MEASURING THE SIZE AND DYNAMICS OF HEAVY ION COLLISIONS WITH FEMTOSCOPY*

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Femtoscopy is a measurement technique used in high energy collisions of hadrons and heavy ions in order to probe their space-time structure and dynamics. It relies on mutual two-particle correlations to extract the size of the region emitting particles. In this paper, we present the theoretical formalism of the method and discuss features observed in the experimental data and their interpretation in hydrodynamic models.

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1. Introduction

The quantum interference of two identical particles has been first used to infer information about an object emitting them (a star) in astronomy. In 1954, Hanbury Brown and Twiss proposed a measurement based on the quantum indistinguishability of photons [1, 2]. Pairs of photons were measured, via the coincidence requirement, in two radio-telescopes separated in space. As the distance between the detectors was decreased, the coincidence rate increased. The width of the coincidence rate distribution as a function of detector spatial separation could be related to the spread of momenta of the incoming photons which, in turn, with the knowledge of the distance to the star, could be converted to its angular size. This type of intensity interferometry was named after the authors: Hanbury Brown–Twiss, or HBT.

In particle physics, an analogous effect was observed by Goldhaber, Goldhaber, Lee, and Pais [3]. They measured an increased production of identically charged pions, with respect to the reference given by pairs of oppositely charged pions. They have correctly interpreted the effect as a Bose–Einstein

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enhancement of the production of identical bosons. Later, Kopylov and Podgoretsky have significantly advanced the technique and developed the modern methodology [4]. It now enables a precision measurement of the size of the particle-emitting region via the construction of the two-particle correlation functions analyzed as a function of the pair relative momentum.

The technique has been extended to measurement of correlations between particles which are not identical [5, 6]. The so-called Final State Interactions (FSI) between pairs of particles, that is Coulomb for charged particles and strong interaction for hadrons, can also be used. The technique then has a formalism which is, to a large extent, mathematically equivalent to the one for identical particles, and enables to perform the measurement of the system size and dynamics. All the techniques, including HBT-like and FSI correlations, are collectively referred to as "femtoscopy", because they enable the measurement of the sizes at the scale of a femtometer, comparable to the hadron size [7].

The HBT technique was developed for stars which are static sources; it can be assumed that they do not change during the measurements. That is not the case for a heavy ion collision which evolves violently as the particle production is taking place. It was soon realized that such evolution produces a specific collective behavior of matter, in particular strong radial and elliptic flows. If the analysis is then performed as a function of the total momentum of the pair, specific patterns arising from this flow can be observed via the so-called "lengths of homogeneity" mechanism. It results in the apparent decrease of the system size with the increase of the pair momentum. The observation of such decrease is a strong argument for the collective nature of matter created in heavy ion collisions. We will explain the effect in detail, when discussing the current interpretation of the experimental femtoscopic data.

2. Femtoscopic formalism

2.1. Definition of the correlation function

The correlation function is defined as a ratio of the conditional probability to observe two particles together, normalized to the product of probabilities to observe each of them separately. In the particular case of femtoscopy, we are interested in mutual two-particle interaction, which is naturally considered in the Pair Rest Frame (PRF), usually as a function of the pair invariant relative momentum $\boldsymbol{q}$. The mutual two-particle correlation comes from the (anti-)symmetrization of the wave function for pairs of identical particles, or from the Final State Interaction (FSI) for charged or neutral hadrons. The femtoscopic correlation function is expressed mathematically as

$$C(\boldsymbol{q}) = \int S(\boldsymbol{r}, \boldsymbol{q}) |\Psi(\boldsymbol{q}, \boldsymbol{r})|^2 d^4\boldsymbol{r} \,, \tag{1}$$

where $\boldsymbol{r}$ is a relative space-time separation of the two particles (usually calculated in PRF) and S is the source emission function which can be interpreted as a probability to emit a particle pair with given momenta from a set of emission points with a given space-time separation. Ψ describes the mutual two-particle correlation, which can have various sources. It is described in detail in Sec. 2.2. The aim of the femtoscopic analysis is to use a measured $C(\boldsymbol{q})$ and a known form of Ψ to deduce information about S. Naturally, this information is limited to the characteristics of the distribution of the relative separation of particles. In particular, if the two particles are identical, the fist moment of S is by definition 0 (S must be symmetric). One can only access the second and higher moments of such distribution. On the other hand, if the two particles are not identical (are distinguishable), the first moment is, in principle, accessible. This is exploited in the so-called "non-identical particle femtoscopy", where the emission asymmetries between particles of different type (usually different mass) are studied [5, 6].

The correlation function is, in the most general case, a six dimensional object (the identities, and therefore the masses of the particles being analyzed must always be known, leaving three independent momentum components for each particle). However, mutual pair interaction only depends on the relative momentum. Therefore, the dependence of the correlation on total momentum is factorized out in the formalism and studied by constructing the correlation separately for pairs in selected ranges of this variable. The remaining correlation is then a function of three components of the relative momentum. It can be analyzed in the traditional Cartesian representation, or in the spherical harmonics decomposition [8–10]. However, the full three-dimensional analysis requires significant statistics, which is often not available, particularly for heavier particles. Then, the correlation is represented in one dimension only, as a function of $q = |\boldsymbol{q}|$. The decomposition of the relative momentum into components is most often done in the Longitudinally Co-Moving System (LCMS), a reference frame in which the total longitudinal (along the beam axis) momentum of the pair vanishes. The three directions are then: longitudinal, outwards (along the pair transverse momentum) and sidewards (perpendicular to the other two). The dependence on pair total momentum reduces then to the dependence on average transverse momentum $k_{\mathrm{T}} = |\boldsymbol{p}_{\mathrm{T},1} + \boldsymbol{p}_{\mathrm{T},2}|/2$ (azimuthal symmetry is implicitly assumed).

2.2. Pair mutual interaction

Particles which are charged will interact via Coulomb, and if they are hadrons, they will also interact via the Strong interaction. This is reflected in the Bethe–Salpeter amplitude for the pair, which corresponds to the solution of the quantum scattering problem, taken with the inverse time direction [11]. Then,

$$\Psi^{(+)}_{-k^*}(\boldsymbol{r}^*) = \sqrt{A_\mathrm{C}(\eta)}\left[e^{-i\boldsymbol{k}^*\boldsymbol{r}^*}F\left(-i\eta,1,i\zeta\right)+f_\mathrm{C}(\boldsymbol{k}^*)\frac{\tilde{G}(\rho,\eta)}{r^*}\right], \quad (2)$$

where A_C is the Gamow factor, $\zeta = k^*r^*(1+\cos\theta^*)$, $\eta = 1/(k^*a_\mathrm{C})$, $\rho = k^*r^*$. F is the confluent hypergeometric function, $\tilde{G}$ is the combination of the regular and the singular s-wave Coulomb function, and f_C is the Coulomb-modified strong interaction scattering amplitude. θ^* is the angle between the pair relative momentum $k^* = |\boldsymbol{k}^*| = q/2$ and relative position $r^* = |\boldsymbol{r}^*|$ in the Pair Rest Frame (PRF), while a_C is the Bohr radius of the pair. In addition, if we are dealing with identical particles, Ψ must also be properly symmetrized. The s-wave approximation is valid in the small k^* region outside of the strong interaction potential. Both conditions are usually met in femtoscopic measurements.

Formula (2) depends on several parameters and its value is usually different in a significant way between various pair types. Pairs of identical pions are most commonly used. For this system, the strong interaction is relatively weak and is usually not considered. Moreover, the Bohr radius is largest of all possible pairs, resulting in a relatively weak Coulomb interaction. The correlation function then has a clear Bose–Einstein enhancement at low q which is damped by the Coulomb as q nears 0. An example correlation function calculated in a model for central Pb–Pb collision is shown in Fig. 1, with the B–E peak appearing below $q_\mathrm{inv} = 0.07$ GeV$/c$, and Coulomb damping strongly affecting only the lowest q points. The sensitivity to the three-dimensional shape is also quite weak in the Coulomb part, so it is mostly the Bose–Einstein effect that enables the measurement of radii in

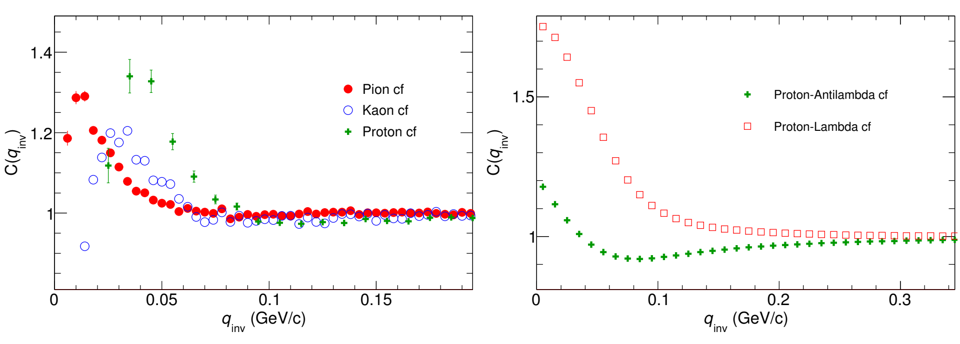

Fig. 1. Example correlation functions calculated in a model for central Pb–Pb collisions for pairs of identical pions, kaons, and protons (left panel) as well as for proton–lambda and proton–antilambda pairs (right panel).

three dimensions. For pairs of identical charged kaons, the situation is similar, although the Coulomb interaction is stronger due to the lower Bohr radius. An example of such correlation function is also shown in Fig. 1, with the B–E peak wider due to a smaller source and the Coulomb damping affecting a significantly wider q region with respect to pions. The picture is more complicated for pairs of protons. They are fermions, and their singlet state contributes an enhancement to the correlation, while the triplet state produces an anti-correlation with the same width. The Coulomb interaction is much stronger than for the lighter mesons and contributes significantly to the correlation shape even at $q = 0.1$ GeV/c. Finally, the strong interaction has a significant positive contribution which forms a peak centered around $q = 40$ MeV/c. All those effects combine into a complicated correlation shape. An example is also shown in Fig. 1. The sensitivity of the system size is now not only in the width, but also in the height of the correlation. For non-identical particles, the effects of Bose–Einstein and Fermi–Dirac statistics are absent, but we still can have a combination of the strong and Coulomb interactions, like for the proton–antiproton pair, or a pure strong interaction correlation, present, for example, for a proton–lambda pair. The case of baryon–antibaryon correlation is especially interesting, since in this case the particle annihilation is represented as imaginary components of the scattering amplitude f, and produces a relatively wide negative correlation, sensitive not only to the system size but also to the magnitude of the imaginary part of the scattering length, which is poorly known for heavier baryon–antibaryon pairs [12].

2.3. Extracting the femtoscopic information

The quantitative analysis of the femtoscopic correlation function amounts to extracting the information about the source from the measured correlation function. In ideal conditions the full information about S can be obtained in this way, for example, via the imaging technique [13]. It does require excellent experimental conditions (large statistics, good two-track resolution, *etc.*) and mathematically advanced analysis tools. A more common approach is to only extract selected properties of the S function, such as its width. We then employ a minimization procedure, to find the parameters of the fitting function which best describe experimental data. But first the fitting function itself must be proposed. We start with the functional form of S, which in heavy ion collision analysis is usually assumed to be a three-dimensional ellipsoid with a Gaussian density profile

$$S(\boldsymbol{r}) \approx \exp\left(-\frac{r_{\mathrm{out}}^2}{4\left(R_{\mathrm{out}}^{\mathrm{G}}\right)^2} - \frac{r_{\mathrm{side}}^2}{4\left(R_{\mathrm{side}}^{\mathrm{G}}\right)^2} - \frac{r_{\mathrm{long}}^2}{4\left(R_{\mathrm{long}}^{\mathrm{G}}\right)^2}\right), \tag{3}$$

where $r_{\rm out}$, $r_{\rm side}$, and $r_{\rm long}$ are components of $\boldsymbol{r}$ calculated in LCMS, and $R_{\rm out}$, $R_{\rm side}$, and $R_{\rm long}$ are single-particle femtoscopic source radii. The S function, together with Ψ from Eq. (2), inserted in Eq. (1) gives the fitting function. However, an analytical form of such function can be given only in selected cases, for example, for pure Quantum Statistics (anti-)symmetrization or for pure strong FSI [14].

For the most common case of identical pions or kaons, additional simplifying assumption is made: the Coulomb part of the charged meson wave-function can be approximately treated as independent from the QS part. It is then integrated separately in a procedure known as Bowler–Sinyukov fitting [15, 16]. Then, Eq. (1) gives the following fit function

$$\begin{aligned} C_{\rm qs}(\boldsymbol{q}) &= (1-\lambda) + \lambda K_{\rm C}(q_{\rm inv}) \\ &\times \left[1 + \exp\left(-\left(R^{\rm G}_{\rm out}\right)^2 q^2_{\rm out} - \left(R^{\rm G}_{\rm side}\right)^2 q^2_{\rm side} - \left(R^{\rm G}_{\rm long}\right)^2 q^2_{\rm long}\right)\right], \end{aligned} \tag{4}$$

where λ accounts for the fact that not all pion pairs are correlated in the source. $K_{\rm C}(q_{\rm inv})$ is the two-pion Coulomb wave-function integrated on a source with Gaussian density profile. The size of this source should be selected to correspond to the size obtained in the fit to the QS part, usually via an iterative procedure. The Bowler–Sinyukov procedure has been shown to be a good approximation for sizes ranging from 1 fm in elementary collisions up to the largest sources measured to date, even up to 9 fm, observed in central Pb–Pb collisions at the LHC [17]. Equation (4) is fitted directly to the experimental correlation functions $C(q)$ to extract the femtoscopic radii.

For heavier particles, a simplified analysis is done in only one dimension in PRF. S is then assumed to be

$$S(\boldsymbol{r}) \approx \exp\left(-\frac{r^{*2}}{4R^2_{\rm inv}}\right), \tag{5}$$

where $R_{\rm inv}$ is the single-particle direction-averaged source size. The equivalent integration is then performed giving the one-dimensional fit function

$$C(\boldsymbol{q}) = (1-\lambda) + \lambda K_{\rm C}(q_{\rm inv}) \left[1 + \exp\left(-R^2_{\rm inv}\, q^2_{\rm inv}\right)\right]. \tag{6}$$

With this function, the quantitative analysis can be performed also with a statistics-limited sample.

The other analytically solvable case is when only the strong interaction is present for a given pair. The one-dimensional correlation function is then [14]

$$C\left(\vec{k^*}\right) = 1 + \sum_S \rho_S \left[\frac{1}{2}\left|\frac{f^S(k^*)}{R_{\rm inv}}\right|^2 \left(1 - \frac{d^S_0}{2\sqrt{\pi}R_{\rm inv}}\right)\right.$$

$$+\frac{2\Re f^S\left(\vec{k^*}\right)}{\sqrt{\pi}R_{\rm inv}}F_1(2k^*R_{\rm inv})-\frac{\Im f^S\left(\vec{k^*}\right)}{R_{\rm inv}}F_2(2k^*R_{\rm inv})\Bigg]\,,\tag{7}$$

where $q = 2k^*$, $F_1(z) = \int_0^z dx e^{x^2-z^2}/z$ and $F_2 = (1 - e^{-z^2})/z$. Summation is done over possible pair spin orientations, with ρ_S the corresponding pair spin fractions. f^S is the strong interaction scattering amplitude (dependent on the spin of the pair), which in this case can be calculated via the effective range approximation

$$f^S(k^*) = \left(\frac{1}{f_0} + \frac{1}{2}d_0 k^{*2} - ik^*\right)^{-1}\,,\tag{8}$$

where f_0 is the scattering length and d_0 is the effective radius of the strong interaction. Both of these parameters are complex numbers, their non-zero imaginary part corresponds to the particle annihilation for a given pair, which appears, for example, for baryon–antibaryon pairs. In Fig. 1, an example of baryon–baryon and baryon–antibaryon correlation functions, calculated according to Eq. (7), is shown for a radius $R_{\rm inv} = 3.5$ fm. For the proton–antilambda pair, a value of $f_0 = 0.49 + i1.00$ fm is used [12] resulting in a wide negative anti-correlation, reflecting the annihilation channel of the interaction.

3. Femtoscopy in experiment

To measure experimentally a correlation that corresponds to the theoretical formula given by Eq. (1) one should isolate only the femtoscopic part of the two-particle correlation. To remove trivial correlations coming from single-particle acceptance the correlation is defined as

$$C(\boldsymbol{q}) = \frac{A(\boldsymbol{q})}{B(\boldsymbol{q})}\,,\tag{9}$$

where A contains pairs of particles coming from the same event, while B contains the "reference" sample. The single particle acceptance for both particles in B should be as close as possible to the one for particles in A. Several methods to construct B in this way are proposed. One consists of taking the A sample and rotating one of the particles by a given angle in the transverse direction. Another, most commonly used, is the so-called "mixing", where the two particles are taken from different events with similar characteristics, such as centrality (final state multiplicity), event plane angle, position of the collision point with respect to the detector (one example: "z-vertex" cut for collider experiments) and so on. The correlation function created in this way contains the two-particle correlations, however it is not guaranteed that all come from femtoscopic effects only. In particular, in small systems significant additional correlations are observed, which then need to be taken into account in the fitting procedure via additional factors [18]. If such background effects are not large, it is usually possible to finally extract the femtoscopic signal and perform the quantitative analysis.

In the experimental femtoscopic analysis, a single measurement of the system size is usually not particularly interesting. It gives a rather limited static picture of the source, and its value can usually be predicted rather easily. What is not trivial, is the dependence of the measured femtoscopic radii (system sizes) on as many variables as possible. Analysis is most commonly done *versus* the pair transverse momentum k_{T}, in three dimensions in LCMS if possible, otherwise in one dimension in PRF. The dependence on event centrality or final state multiplicity is also done very often. Finally, the size is studied *versus* the colliding system and collision energy $\sqrt{s_{NN}}$, ideally in the same accelerator facility and detector system. For non-central collisions at ultra-relativistic energies, the analysis can also be done *versus* the orientation of pair transverse momentum with respect to the reaction plane. In the following section, we briefly describe the theoretical motivation for such studies. A large set of recent experimental results can be found in [17, 19–24] and references therein.

4. Theoretical expectations for femtoscopic radii

The important discovery of heavy ion collisions at ultra-relativistic energies, at Relativistic Heavy-Ion Collider (RHIC), at the Large Hadron Collider (LHC) and, to some extent, at the Super Proton Synchrotron (SPS) was the discovery and the characterization of the new state of strongly interacting matter, the Quark–Gluon Plasma (QGP) [26–29]. Such system is well described by hydrodynamic models with small viscosity. Surprisingly, early calculations by such models failed to quantitatively describe the pion femtoscopy data. It turned out that a correct description of the system sizes required a significant modification of the original assumptions [30, 31]. Firstly, a first-order phase transition between QGP and normal hadronic matter was excluded. Instead, a smooth cross-over had to be used. A non-zero transverse flow at the beginning of the hydro calculation was required. Also a careful treatment of resonance decay and propagation as well as introduction of a small viscosity in the calculation was needed. As a result, a satisfactory description of experimental data was achieved.

As an example of a hydrodynamic model predictions, a recent full calculation for the LHC can be analyzed [25]. The pion radii are extracted as a function of centrality and m_{T}. A strong power-law-like decrease of the radii is observed. This is understood as a non-trivial consequence of strong collective flows, a manifestation of the so-called "lengths of homogeneity" mechanism [32]. A velocity of a particle in a collective system is a convolution of the common "flow" one, always pointing from the center of the source outwards, and a random "thermal" one. For particles with small momentum, the thermal velocity dominates and two particles can be emitted with the same velocity (if their mass is the same, this also means small relative momentum) essentially from the whole volume of the source. The observed radius is then large. However, in order to observe two particles, both with large velocities which have the same direction, one needs two particles for

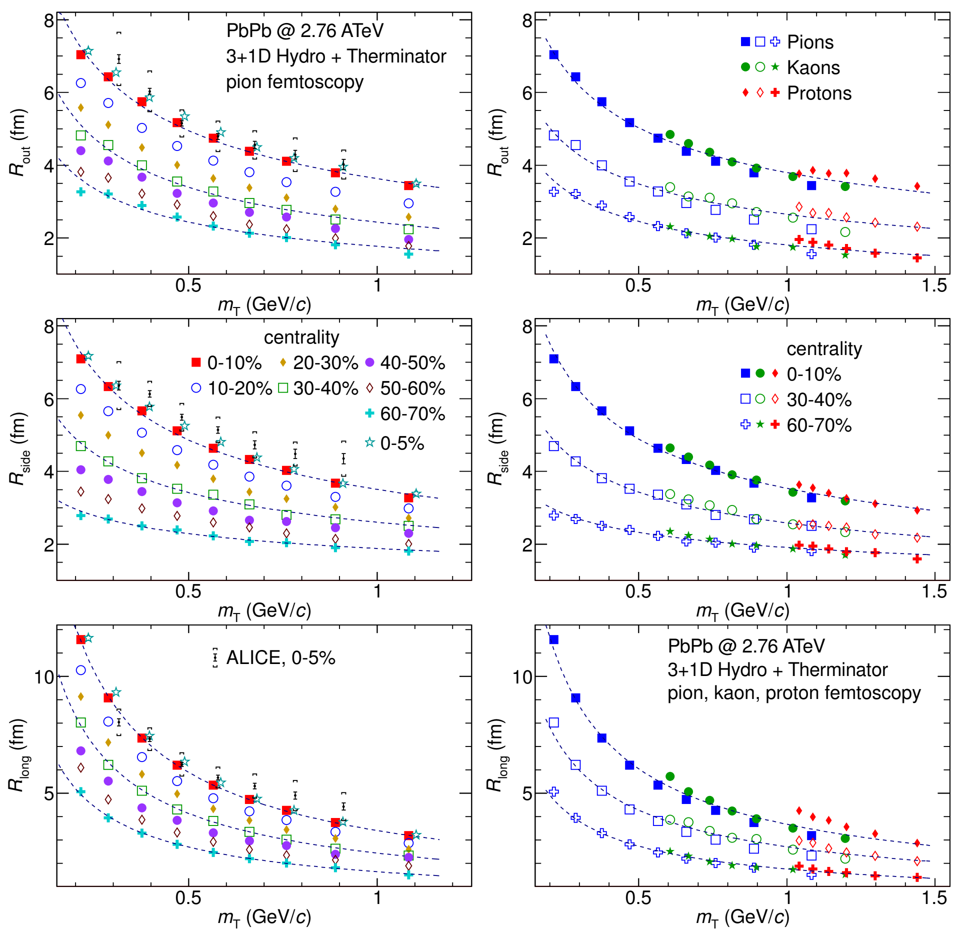

Fig. 2. (Left panel) Pion femtoscopic radii from a hydrodynamic model of Pb–Pb collisions at the LHC, calculated as a function of pair transverse mass $m_{\mathrm{T}} = \sqrt{k_{\mathrm{T}}^2 + m^2}$ and collision centrality. (Right panel) The radii for pions, kaons, and protons as function of pair transverse mass, for selected centralities. From [25].

which the flow and thermal velocities happen to be aligned. But the flow velocity of the two particles is only aligned when they come from parts of the source which are relatively close to each other. Therefore, an effective volume from which such particle pairs can be emitted is much smaller — the radius observed for large m_{T} is small. Such decrease is clearly seen in Fig. 2, taken from [25]. The very nature of the collective flow means that it should be the same for all particle types, regardless of their mass. One should therefore expect that similar behavior of radii is also observed for heavier particles. This is seen in the second panel of Fig. 2, where a universal approximate m_{T} dependence is seen for pions, kaons, and protons.

A hydrodynamic calculation is performed with a set of initial conditions. They are tailored separately for each collision system and centrality range, to correctly describe the transverse momentum spectra of a set of particles species (usually at least pions, kaons, and protons) as well as the transverse momentum dependence of the elliptic flow. As a result, no free parameters remain, and the calculation of the femtoscopic radii for such events can be treated as a prediction and an important cross-check of the consistency of the simulation. The hydrodynamic calculation for the LHC predicts, that the radii scale linearly with $\langle dN_{\mathrm{ch}}/d\eta\rangle^{1/3}$ [25]. Similar scaling was indeed observed for heavy-ion data at lower energies [33]. In summary, the hydrodynamic models allow to predict, via simple scaling laws, femtoscopic radii for a number of particle species (at least pions, kaons, and protons, and possibly heavier baryons) in ultra-relativistic heavy ion central and mid-central collisions. The upcoming experimental verification of such predictions will show whether our detailed understanding of the system created in such collisions is valid.

5. Summary

We have given a short historical introduction to the technique of the femtoscopic measurement. We briefly describe the formalism, giving the important theoretical underpinnings and deriving important formulas such as fitting functions which enable the extraction of the femtoscopic information. We shortly describe the experimental techniques needed to obtain femtoscopic correlation functions. We also summarize the current theoretical understanding of the measured values and discuss how the femtoscopic radii give us insight into the space-time characteristics and dynamics of the source created in the ultra-relativistic heavy ion collisions.

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