BASIC PHENOMENOLOGY FOR RELATIVISTIC HEAVY ION COLLISIONS*

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Basic concepts used to interpret the soft-hadronic data collected in relativistic heavy ion collisions are reviewed at the elementary level.

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1. Introduction

Physics of relativistic heavy ion collisions is a very broad, interdisciplinary field of physics. It is impossible to cover its all important aspects in a short text. Therefore, one has to decide which topics are selected for presentation. The idea behind this article is to give a possibly general overview which concentrates on soft-hadronic observables and may serve as the background for other advanced lectures presented during the School.

The omitted discussions are partly compensated by references given to many original papers. They will guide the reader in further studies. We also note that at present there are several textbooks available which discuss heavy ion collisions and physics of the quark–gluon plasma [1–5]. We refer to them and to the collected review articles [6] for additional information.

In the remaining part of Introduction, we give a historical outline of the development of the heavy ion physics, present the main theoretical tools, and discuss the concepts of the quark–gluon plasma. In Sec. 2 we discuss

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the main physics terminology used in our field. Section 3 is devoted to the concept of the limiting Hagedorn temperature. This concept has turned out to be very inspiring in the physics of strong interactions and led us directly to the idea of the phase transition from ordinary hadronic matter to quark matter (later called the quark–gluon plasma [7, 8]). In Sec. 4, we discuss shortly the coefficients of the Fourier expansion of the momentum distribution in the azimuthal angle. The first three coefficients are known as the directed, elliptic and triangular flows. The large values of the elliptic flow have been reproduced within the hydrodynamic calculations and suggest the small viscosity to entropy density ratio of the quark–gluon plasma. In Sec. 5, we present the basics of the Glauber model, which is commonly used to determine initial distributions of the entropy or energy density in the transverse plane. The Glauber model serves also as a tool in comparisons between heavy ion and more elementary proton–nucleus and proton–proton collisions. The hydrodynamic approaches are discussed in Sec. 6, where we first recall the famous Fermi, Landau and Bjorken models and then switch to the characteristics of the perfect-fluid and viscous-fluid dynamics. The final stage of the space-time evolution of matter is shortly called a freeze-out. Several approaches to deal with this rather complicated process are presented in Sec. 7. The space-time dimensions of the produced system at freeze-out can be inferred by the study of the identical particle correlations, which we shortly discuss in Sec. 8. The paper is closed with short conclusions. Throughout the text we use natural units, where $c = \hbar = k_{\rm B} = 1$.

1.1. Historical background

The name "heavy ions" is used for heavy atomic nuclei, and the term "relativistic energy" denotes the energy regime where the kinetic energy is much larger than the rest energy. The first experiments with the relativistic heavy ions (energies larger than 10 GeV per nucleon in the projectile beam) took place at the Brookhaven National Laboratory (BNL) and at the European Organisation for Nuclear Research (CERN) in 1986. The Alternating Gradient Synchrotron (AGS) at BNL accelerated ^{28}Si at 14 GeV per nucleon. At CERN, the Super Proton Synchrotron (SPS) accelerated ^{16}O at 60 and 200 GeV per nucleon in 1986, and ^{32}S at 200 GeV per nucleon in 1987. In 1990 the next project on heavy ion physics was organized at CERN with ^{32}S beams. In 1992 the experiments with ^{197}Au beams at 11 GeV per nucleon were initiated at BNL. In 1995 the new experiments took place at CERN with ^{208}Pb beams at 158 GeV per nucleon.

In 2000 the first data from the Relativistic Heavy Ion Collider (RHIC) at BNL were taken. During the first run, the maximum energy of 130 GeV per nucleon pair was achieved. In the next years, new runs took place with

the maximum energy of 200 GeV per nucleon pair. One of those runs was devoted to study deuteron–gold collisions which were analysed in order to get a reference point for more complicated gold–gold collisions.

At present, the main activity in the field is connected with Large Hadron Collider (LHC) at CERN (Pb on Pb reactions at $\sqrt{s_{NN}} = 2.76$ TeV, first run in Nov.–Dec., 2010). However, new experiments at lower energies (NA61 at CERN, STAR at BNL) are also very important, since they allows us to study the energy dependence of many characteristics of the particle production and analyse the systems at finite baryon chemical potential (see Ref. [9]).

1.2. Theoretical tools

In the relativistic heavy ion collisions, very large numbers (multiplicities) of particles are produced. For instance, in the central Au–Au collisions at RHIC, at the beam energy $\sqrt{s_{NN}} = 200$ GeV, the total charged particle multiplicity is about 5000. Hence, the number of produced particles exceeds the number of initial nucleons by a factor of 10. In this situation, different theoretical methods are used, which are appropriate for description of large macroscopic systems, *e.g.*, thermodynamics, hydrodynamics, kinetic (transport) theory, field theory at finite temperature and density, non-equilibrium field theory, Monte-Carlo simulations. More importantly, we may also apply the fundamental theory of strong interactions.

In high-energy nuclear collisions, a many-body system of strongly interacting particles is produced. The fundamental theory of strong interactions is Quantum Chromodynamics (QCD), the theory of quarks and gluons which are confined in hadrons, *i.e.*, baryons and mesons. The discovery of asymptotic freedom in the strong interactions in 1973 by Gross, Politzer, and Wilczek [10, 11] allowed for making precise predictions of the results of many high-energy experiments in the framework of the perturbative quantum field theory — the asymptotic freedom is the property that the interaction between particles becomes weaker at shorter distances (see Ref. [12]).

Probably the most striking feature of QCD is colour confinement, which is the other side of the asymptotic freedom. This is the phenomenon that colour charged particles (such as quarks and gluons) cannot be isolated as separate objects. In other words, quarks and gluons cannot be directly observed. The physical concept of confinement may be illustrated by a string which is spanned between the quarks when we try to separate them. If the quarks are pulled apart too far, large energy is deposited in the string and it breaks into smaller pieces.

1.3. Quark–gluon plasma

The main aim of the relativistic heavy ion collisions is the observation of the two phase transitions predicted by QCD, *i.e.*, the deconfinement and chiral phase transitions. At Earth conditions (*i.e.*, at low energy densities), quarks and gluons are confined in hadrons. However, with increasing temperature (heating) and/or increasing baryon density (compression), a phase transition may occur to the state where ordinary hadrons do not exist any longer; quarks and gluons become the right degrees of freedom, and their motion is not confined to hadrons.

This popular picture is based on the asymptotic freedom — QGP is considered as an asymptotic state available at extremely high energies. Most likely, such a state has not been reached in the present experiments and, more importantly, it is very difficult to find an experimental evidence for its formation. On the other hand, we can accept a more pragmatic point of view and consider QGP as a new state of strongly interacting matter, whose properties can be inferred from experimental and theoretical investigations carried out at the currently available energies (with direct connections to QCD wherever it is possible). The present evidence suggests that the matter produced in heavy ion collisions consists of quarks and gluons (due to the strong coupling these might not be elementary excitations in the system), it is locally well equilibrated, and characterized by the small (shear) viscosity to entropy density ratio. These striking experimental and theoretical findings suggest that QGP behaves more like a fluid than a gas [13, 14].

In the limit of vanishing masses, the left- and right-handed quarks become decoupled from each other and QCD becomes invariant under their interchange — left- and right-handed quark currents are separately conserved, each state of the theory should have a degenerate partner of the opposite parity. On the other hand, we know that hadrons have well defined parity, and no such parity partners are observed. This paradox is resolved by the phenomenon of the spontaneous breakdown of chiral symmetry [15, 16]: the chiral symmetry of the interaction is broken by the true ground state of the theory. One expects that this symmetry is restored at high energies where quarks and gluons become the correct degrees of freedom. This is a very exciting subject but we are not going to follow it any longer in this paper.

2. Basic physics terms

2.1. Participants, spectators, and wounded nucleons

At high energies, simple geometric concepts are often used, for example, one distinguishes participants from spectators — if we assume that all nucleons propagate initially (*i.e.*, before a collision) along parallel, straight line trajectories, then the nucleons which do not strike any other nucleons

on their way are called spectators. Other nucleons which interact with each other are called participants. The participants which suffered at least one inelastic collision are called the wounded nucleons. A two-dimensional vector connecting centres of the colliding nuclei is called the impact vector (its length is the impact parameter). In particle and nuclear physics, one introduces a coordinate system, where the spatial z-axis is parallel to the beam, and the impact vector $\boldsymbol{b}$ points in x-direction. The axes x and z span the reaction plane of a given collision.

The simple picture outlined above is most convenient for theoretical investigations where we control the (initial) geometry of the collision process. On the other hand, the quantities such as the impact parameter or the reaction plane are not directly measured observables and we have to introduce them in a more sophisticated way. Before we do it, let us define the popular ways to parametrise the four-momenta of the produced particles.

2.2. Transverse mass and rapidity

The component of a three-vector $\boldsymbol{A}$ parallel to z-axis is usually denoted by $\boldsymbol{A}_\parallel$, and the transverse component is $\boldsymbol{A}_\perp = \boldsymbol{A} - \boldsymbol{A}_\parallel$. The transverse mass of a particle is defined as $m_\perp = \sqrt{m^2 + \boldsymbol{p}_\perp^2}$, where m and $\boldsymbol{p}$ are the particle's mass and three-momentum (the "transverse" quantities are also denoted by T, *e.g.*, m_T or p_T, the "longitudinal" quantities are then denoted by L, *e.g.*, p_L).

Since we deal with relativistic energies, it is useful to use the rapidity instead of the standard velocity

$$y = \frac{1}{2} \ln \frac{\big(E + p_\parallel\big)}{\big(E - p_\parallel\big)} = \operatorname{arctanh}\left(\frac{p_\parallel}{E}\right) = \operatorname{arctanh}\big(v_\parallel\big) \ . \tag{1}$$

Here, E is the energy of a particle, $E = \sqrt{m^2 + \boldsymbol{p}^2}$, and $v_\parallel = p_\parallel / E$ is the longitudinal component of the velocity.

Rapidity is additive under Lorentz boosts along the z-axis. This means that the difference dy as well as the rapidity density dN/dy do not change under Lorentz boosts along the collision axis. The invariance under this type of transformation (corresponding to a constant dN/dy) is shortly called the boost-invariance.

Using the rapidity and the transverse mass, we can calculate the energy and the longitudinal momentum of a particle from the two equations: $E = p^0 = m_\perp \cosh y$ and $p_\parallel = m_\perp \sinh y$. Experimentalists distinguish between rapidity and pseudorapidity. The latter is defined by the formula

$$\eta = \frac{1}{2} \ln \frac{\big(|\boldsymbol{p}| + p_\parallel\big)}{\big(|\boldsymbol{p}| - p_\parallel\big)} = \ln\left(\cot\frac{\theta}{2}\right) = -\ln\left(\tan\frac{\theta}{2}\right) \ , \tag{2}$$

where θ is the scattering angle. Pseudorapidity is easier to measure than rapidity (it is just a measure of the angle at which a particle has been emitted). To measure rapidity, one has to identify the particle. Since at large energies $E \approx |\boldsymbol{p}|$, one is often tempted to assume that $dN/dy \approx dN/d\eta$. In practice, this approximation is poor, especially in the region where rapidity is close to zero.

In theoretical calculations, one usually uses the space-time rapidity

$$\eta_{\parallel} = \frac{1}{2} \ln \frac{t+z}{t-z} \,. \tag{3}$$

The space-time rapidity $\eta_{\parallel}$ together with the (longitudinal) proper time $\tau = \sqrt{t^2 - z^2}$ are used instead of the coordinates t and z to parametrise the interior of the light cone $t^2 = z^2 \, (t > 0)$. One has to be careful to distinguish between rapidity, pseudorapidity, and space-time rapidity. In realistic calculations they are usually quite different. Only in very simple boost invariant models one may assume that these three quantities are equal.

3. Hagedorn limiting temperature

In 1960s, the statistical bootstrap model (SBM) was introduced [17, 18] that was based on the observation that hadrons form bound and resonance states. This led to the concept of a possibly unlimited sequence of heavy resonance states, each being a constituent of a still heavier resonance. The number of such states in the mass interval $(m, m + dm)$ is denoted by $\rho(m)dm$, and the function $\rho(m)$ is known as the SBM mass spectrum. The requirement that resonances are formed from other resonances in the self-consistent manner leads to the bootstrap condition for the mass spectrum $\rho(m)$. The solution of the bootstrap equation shows that the mass spectrum for large masses m grows exponentially, as found by Hagedorn already in 1965 [17]. As a consequence, any thermodynamics employing this mass spectrum has a singular temperature $T_{\rm H}$ generated by the asymptotics $\rho(m) \sim \exp(m/T_{\rm H})$. At present, $T_{\rm H}$ (Hagedorn temperature) is interpreted as the temperature where the phase transition from the hadron gas to the quark–gluon plasma occurs [19].

The subject of the limiting temperature is still an intriguing issue. More recent studies of the hadron mass spectrum have revealed that the Hagedorn temperatures of mesons and baryons are different [20, 21]. The concepts that we are still missing some resonance states and they may be responsible for (fast) thermalization of the produced matter are widely discussed [22, 23].

4. Harmonic flows

At present, the extraction of the reaction plane is one aspect of the very advanced flow analysis of the collisions [24, 25]. In this type of the investigations, one represents the momentum distribution of the produced particles in the form

$$\frac{dN}{dyd^{2}p_{\perp}} = \frac{dN}{2\pi p_{\perp} dp_{\perp} dy} \left[1 + \sum_{k=1}^{\infty} 2v_k \cos\left(k\left(\phi_p - \Psi_k\right)\right)\right], \tag{4}$$

where Ψ_k is the reference angle defined by the condition $\langle \sin(k\Psi_k) \rangle = 0$, where the averaging is done over all particles in one event. Until very recently, it has been common to assume $\Psi_k = \Psi_{\mathrm{RP}}$ for all ks and to identify the angle Ψ_{RP} with the angle which specifies the position of the reaction plane.

Averaging of (4) over the azimuthal angle gives the transverse-momentum distribution. The coefficients v_k characterize the momentum anisotropy. The coefficient v_1 is called the directed flow, whereas the coefficient v_2 is called the elliptic flow. In general, the coefficients v_k are functions of rapidity and transverse momentum, $v_k = v_k(y, p_{\perp})$, and in this form often called the k^{th} harmonic differential flow.

4.1. Directed flow

At low energies, the directed flow is manifested by the reflection of incoming particles by the first produced regions of highly compressed nuclear matter. At the SPS energies, the situation is already quite complex [26]. At positive rapidities, the proton directed flow is positive, while the pion directed flow is negative. This suggests a different origin of v_1 of protons and pions. At the RHIC energies, the directed flow of charged particles is negative, whereas the v_1 of the spectator neutrons is positive. This trend in the data suggests different behaviour of the matter created in the central region and in the target/projectile fragmentation regions [27, 28].

4.2. Elliptic flow and thermalisation

In a non-central collision (characterised by a finite impact parameter b), the particles are produced in an almond-like region in the transverse plane. This region can be characterised by the spatial anisotropy parameter ϵ. The initial longitudinal momenta of the produced particles are very large (due to the initial impact). On the other hand, the typical initial transverse momenta are small and distributed isotropically. If the produced particles do not interact, their final transverse momenta should be also isotropic.

Contrary, if the particles do interact, the spatial anisotropy is being transferred into the momentum anisotropy [24]. This leads to non-zero values of the coefficient v_2 in Eq. (4), which becomes proportional to ϵ.

At RHIC energies, the measured values of the elliptic flow were explained first by the approaches based on the perfect-fluid hydrodynamics [29]. This led to the conclusion that the system produced in such collisions is characterised by the very small shear viscosity η [13, 14]. Further studies done within dissipative hydrodynamics set an upper limit on the ratio of the shear viscosity η to the entropy density s, which is still very low [30, 31], $1 \leq 4\pi\eta/s \leq 3$ (smaller than for helium at its critical temperature). One has to clarify that the shear viscosity itself is not a good measure of the viscous effects as it is a dimensional quantity. A better measure is the ratio η/s as it is a dimensionless observable (in the natural system of units). Therefore, the recent hydrodynamic studies aim at fixing η/s.

Originally, the large values of v_2 were treated as the evidence of fast thermalisation of the produced matter. A frequently repeated statement was that v_2 could be generated only in the very early times, smaller than 1 fm/c. Model calculations show, however, that the elliptic flow can be generated during the whole time evolution of the system [32, 33] (although the initial growth is the strongest). The realistic values of v_2 can be obtained with scenarios assuming initial free streaming of patrons or large anisotropy of the initial pressure [34, 35]. This allows for delayed thermalisation taking place at the times 1–2 fm/c, which is compatible with the results of different microscopic calculations (see Refs. [36, 39]).

4.3. Triangular flow

Recent hydrodynamic calculations include event-by-event fluctuations in the distribution of the initial energy or entropy density [40, 41] (this effect follows from the fluctuations of the nucleon–nucleon collisions which are described below in the framework of the Glauber model). The fluctuations make this distribution less symmetric and this gives rise to both cosine and sine terms in the Fourier decomposition of the particle momenta in the azimuthal angle. Equivalently, different reference angles Ψ_k should be used for each value of k in the decomposition (4). In event-by-event hydrodynamics, one performs the hydrodynamic calculation for a single event first, extracts v_ks, and then the averaging over the events is done. This leads to non-trivial results for the odd harmonics such as, for example, v_3 which is called the triangular flow [42]. The combined measurements of the elliptic and triangular flow enhance our ability to determine the value of the shear viscosity.

5. Glauber model

Glauber model is used to determine the initial energy (or entropy) density of matter produced in heavy ion collisions[1]. With an additional assumption that the matter is thermalised, one can use the results obtained with the Glauber model as the input for hydrodynamic calculations which determine the subsequent space-time evolution of matter until the freeze-out point.

Originally, the Glauber model was applied only to elastic collisions. In this case, a nucleon does not change its properties in the individual collisions, so all nucleon interactions can be well described by the same (elastic) cross section. Applying the Glauber model to inelastic collisions, we assume that after a single inelastic collision an excited nucleon-like object is created that interacts basically with the same inelastic cross section with other nucleons.

An alternative to the Glauber model calculations are theoretical studies related more directly to QCD. The most common approach of this type is the colour-glass-condensate (CGC) theory [43, 44] which is based on the concept of gluon saturation [45, 46] (see also [36–38]).

5.1. Nucleon–nucleon collisions

At high energies, the inelastic nucleon–nucleon cross section σ_{in} gives the main contribution to the total cross section. A certain subclass of the inelastic processes is the diffractive dissociation process. In this process, a nucleon is only slightly excited and a small number of particles is produced. The diffractive processes contribute to about 10% of all inelastic collisions. In non-diffractive inelastic nucleon–nucleon collisions, a certain number of charged particles is produced. The average charged particle multiplicity $\overline{N}_{NN}$ is described by phenomenological formulas which give $\overline{N}_{NN}$ as a function of the energy $\sqrt{s}$. Such formulas can be used in the comparisons done between heavy ion and hadronic collisions.

Let us consider a nucleon–nucleon collision at a given energy $\sqrt{s}$ and at an impact parameter b. In the eikonal approximation, we may introduce the probability of having a nucleon–nucleon inelastic collision

$$p\left(\boldsymbol{b}\right) = \left(1 - \left|e^{i\chi(\boldsymbol{b})}\right|^2\right) \equiv t\left(\boldsymbol{b}\right)\sigma_{\text{in}}\,, \tag{5}$$

where $\chi(\boldsymbol{b})$ is the phase shift (times a factor of two). The function $t\left(\boldsymbol{b}\right)$, defined by (5), is called the nucleon–nucleon thickness function. The integral of $p\left(\boldsymbol{b}\right)$ over the whole range of the impact parameter should be normalized to σ_{in}. Thus, the thickness function is normalized to unity.

[1] For a recent review of the applications of the Glauber model to describe initial stages of relativistic heavy ion collisions see, for example, Sec. III in Ref. [5].

5.2. Nucleon–nucleus collisions

The probability of finding a nucleon in the nucleus with the atomic mass number A is the usual baryon density divided by the number of baryons in the nucleus. For large nuclei, one commonly uses the Woods–Saxon function (our definition of $\rho_A(r)$ includes A in the denominator, because we want to interpret $\rho_A(r)$ as the probability distribution)

$$\rho_A(r) = \frac{\rho_0}{A}\left(1+\exp\left[\frac{r-r_0}{a}\right]\right)^{-1}, \tag{6}$$

with the parameters: $r_0 = (1.12A^{1/3} - 0.86A^{-1/3})$ fm, $a = 0.54$ fm, $\rho_0 = 0.17$ fm^{-3}. The parameter ρ_0 is the nuclear saturation density.

The nucleon–nucleus thickness function for the nucleus A is obtained from a simple geometric consideration and the assumption that the nucleon positions in the nucleus A are not changed during the collision process

$$T_A(\boldsymbol{b}) = \int dz_A \int d^2 s_A\, \rho_A(\boldsymbol{s}_A, z_A)\; t(\boldsymbol{s}_A - \boldsymbol{b})\;. \tag{7}$$

Here, the transverse coordinates in the nucleus A are denoted by the vector $\boldsymbol{s}_A$ and

$$\rho_A(\boldsymbol{s}_A, z_A) = \rho_A\left(\sqrt{\boldsymbol{s}_A^2 + z_A^2}\right). \tag{8}$$

Our definition of the Woods–Saxon distribution implies the normalization condition

$$\int d^2 b\, T_A(\boldsymbol{b}) = 1\,. \tag{9}$$

The quantity $T_A(\boldsymbol{b})\,\sigma_{\rm in}$ is the probability that a single nucleon–nucleon collision takes place in a nucleon–nucleus collision at the impact parameter $\boldsymbol{b}$. Treating all possible nucleon–nucleon collisions in the nucleon–nucleus collision as completely independent and characterized by the same cross section, we easily find the probability of having n such collisions

$$P(n; A; \boldsymbol{b}) = \left(\begin{array}{c} A \\ n \end{array}\right)\left[1 - T_A(\boldsymbol{b})\,\sigma_{\rm in}\right]^{A-n}\left[T_A(\boldsymbol{b})\,\sigma_{\rm in}\right]^n. \tag{10}$$

The average number of binary nucleon–nucleon collisions may be calculated from the expression $\overline{n}(A;\boldsymbol{b}) = A\,T_A(\boldsymbol{b})\;\sigma_{\rm in}$.

5.3. Nucleus–nucleus collisions

The thickness function for the nucleus–nucleus collision follows also from the geometric considerations which give in this case

$$T_{AB}(\boldsymbol{b}) = \int dz_A \int d^2 s_A \rho_A(\boldsymbol{s}_A, z_A) \int dz_B \int d^2 s_B \rho_B(\boldsymbol{s}_B, z_B) t(\boldsymbol{b}+\boldsymbol{s}_B-\boldsymbol{s}_A)\,, \tag{11}$$

with the corresponding normalization condition $\int d^2 b\, T_{AB}(\boldsymbol{b}) = 1$.

The quantity $T_{AB}(\boldsymbol{b})\,\sigma_{\rm in}$ is the averaged probability that a nucleon–nucleon collision takes place in a nucleus–nucleus collision characterized by the impact parameter $\boldsymbol{b}$. In the limit $t(\boldsymbol{b}) \to \delta^{(2)}(\boldsymbol{b})$, we may write

$$T_{AB}(\boldsymbol{b}) \;=\; \int d^2 s_A\, T_A(\boldsymbol{s}_A)\, T_B(\boldsymbol{s}_A - \boldsymbol{b})\,. \tag{12}$$

The nucleus–nucleus thickness function $T_{AB}(\boldsymbol{b})$ can be used to calculate the probability of having n inelastic binary nucleon–nucleon collisions in a nucleus–nucleus collision at the impact parameter $\boldsymbol{b}$

$$P(n; AB; \boldsymbol{b}) = \left(\begin{array}{c} AB \\ n \end{array} \right) \left[1 - T_{AB}(\boldsymbol{b})\,\sigma_{\rm in}\right]^{AB-n} \left[T_{AB}(\boldsymbol{b})\,\sigma_{\rm in}\right]^n \,. \tag{13}$$

The average number of the collisions is $\overline{n}(AB; \boldsymbol{b}) = AB\, T_{AB}(\boldsymbol{b})\;\sigma_{\rm in}$.

The total probability of an inelastic nuclear collision is the sum over n from $n = 1$ to $n = AB$

$$P_{\rm in}(AB; \boldsymbol{b}) = \sum_{n=1}^{AB} P(n; AB; \boldsymbol{b}) = 1 - \left[1 - T_{AB}(\boldsymbol{b})\,\sigma_{\rm in}\right]^{AB} \,. \tag{14}$$

In more realistic calculations, the positions of nucleons in the target and projectile nucleus are fixed, and the averaging should be done later. The probability of an inelastic collision for a fixed nucleon configuration equals

$$1 - \prod_{j=1}^{A} \prod_{i=1}^{B} \left[1 - t\left(\boldsymbol{b} + \boldsymbol{s}_i^B - \boldsymbol{s}_j^A\right) \sigma_{\rm in}\right] \,. \tag{15}$$

The probability of an inelastic nuclear collision at the impact parameter $\boldsymbol{b}$ is then

$$P_{\rm in}(AB; \boldsymbol{b}) = \int d^2 s_1^A T_A\left(\boldsymbol{s}_1^A\right) \ldots d^2 s_A^A T_A\left(\boldsymbol{s}_A^A\right) \int d^2 s_1^B T_B\left(\boldsymbol{s}_1^B\right) \ldots d^2 s_B^B T_B\left(\boldsymbol{s}_B^B\right)$$
$$\times \left\{ 1 - \prod_{j=1}^{A} \prod_{i=1}^{B} \left[1 - t\left(\boldsymbol{b} + \mathbf{s}_i^B - \boldsymbol{s}_j^A\right) \sigma_{\rm in}\right] \right\} \,. \tag{16}$$

The integration of (16) over b gives $\sigma_{\rm in}^{AB}$. Equations (14) and (16) differ from each other! The more accurate formula (16) is much more complicated to handle and cannot be simply reduced to (14). Only for nucleon–nucleus collisions the two methods are equivalent. Since there is no good analytic method to evaluate (16) for large values of A and B, one is most often satisfied with Eq. (14) only. It is called the optical limit of the Glauber model. In order to have a more reliable distributions, one does the Monte-Carlo calculations [47, 48].

5.4. Wounded nucleons

The Glauber model can be used also to calculate the number of the participants. To be more precise, we distinguish between the participants which may interact elastically and the participants which interact only inelastically. The latter are called the wounded nucleons [49]. The number of nucleons in the nucleus A is $A\int d^2s\, T_A(\boldsymbol{s})$. Probability, that the nucleon from A at the position $\boldsymbol{s}$ collides one or more times with the nucleons in B (in an AB collision at the impact parameter $\boldsymbol{b}$) is

$$\sum_{n=1}^{B} P\left(n;B;\boldsymbol{b}-\boldsymbol{s}\right) = 1-\left[1-\sigma_{\rm in}T_B\left(\boldsymbol{b}-\boldsymbol{s}\right)\right]^B \ . \tag{17}$$

The number of wounded nucleons in A can be obtained from the expression

$$\overline{w}_A\left(A;B;\boldsymbol{b}\right) \ = \ A\int d^2s\, T_A\left(\boldsymbol{s}\right)\left(1-\left[1-\sigma_{\rm in}T_B\left(\boldsymbol{b}-\boldsymbol{s}\right)\right]^B\right) . \tag{18}$$

An analogous expression holds for the number of wounded nucleons in B. Since the number of wounded nucleons in the collision of A and B is the sum of the wounded nucleons in the nucleus A and B, we obtain [49] (after making the appropriate shifts in the integration over positions $\boldsymbol{s}$)

$$\begin{aligned}\overline{w}\left(A;B;\boldsymbol{b}\right) \ = \ & A\int d^2s\, T_A\left(\boldsymbol{b}-\boldsymbol{s}\right)\left(1-\left[1-\sigma_{\rm in}T_B\left(\boldsymbol{s}\right)\right]^B\right) \\ & + B\int d^2s\, T_B\left(\boldsymbol{b}-\boldsymbol{s}\right)\left(1-\left[1-\sigma_{\rm in}T_A\left(\boldsymbol{s}\right)\right]^A\right) .\end{aligned} \tag{19}$$

5.5. Nuclear modification factor

A simple way to quantify the differences between the nucleus–nucleus collisions and the nucleon–nucleon collisions is to calculate the nuclear modification factor

$$R_{AB}(p_\perp) \ = \ \frac{1}{\overline{n}_{AB}}\frac{d^2\overline{N}_{AB}}{dp_\perp d\eta}\Big/\frac{1}{\sigma_{\rm tot}^{pp}}\frac{d\sigma_{\rm incl}^{pp}}{dp_\perp d\eta} , \tag{20}$$

where $\overline{N}_{AB}$ — average number of particles produced in the collisions of the nuclei A and B, $\overline{n}_{AB}$ — number of the binary nucleon–nucleon collisions obtained in the framework of the Glauber model.

The denominator of (20) is the inclusive cross section for pp collisions divided by the total cross section. This quantity is equal to the average number of particles produced in pp collisions in the appropriate phase-space interval

$$\frac{dN_{pp}}{dp_\perp d\eta} = \frac{1}{\sigma_{\rm tot}^{pp}} \frac{d\sigma_{\rm incl}^{pp}}{dp_\perp d\eta}\,. \tag{21}$$

If the collisions of the nuclei A and B are simple superpositions of the elementary pp collisions, the scaling with the number of binary collisions should hold, and the nuclear modification factor is expected to be equal to unity. This is the reason why the nuclear modification factor is so frequently used in direct searches for non-trivial dynamics.

6. Hydrodynamic description of heavy ion collisions

6.1. Early statistical and hydrodynamic models

The use of relativistic hydrodynamics to describe particle production in hadronic collisions has a long history which starts with the famous work of Landau in the early 1950s [50], see also [51, 52]. Landau's considerations were preceded, however, by a few approaches that used pure statistical and thermodynamic concepts in the analysis of hadronic collisions [53–56]. Such approaches may be regarded as pre-hydrodynamic models and it is important to present them before we turn to the discussion of the genuine hydrodynamic models.

6.1.1. Fermi model

Fermi assumed that when two relativistic nucleons collide, the deposited energy is released in a very small volume V, whose magnitude corresponds to the Lorentz contracted characteristic pion field volume V_0, *i.e.*, $V = 2m_N V_0/\sqrt{s}$, where $V_0 = (4/3)\,\pi R_\pi^3$ with $R_\pi = 1/m_\pi$, and where $\sqrt{s}$ is the centre-of-mass energy. Subsequently, such a dense system decays into one of many accessible multiparticle states. The decay probability is calculated in the framework of the standard statistical physics.

The main reason for the introduction of the statistical concepts was the breakdown of the perturbation theory in description of strongly interacting systems. Clearly, the large values of the coupling constant prohibit the

application of the perturbation theory. On the other hand, the large coupling is responsible for the phenomenon of multiparticle production, which is a very characteristic feature of strong interactions.

The probability of the transition into a given state is proportional to the square of the matrix element and to the density of states. In the statistical description, the matrix elements are treated as constants and the main effect comes from the phase space. Thus, the statistical approach represents a simple theoretical modelling of collisions which may be regarded as the complementary approach to the perturbation schemes which typically break down at a certain scale. The main heuristic argument for the justification of the use of the statistical approach is that the role of the phase space naturally grows with the increasing energy of the collisions.

According to Fermi, the probability for the formation of the state with n particles is proportional to the factor

$$S(n) = \left[\frac{V}{(2\pi)^3}\right]^{(n-1)} \frac{dQ(W)}{dW}\,. \tag{22}$$

Here, W is the total energy of the colliding system, dQ/dW is the number of states per unit energy, and V is the interaction volume. The power $n-1$ arises from the fact that the momenta of only $n-1$ particles are independent.

In the last chapter of his seminal paper from 1950, Fermi considers the collisions at extremely high energies. He argues that in this case a detailed statistical considerations may be replaced by the simple thermodynamic arguments. Assuming that the matter is thermalized, one can calculate the temperature of the produced hadronic system from the thermodynamic relations valid for massless particles. Fermi took into account only the production of pions, nucleons and antinucleons, and used the formula

$$\left(\varepsilon_\pi + \varepsilon_{N+\bar{N}}\right) V = \frac{\pi^2 V T^4}{3} = W\,. \tag{23}$$

Using the expression for the Lorentz contracted volume, we rewrite (23) in the following form

$$T^4 = \frac{3}{2\pi^2}\frac{W^2}{V_0 m_N} = \frac{9}{8\pi^3}\frac{W^2 m_\pi^3}{m_N}\,. \tag{24}$$

This equation may be used to calculate the abundances of the produced pions, nucleons and antinucleons from the thermodynamic relations giving the particle densities in terms of the temperature. It is interesting to note that Fermi's idea forms the ground for present thermal model analyses discussed below in Sec. 7.2.

6.1.2. Landau model

In the Landau model [50], an expansion of matter before the hadron decoupling is included. The idea of modification of the Fermi approach was indicated by Pomeranchuk [56]. He argued that the particles in the system should interact until the average distance between them becomes larger than the typical interaction distance. Landau proposed his hydrodynamic approach to describe proton–proton collisions. Following Fermi, he assumed that the two colliding protons released their energy in the volume corresponding to the Lorentz-contracted size of a proton. Under the influence of the longitudinal gradient, the system starts expanding. The transverse gradient is also present but, initially, the gradient in the longitudinal direction is much larger and the early expansion may be regarded as one-dimensional.

6.1.3. Bjorken model

In the Landau model, the initial conditions are specified for a given laboratory time in the centre-of-mass frame, when the matter is highly compressed and at rest. Landau's description does not include one aspect of high-energy processes — fast particles are produced later and further away from the collision centre than the slow particles. It is possible to include this effect by imposing special initial conditions. This idea was proposed and developed by Bjorken [57].

The Bjorken hydrodynamic model [57] is based on the assumption that the rapidity distribution dN_{ch}/dy is constant in the mid-rapidity region. This fact means that the central region is boost invariant. In this case, the longitudinal flow has the form $v_z = z/t$ and all thermodynamic quantities characterizing the central region depend only on the longitudinal proper time $\tau = \sqrt{t^2 - z^2}$ and the transverse coordinates x and y.

The main success of the Bjorken model is that it allows for simple and realistic estimates of the initial energy density available at the early stages of heavy ion collisions. Such estimates always indicate that the produced matter has the energy density much larger than the typical energy density characterising the phase transition from the hadron gas to the quark–gluon plasma. Thus, using Bjorken's simple estimates we expect a formation of a new state of matter in heavy ion collisions at the relativistic energies.

6.1.4. Towards modern hydro approaches

Modern hydrodynamic calculations follow general concepts introduced in the Landau and Bjorken models. However, they differ from the Landau original description in the way how they treat the initial conditions. They also go beyond the simple Bjorken approach by including transverse expansion [58] and, eventually, by breaking the boost-invariance. Additionally, the

recent hydrodynamic codes use modern equations of state inspired by the lattice simulations of QCD and advanced hadron-gas calculations. In addition, the hydrodynamic simulations are performed on the event-by-event basis [40, 41].

In the Landau and Bjorken models, the thermalized matter was a gas of ultra-relativistic particles, mainly pions, satisfying the extreme relativistic equation of state $P = \varepsilon/3$. At present, more accurate equations of state for hot and dense matter are known (for the perturbative QCD calculation, see [59, 60] and for the lattice QCD calculations, see [61, 62] and Hoelbling lecture [63]). Expecting the phase transition to the quark–gluon plasma, one can use the plasma equation of state including the phase transition back to ordinary hadronic matter, see, for example, Refs. [64, 65]. In this way, the phase transition is incorporated in the hydrodynamic frameworks in a very elegant and thermodynamically consistent way.

It is interesting to stress that the use of the appropriate QCD equation of state was crucial for the correct description of the HBT radii (see Kisiel [66]). Moreover, determination of the real character of the phase transition may be helpful to perform realistic calculations within the cosmological models based on the Friedman equation [67].

6.2. Initial conditions

For boost-invariant systems with vanishing baryon chemical potential, one usually assumes that either the initial entropy density, $\sigma_{\rm i}(\boldsymbol{x}_\perp) = \sigma(\tau_{\rm i}, \boldsymbol{x}_\perp)$, or the initial energy density, $\varepsilon_{\rm i}(\boldsymbol{x}_\perp) = \varepsilon(\tau_{\rm i}, \boldsymbol{x}_\perp)$, are directly related to the density of sources of particle production, $\rho_{\rm sr}(\boldsymbol{x}_\perp)$.

The sources considered in this context are wounded nucleons or binary collisions. The symmetry with respect to the Lorentz boosts along the collision axis means that it is sufficient to consider all these quantities in the plane $z = 0$. In general, a mixed model is used, with a linear combination of the wounded-nucleon density $\overline{w}\,(\boldsymbol{x}_\perp)$ and the density of binary collisions $\overline{n}\,(\boldsymbol{x}_\perp)$. This leads to the following expression for the initial entropy [68, 69]

$$\sigma_{\rm i}(\boldsymbol{x}_\perp) \;\propto\; \rho_{\rm sr}(\boldsymbol{x}_\perp) = \frac{1-\kappa}{2}\,\overline{w}\,(\boldsymbol{x}_\perp) + \kappa\,\overline{n}\,(\boldsymbol{x}_\perp)\;, \tag{25}$$

where κ is a fit parameter. The initial longitudinal profiles are less known. One usually uses Gaussian parametrisation of the entropy density in space-time rapidity (with a possible flat insert in the middle). The width of this distribution is chosen in such a way as to reproduce the measured rapidity distribution.

6.3. Hydrodynamics of perfect fluid

The *perfect fluid* is defined formally by the form of its energy-momentum tensor, namely

$$T^{\mu\nu}_{\rm eq} = (\varepsilon + P)u^\mu u^\nu - Pg^{\mu\nu}\,, \tag{26}$$

where $g^{\mu\nu}$ is the metric tensor with $g^{00} = 1$, ε is the energy density, P is the pressure, and u^μ is the four-velocity of the fluid element.

The form (26) follows from the assumption of local thermal equilibrium. Equations of motion of the perfect fluid are then obtained from the conservation laws

$$\partial_\mu T^{\mu\nu}_{\rm eq} = 0\,. \tag{27}$$

In (27) we have four equations. On the other hand, we search for five unknown functions: three independent components of the fluid four-velocity, the energy density, and pressure. The system of equations becomes closed if (27) is supplemented with the equation of state, for example, in the form $P = P(\varepsilon)$. Using (27), one may check that the entropy is conserved, hence the flow is adiabatic. If the fluid has a non-zero baryon density, one should add the baryon number conservation law to (27) as an extra equation.

6.4. Hydrodynamics of viscous fluid

In viscous hydrodynamics, the energy-momentum tensor has the form

$$T^{\mu\nu} = T^{\mu\nu}_{\rm eq} + \Pi^{\mu\nu}\,, \tag{28}$$

where $T^{\mu\nu}_{\rm eq}$ is the perfect-fluid part given by (26) and $\Pi^{\mu\nu}$ describes dissipation

$$\Pi^{\mu\nu} = \pi^{\mu\nu} + \Pi\Delta^{\mu\nu}\,, \qquad \Delta^{\mu\nu} = g^{\mu\nu} - u^\mu u^\nu\,. \tag{29}$$

Here, $\pi^{\mu\nu}$ is the shear tensor and Π describes the viscous bulk pressure. The equations of hydrodynamics follow from the conservation laws for energy and momentum, and from the requirement that the entropy production is positive. These conditions determine the form of equations to be satisfied by the dissipative terms $\pi^{\mu\nu}$ and Π.

From the formal point of view, the inclusion of the dissipative terms in (28) follows from the gradient expansion around the local equilibrium. In the first order in gradients, one finds the Navier–Stokes expressions

$$\pi^{\mu\nu} = \eta\nabla^{\langle\mu}u^{\nu\rangle}\,, \qquad \Pi = \zeta\partial_\alpha u^\alpha\,, \tag{30}$$

where the angle brackets project out the traceless symmetric part (the symmetric part is denoted by round brackets)

$$\nabla^{\langle\mu}u^{\nu\rangle} = 2\nabla^{(\mu}u^{\nu)} - \tfrac{2}{3}\Delta^{\mu\nu}\nabla_\alpha u^\alpha\,, \qquad \nabla^\alpha = \Delta^{\alpha\beta}\partial_\beta\,. \tag{31}$$

The quantities η and ζ in (30) are the shear and bulk viscosity, respectively. Unfortunately, the relativistic fluid dynamics based on the Navier–Stokes prescription suffers from problems connected with the acausal transmission of signals. This is why the second-order theory had been developed by Israel and Stewart [70]. Within the second-order theory, the shear tensor $\pi^{\mu\nu}$ and the bulk pressure Π satisfy non-trivial dynamic equations. They are not any longer expressed by simple formulas such as (30). Moreover, the second-order theory requires that higher-order kinetic coefficients should be introduced.

At the moment, the formalism developed by Israel and Stewart is the most popular version of the dissipative hydrodynamics used to describe heavy ion collisions. Usually, only the shear viscosity is included in such calculations. There are, however, suggestions that the bulk viscosity may also play an important role [71]. More importantly, the second-order formalism may lead to unphysical behaviour at the early stages of the collisions or at the edges of the produced system. Such issues are discussed in the lectures by Strickland [39] in the context of a new formulation of dissipative fluid dynamics (anisotropic hydrodynamics [39, 72, 73]).

7. Freeze-out

7.1. Kinetic freeze-out

The thermal or kinetic freeze-out is the stage in the evolution of matter when the hadrons practically stop to interact. In other words, the thermal freeze-out is a transition from a strongly coupled system (very likely evolving from one local equilibrium state to another) to a weakly coupled one (consisting of essentially free-streaming particles).

It is triggered by the expansion of matter, which causes a rapid growth of the mean free path, $\lambda_{\rm mfp}$, of particles. The thermal freeze-out happens when the timescale connected with the collisions, $\tau_{\rm coll} \sim \lambda_{\rm mfp}$, becomes larger than the expansion timescale, $\tau_{\rm exp}$. In this case, the particles depart from each other so fast that the collision processes become ineffective. We may formulate this condition as the inequality [74]

$$\tau_{\rm coll} \geq \tau_{\rm exp} \,. \tag{32}$$

The magnitude of the collision time is determined by the product of the average cross section and the particle density,

$$\tau_{\rm coll} \sim \frac{1}{\sigma\, n} \,, \tag{33}$$

whereas the magnitude of the expansion time is characterized by the divergence of the four-velocity field, u^μ, describing the hydrodynamic flow of

matter,

$$\tau_{\rm exp} \sim \frac{1}{\partial_\mu u^\mu} \,. \tag{34}$$

Very often a simplified criterion is assumed which says that the thermal freeze-out happens at the time when the mean free path of hadrons is of the same order as the size of the system.

7.2. Chemical freeze-out and thermal models

Chemical freeze-out defines the stage where the hadron abundances are fixed — it should precede the kinetic freeze-out. The concept of the chemical freeze-out is used in thermal models of particle production. In such models, one assumes that a gas of stable hadrons and resonances is formed (at the chemical freeze-out). The final (measured) multiplicities of hadrons consist of primary particles, present in the hot fireball, and of secondary particles coming from the decays of resonances.

There exist several versions of the thermal approach in the literature, for example, see [75–83]. The most popular is the grand canonical version, where one assumes that all hadron species are formed in local thermal and chemical equilibrium. Other versions of the thermal approach assume chemical non-equilibrium in the strange sector of particles, or chemical non-equilibrium in both the strange and non-strange sectors[2].

There is also a possibility that the chemical and thermal freeze-out coincide. Such a single freeze-out model was constructed in Ref. [85] and was successfully used to describe the RHIC soft hadronic observables. Its Monte Carlo version is defined in Refs. [86, 87].

The recent LHC data on heavy ion collisions show that the predictions of two popular versions of the statistical model (the chemical equilibrium model and the strangeness non-equilibrium model) give too large values for the kaon to pion ratio, $(K^+ + K^-)/(\pi^+ + \pi^-)$, and, especially, for the ratio of protons to pions $(p + \bar{p})/(\pi^+ + \pi^-)$ [88, 89]. The recent fit [83] gives almost three standard deviations higher values for protons and anti-protons compared to the LHC data. Besides the problems with thermal interpretation of the hadron abundances at the LHC, one encounters also the problems with the hydrodynamic interpretation of the transverse-momentum spectra of pions, kaons and protons. It turns out that one can connect the proton puzzle with the anomalous behaviour of the pion $p_{\rm T}$ spectra and solve the two problems within the chemical non-equilibrium version of the single freeze-out model [90, 91].

[2] The results of different versions of the thermal model applied at the LHC energy have been recently reviewed in [84].

In the hydrodynamic calculations, the freeze-out process is modelled in two alternative ways: either one uses the concept of a single freeze-out and assumes that the hadrons are completely decoupled on a specific freeze-out hypersurface or one switches from the hydrodynamic description to the hadronic cascade model which relies on the kinetic theory [92].

8. Hanbury–Brown-Twiss interferometry

The fundamental object in the HBT interferometry is the two-particle correlation function $C(\boldsymbol{p}_1, \boldsymbol{p}_2)$, measured for pairs of identical particles such as $\pi^+\pi^+$, $\pi^-\pi^-$, or K^+K^+. In general, it is defined by the expression

$$C(\boldsymbol{p}_1, \boldsymbol{p}_2) \;=\; \frac{\mathcal{P}_2(\boldsymbol{p}_1, \boldsymbol{p}_2)}{\mathcal{P}_1(\boldsymbol{p}_1)\mathcal{P}_1(\boldsymbol{p}_2)}\,, \tag{35}$$

where $\mathcal{P}_1(\boldsymbol{p})$ is the invariant inclusive one-particle distribution function in the space of rapidity and transverse-momentum,

$$\mathcal{P}_1(\boldsymbol{p}) \;=\; E_p \frac{dN}{d^3p} = \frac{dN}{dy d^2p_\perp}\,, \tag{36}$$

and $\mathcal{P}_2(\boldsymbol{p}_1, \boldsymbol{p}_2)$ is the analogous two-particle distribution

$$\mathcal{P}_2(\boldsymbol{p}_1, \boldsymbol{p}_2) \;=\; E_{p_1} E_{p_2} \frac{dN}{d^3p_1 d^3p_2} = \frac{dN}{dy_1 d^2p_{1\perp} dy_2 d^2p_{2\perp}}\,. \tag{37}$$

Equations (36) and (37) imply that the correlation function (35) transforms like a Lorentz scalar. In (35), we may use the average momentum

$$\boldsymbol{k} = \tfrac{1}{2}\,(\boldsymbol{p}_1 + \boldsymbol{p}_2)\,, \tag{38}$$

and the difference of the two momenta

$$\boldsymbol{q} = \boldsymbol{p}_1 - \boldsymbol{p}_2\,. \tag{39}$$

In the analyses of the correlation functions, one uses commonly the out-side-long coordinate system. First, by making the Lorentz boost along the collision axis, we may set $\boldsymbol{k}_\parallel = 0$. In this way, we change to the special frame that is called the longitudinally comoving system (LCMS). In this frame, the direction of the beam is called the long direction. The direction of the three-momentum of the pair is the out direction, and the third orthogonal direction is called the side direction. The measured correlation functions are usually fitted with the Gaussian of the following form[3]

$$C(k_\perp, \boldsymbol{q}) = 1 + \lambda \exp\left[-R^2_{\rm long}(k_\perp) q^2_{\rm long} - R^2_{\rm out}(k_\perp) q^2_{\rm out} - R^2_{\rm side}(k_\perp) q^2_{\rm side}\right]\,. \tag{40}$$

[3] Strictly speaking, the parametrisation (40) is suitable for boost-invariant and azimuthally symmetric systems. In more general cases, one should use more complex formulas.

The parameters $R_{\rm side}$, $R_{\rm out}$, and $R_{\rm long}$ are called the HBT radii. The measurements of the HBT radii as functions of the mean transverse momentum of the pion pairs gives us information about the space-time sizes of the system at the kinetic freeze-out (for more details, see [66]).

9. Conclusions

Successful applications of relativistic hydrodynamics in description of ultra-relativistic heavy ion collisions allowed us to establish a uniform picture of these complicated processes. In some sense, we are in a fortunate situation that such complex systems and processes may be described within a concise and well-defined framework.

The hydrodynamic approach, combined with the modelling of the initial state by the Glauber model or the colour glass condensate on the one side, and supplemented by the kinetic simulations of the freeze-out process on the other side, forms the foundation of an approach that may be regarded as the Standard Model of ultra-relativistic heavy ion collisions. Nevertheless, many details of this picture should be improved and surprises may wait for us just around the corner.

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