,, \tag{A.7}$$

respectively, and the lepton-number-like baryon-minus-lepton number $B-L$

$$-\tfrac{1}{3} - \tfrac{2}{3} = -1\ , \qquad -\tfrac{1}{3} - \tfrac{2}{3} = -1\ , \tag{A.8}$$

respectively. Thus, the construction (A.6) may be tried as a tentative model of composite leptons $\nu_N = \nu_1, \nu_2, \nu_3$ and $e_N = e^-, \mu^-, \tau^-$ $(N = 1, 3, 5)$. In this case, $(B-L)_{\nu_N} = -1$ and $(B-L)_{e_N} = -1$ are to be identified with minus the conventional lepton numbers $L_{\nu_N} = 1$ and $L_{e_N} = 1$ (then $B_{\nu_N} = 0$ and $B_{e_N} = 0$).

However, for the construction (A.6), the values $\frac{2}{9}$ and $\frac{5}{9}$ obtained in Eqs. (A.4) for the bound states (A.1) do not change, so our proportionality argumentation cannot explain the experimental smallness of $m_{\nu_1} \ll m_e$, not consistent with the proportion

$$m_{\nu_1} \colon m_e = \mu^{(\nu)}\varepsilon^{(\nu)} \colon \mu^{(e)}\varepsilon^{(e)} = \tfrac{2}{9} \colon \tfrac{5}{9} = 1 \colon 2.5 \tag{A.9}$$

(*cf.* $m_{\nu_1} \to 0$ in Eq. (13)). Thus, the better understanding of the role of charges in the close binding of charged preons is needed to solve the problem of $m_{\nu_1} \to 0$ for our orbitally composite leptons.

To this end, in order to fulfill formally the experimental requirement of $m_{\nu_1} \ll m_e$ (and not to spoil our explanation of the experimental value $m_d/m_u = 2.1$), we may try — at the purely phenomenological level — to replace in our argumentation the values (A.4) by

$$\begin{aligned}
\left(\frac{1}{3}\right)^2 + \left(\frac{1}{3}\right)^2 + \lambda\left(\frac{1}{3}\right)\left(\pm\frac{1}{3}\right) &= \frac{2\pm\lambda}{9} \left\{ \begin{array}{l} \text{for quarks } u_N \\ \text{for leptons } \nu_N \end{array} \right. , \\
\left(\frac{2}{3}\right)^2 + \left(\frac{1}{3}\right)^2 + \lambda\left(-\frac{2}{3}\right)\left(\pm\frac{1}{3}\right) &= \frac{5\mp 2\lambda}{9} \left\{ \begin{array}{l} \text{for quarks } d_N \\ \text{for leptons } e_N \end{array} \right.
\end{aligned} \tag{A.10}$$

with

$$\lambda \to \left\{ \begin{array}{ll} 0 & \text{for quarks} \\ 2 & \text{for leptons} \end{array} \right. . \tag{A.11}$$

Explicitly, the values (A.10) are

$$\left\{ \begin{array}{lll} \frac{2}{9}\ , & \frac{5}{9} & \text{for quarks} \\ 0\ , & 1 & \text{for leptons} \end{array} \right. , \tag{A.12}$$

respectively, being consistent both with $m_d/m_u = 2.1$ and $m_{\nu_1}/m_e \to 0$. The new term on the l.h.s. of Eqs. (A.10), proportional to λ, corresponds formally to a phenomenological electromagnetic interaction between differently located charges $\frac{1}{3}$ and $\pm\frac{1}{3}$ within bound states u_N and ν_N, and between $-\frac{2}{3}$ and $\pm\frac{1}{3}$ within d_N and e_N (it vanishes for u_N and d_N).

The plausible interpretation of ansatz (A.10) with (A.11) is that the role of interaction of differently located charges increases, when the inner radius of the bound states (involving these charges) decreases, leading to the coherent interaction of the sum of all involved charges with itself. This happens in the limit of $\lambda \to 2$ that is realized in the case of leptons (they are here much closer bound states than are quarks because of their color-singlet binding). In the sense of this limit, leptons can be considered as the closest bound states of the charged preons involved.

In our discussion, the electromagnetic interaction dominates in determining the composite masses for $N = 1$, $m_u = m_{u_1}$, $m_d = m_{d_1}$ and m_{ν_1}, $m_e = m_{e_1}$. The Higgs mechanism may be responsible for the preon masses m_{U_N}, m_{D_N} and m_S (m_{U_1}, m_{D_1} and m_S are expected to be tiny).

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