

# PHENOMENOLOGY OF THE $ppK^+K^-$ SYSTEM NEAR THRESHOLD \*

M. SILARSKI

The Marian Smoluchowski Institute of Physics, Jagiellonian University  
Reymonta 4, 30-059 Kraków, Poland

(Received January 7, 2014)

In this article, studies of the near threshold  $pp \rightarrow ppK^+K^-$  reaction conducted with the COSY-11 and the ANKE detectors are reviewed. In particular, recent investigations on the  $K^+K^-$  final state interaction are revisited taking into account updated cross sections of the COSY-11 experiment. These studies resulted in the new value of  $K^+K^-$  effective range amounting to:  $\text{Re}(b_{K^+K^-}) = -0.2^{+0.8_{\text{stat}}+0.4_{\text{sys}}}_{-0.6_{\text{stat}}-0.4_{\text{sys}}}$  fm and  $\text{Im}(b_{K^+K^-}) = 1.2^{+0.5_{\text{stat}}+0.3_{\text{sys}}}_{-0.3_{\text{stat}}-0.3_{\text{sys}}}$  fm. The determined real and imaginary parts of the  $K^+K^-$  scattering length were estimated to be:  $|\text{Re}(a_{K^+K^-})| = 10^{+17_{\text{stat}}}_{-10_{\text{stat}}}$  fm and  $\text{Im}(a_{K^+K^-}) = 0^{+37_{\text{stat}}}_{-10_{\text{stat}}}$  fm.

DOI:10.5506/APhysPolB.45.775

PACS numbers: 13.75.Lb, 14.40.Aq

## 1. Introduction

The low energy  $ppK^+K^-$  system provides opportunity to study both the  $pK^-$  and  $K^+K^-$  final state interactions. The latter is of great importance in the still ongoing discussion about the possible formation of the  $K\bar{K}$  bound states [1, 2] which requires a strong attractive potential. The  $pK^-$  final state interaction (FSI) is also very important in view of the unknown structure of the  $\Lambda(1405)$  hyperon which is often considered as the  $NK^-$  molecule, and could provide some hints for the existence of the deeply bound  $ppK^-$  kaonic states [3, 4].

The dynamics of the  $ppK^+K^-$  system has been studied mainly in the proton-proton collisions at the cooler synchrotron COSY at the Research Center in Jülich, Germany [5]. COSY, providing proton and deuteron beams with low emittance and small momentum spread, is an ideal facility for

---

\* Presented at the II International Symposium on Mesic Nuclei, Kraków, Poland, September 22–25, 2013.

measurements at threshold where the cross sections rise rapidly. First measurements of the  $pp \rightarrow ppK^+K^-$  reaction were performed by the COSY-11 Collaboration to study the properties of  $f_0$  and  $a_0$  scalar resonances which are proposed to be a bound state of  $K^+$  and  $K^-$  mesons<sup>1</sup> [1, 2]. These measurements revealed however that the total cross sections for this reaction near threshold are in the order of nanobarns making these studies difficult due to low statistics [10–12]. Moreover, the possible  $f_0$  or  $a_0$  signal was too weak to be observed with COSY-11 in the proton–proton collisions [12, 13]. However, COSY-11 data showed unambiguous signs of the  $pK^-$  final state

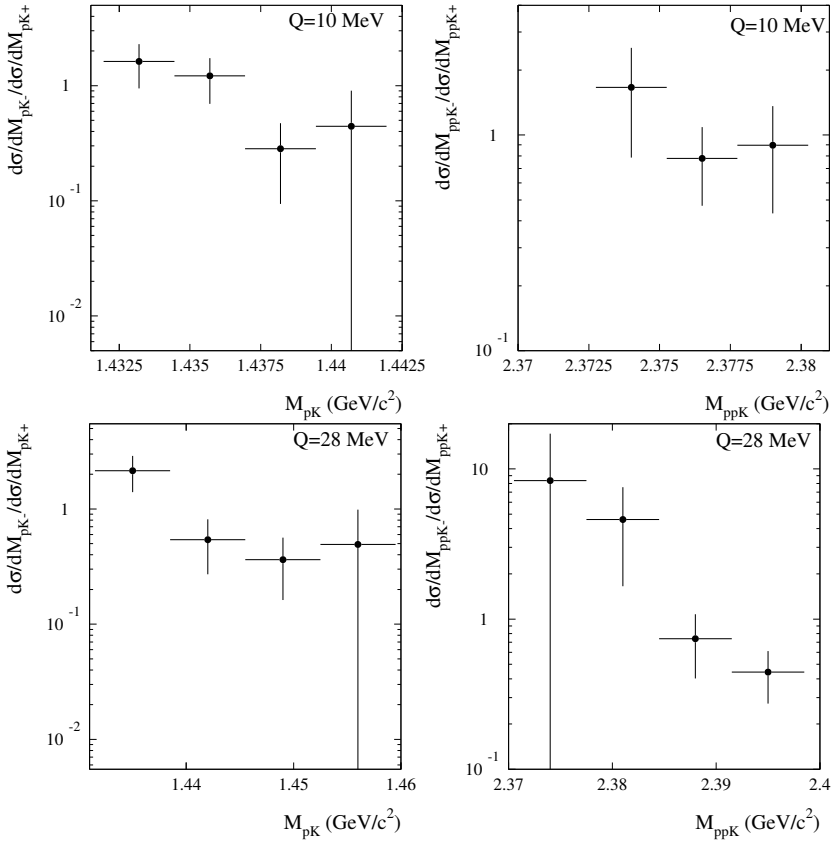


Fig. 1. Ratios of differential cross sections as a function of  $pK^-$  invariant mass ( $M_{pK^-}$ ) and  $ppK^-$  invariant mass ( $M_{ppK^-}$ ) measured by the COSY-11 experiment at excess energies of  $Q = 10$  MeV and  $Q = 28$  MeV [10, 14].

<sup>1</sup> Besides that interpretation, these particles were also considered to be ordinary  $q\bar{q}$  mesons [6], tetraquark states [7], hybrid  $q\bar{q}$ /meson–meson systems [8] or even gluballs [9].

interaction. It manifested itself particularly strongly in the  $pK^-$  and  $ppK^-$  invariant mass distributions measured at excess energies of  $Q = 10$  MeV and  $Q = 28$  MeV. The following ratios:

$$R_{pK} = \frac{d\sigma/dM_{pK^-}}{d\sigma/dM_{pK^+}},$$

$$R_{ppK} = \frac{d\sigma/dM_{ppK^-}}{d\sigma/dM_{ppK^+}},$$

showed a significant enhancement in the region of both low  $pK^-$  invariant mass  $M_{pK^-}$ , and the low  $ppK^-$  invariant mass  $M_{ppK^-}$  [10, 14] (see Fig. 1). Since the  $pK^+$  interaction is known to be very weak, this enhancement indicates a strong influence of the  $pK^-$  final state interaction. This effect has

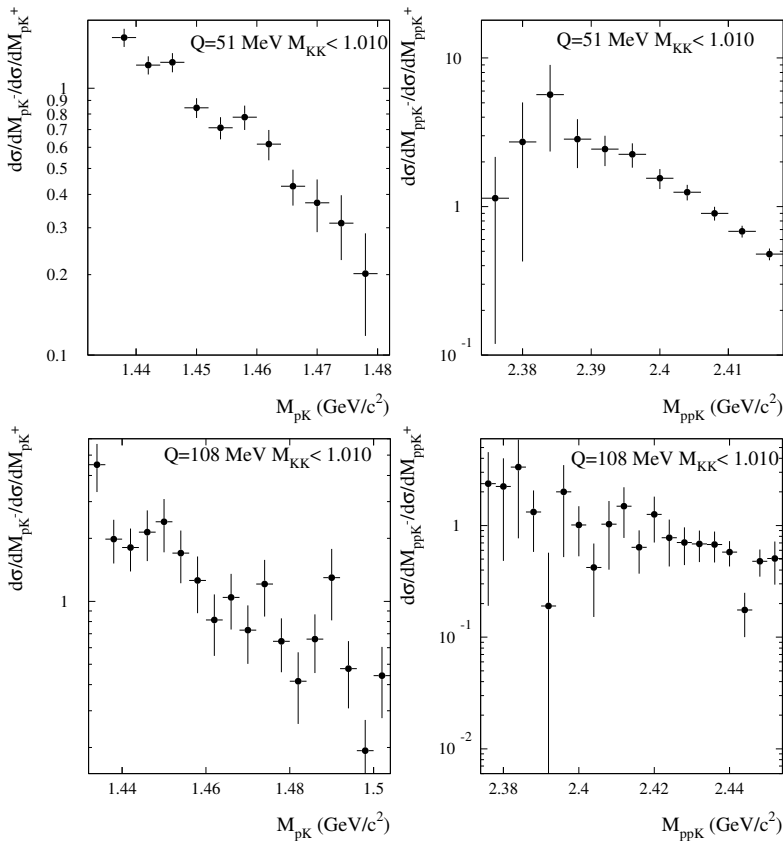


Fig. 2. Ratios of differential cross sections as a function of  $pK^-$  invariant mass  $M_{pK^-}$  and  $ppK^-$  invariant mass  $M_{ppK^-}$  measured by the ANKE experiment at excess energies of  $Q = 51$  MeV and  $Q = 108$  MeV [15].

been then observed also by the ANKE Collaboration at higher energies with data of much better statistics [15–17]. Examples of  $R_{pK}$  and  $R_{ppK}$  distributions measured by the ANKE Collaboration are presented in Fig. 2. The influence of the final state interaction in the low energy  $ppK^+K^-$  system manifests itself also in the shape of the  $pp \rightarrow ppK^+K^-$  excitation function, where one observes a strong deviation from the pure phase space expectations.

## 2. Description of the dynamics in the low energy $ppK^+K^-$ system

Since shapes of the ratios presented in the previous section indicated a strong  $pK^-$  attraction, the  $pp \rightarrow ppK^+K^-$  reaction near threshold was described in terms of the final state interaction. As we are dealing with the close-to-threshold region, the complete transition matrix element for this reaction may be factorized approximately as [18]

$$|M_{pp \rightarrow ppK^+K^-}|^2 \approx |M_0|^2 |M_{\text{FSI}}|^2, \quad (1)$$

where  $|M_0|^2$  represents the total short range production amplitude, and  $|M_{\text{FSI}}|^2$  denotes the final state interaction enhancement factor. The ANKE Collaboration proposed a simple ansatz assuming factorization of  $M_{\text{FSI}}$  to the two-particle scattering amplitudes [19], taking into account strong proton–proton and  $pK^-$  interactions and neglecting the  $K^+$  influence<sup>2</sup>

$$M_{\text{FSI}} = F_{pp}(k_1) \times F_{p_1K^-}(k_2) \times F_{p_2K^-}(k_3), \quad (2)$$

where  $k_1$ ,  $k_2$  and  $k_3$  denote the relative momentum of particles in the proton–proton and two proton– $K^-$  subsystems. Using this approximation one can describe well all the measured differential distributions using an effective scattering length  $a_{pK^-} = i1.5 \text{ fm}$  [15].

This model, however, underestimates COSY-11 total cross sections near threshold, which indicates that in the low energy region the influence of the  $K^+K^-$  final state interaction may be significant. Motivated by this observation, the COSY-11 Collaboration has performed analysis of the low energy  $pp \rightarrow ppK^+K^-$  Goldhaber Plot distributions measured at excess energies of  $Q = 10 \text{ MeV}$  and  $28 \text{ MeV}$  [14]. The final state interaction model used in that analysis was based on the factorization ansatz in Eq. (2), with an additional term describing the interaction of the  $K^+K^-$  pair. The proton–proton scattering amplitude was taken into account using the following parametrization

$$F_{pp} = \frac{e^{i\delta_{pp}(^1S_0)} \sin \delta_{pp}(^1S_0)}{Ck_1}, \quad (3)$$

---

<sup>2</sup> This is a very rough approximation, but more realistic calculations for four-body final states are not available.

where  $C$  stands for the square root of the Coulomb penetration factor [20]. The parameter  $\delta_{pp}(^1S_0)$  denotes the phase shift calculated according to the modified Cini–Fubini–Stanghellini formula with the Wong–Noyes Coulomb correction [21–23]. Moreover, factors describing the enhancement originating from the  $pK^-$  and  $K^+K^-$ -FSI were parametrized using the scattering length approximation

$$F_{pK^-} = \frac{1}{1 - ik a_{pK^-}}, \quad F_{K^+K^-} = \frac{1}{1 - ik_4 a_{K^+K^-}}, \quad (4)$$

where  $a_{pK^-} = i1.5$  fm and  $a_{K^+K^-}$  is the scattering length of the  $K^+K^-$  interaction treated as a free parameter in the analysis. As a result of these studies,  $a_{K^+K^-}$  was estimated to be:  $|\text{Re}(a_{K^+K^-})| = 0.5^{+4}_{-0.5}$  fm and  $\text{Im}(a_{K^+K^-}) = 3 \pm 3$  fm.

This model neglects any coupled channel effects, like *e.g.* the charge-exchange interaction allowing for the  $K^0\bar{K}^0 \rightleftharpoons K^+K^-$  transitions or rescattering to scalar mesons:  $K^+K^- \rightarrow f_0(980)/a_0(980) \rightarrow K^+K^-$ , which would generate a significant cusp effect in the  $K^+K^-$  invariant mass spectrum near the  $K^0\bar{K}^0$  threshold [24], and the  $a_{K^+K^-}$  isospin dependence. The detailed analysis of the  $K^+K^-$  invariant mass distributions measured by the ANKE experiment showed however, that these effects cannot be distinguished from the pure kaons elastic scattering and the production with isospin  $I = 0$  is dominant in the  $pp \rightarrow ppK^+K^-$  reaction independently on the exact values of the scattering lengths [24].

Since the shape of the excitation function for the  $pp \rightarrow ppK^+K^-$  reaction appeared to be quite sensitive to the final state interaction in the close-to-threshold region, we have extended the analysis of differential cross sections measured by the COSY-11 Collaboration at  $Q = 10$  and  $Q = 28$  MeV taking into account in the fit also all the  $pp \rightarrow ppK^+K^-$  total cross sections measured near threshold [25, 26]. Moreover, since the  $pK^-$  scattering length estimated by the ANKE group is rather an effective parameter [15], in this analysis we have used more realistic  $a_{pK^-}$  value estimated independently as a mean of all the scattering length values summarized in Ref. [27]:  $a_{pK^-} = (-0.65 + 0.78i)$  fm. The energy range for the experimental excitation function is rather big, thus the  $K^+K^-$  final state enhancement factor was parametrized using the effective range expansion

$$F_{K^+K^-} = \frac{1}{\frac{1}{a_{K^+K^-}} + \frac{b_{K^+K^-}k_4^2}{2} - ik_4}, \quad (5)$$

where  $a_{K^+K^-}$  and  $b_{K^+K^-}$  are the scattering length and the effective range of the  $K^+K^-$  interaction, respectively. As a result of these studies, we have ob-

tained the following values of  $K^+K^-$  final state interaction parameters [26]:

$$\begin{aligned}\operatorname{Re}(b_{K^+K^-}) &= -0.1 \pm 0.4_{\text{stat}} \pm 0.3_{\text{sys}} \text{ fm}, \\ \operatorname{Im}(b_{K^+K^-}) &= 1.2^{+0.1_{\text{stat}}}_{-0.2_{\text{stat}}} {}^{+0.2_{\text{sys}}}_{-0.0_{\text{sys}}} \text{ fm}, \\ |\operatorname{Re}(a_{K^+K^-})| &= 8.0^{+6.0_{\text{stat}}}_{-4.0_{\text{stat}}} \text{ fm}, \\ \operatorname{Im}(a_{K^+K^-}) &= 0.0^{+20.0_{\text{stat}}}_{-5.0_{\text{stat}}} \text{ fm}.\end{aligned}$$

The fit is, in principle, sensitive to both the scattering length and effective range, however, with the available low statistics data the sensitivity to  $a_{K^+K^-}$  is very weak.

### 3. Update of the COSY-11 total cross sections measured at $Q = 6$ MeV and $Q = 17$ MeV

In all the COSY-11 measurements of the  $pp \rightarrow ppK^+K^-$  reaction, the luminosity needed for evaluation of cross sections was determined based on the simultaneous registration of elastically scattered protons. The differential counting rates of elastic protons scattering measured together with the  $pp \rightarrow ppK^+K^-$  reaction were then compared to data obtained by the EDDA Collaboration. The luminosity for measurements at  $Q = 6$  MeV and  $Q = 17$  MeV was calculated using EDDA data gathered in 1997 [29], while for measurements at the two other excess energies the updated and much more precise EDDA differential cross sections were used [30]. Therefore, we have reevaluated the COSY-11 luminosities at  $Q = 6$  MeV and  $Q = 17$  MeV which resulted in new total cross section values for these excess energies [31]. The updated values of the cross sections are gathered in Table I. One can see that they are slightly higher than the old published total cross sections [11, 12] which increases the observed enhancement at threshold. Therefore, it is worth to check how the values of scattering length and effective range obtained in [26] change for a fit which takes into account the updated COSY-11 cross sections.

TABLE I

Total cross sections measured by the COSY-11 experiment at excess energies of  $Q = 6$  MeV and  $Q = 17$  MeV determined with the old values of luminosity ( $\sigma_{\text{old}}$ ) and taking into account the newest EDDA cross sections ( $\sigma_{\text{new}}$ ).

$Q$ [MeV]	$\sigma_{\text{old}}$ [nb]	$\sigma_{\text{new}}$ [nb]
6	$0.49 \pm 0.40$	$0.51 \pm 0.42$
17	$1.80 \pm 0.27$	$1.88 \pm 0.28$

#### 4. Determination of the $K^+K^-$ -FSI parameters taking into account updated COSY-11 cross sections

In the new fit, we have taken into account not only the updated COSY-11 cross section but also the newest measurement of the ANKE group done at  $Q = 24$  MeV [17]. As in the previous analysis [26], we have preformed combined fit to Goldhaber plots measured at excess energies of  $Q = 10$  MeV and  $Q = 28$  MeV and to the excitation function determined near the threshold. To determine  $a_{K^+K^-}$  and  $b_{K^+K^-}$ , we have constructed the following  $\chi^2$  statistics

$$\chi^2(a_{K^+K^-}, b_{K^+K^-}, \alpha) = \sum_{i=1}^8 \frac{(\sigma_i^{\text{expt}} - \alpha \sigma_i^m)^2}{(\Delta \sigma_i^{\text{expt}})^2} + 2 \sum_{j=1}^2 \sum_{k=1}^{10} \left[ \beta_j N_{jk}^s - N_{jk}^e + N_{jk}^e \ln \left( \frac{N_{jk}^e}{\beta_j N_{jk}^s} \right) \right], \quad (6)$$

where the first term was defined following the Neyman's  $\chi^2$  statistics, and accounts for the excitation function near the threshold for the  $pp \rightarrow ppK^+K^-$  reaction.  $\sigma_i^{\text{expt}}$  denotes the  $i^{\text{th}}$  experimental total cross section measured with uncertainty  $\Delta \sigma_i^{\text{expt}}$  and  $\sigma_i^m$  stands for the calculated total cross section normalized with a factor  $\alpha$  which is treated as an additional parameter of the fit.  $\sigma_i^m$  was calculated for each excess energy  $Q$  as a phase space integral over five independent invariant masses [33]

$$\sigma^m = \int \frac{\pi^2 |M|^2}{8s\sqrt{-B}} dM_{pp}^2 dM_{K^+K^-}^2 dM_{pK^-}^2 dM_{ppK^-}^2 dM_{ppK^+}^2.$$

Here,  $s$  denotes the square of the total energy of the system determining the value of the excess energy, and  $B$  is a function of the invariant masses with the exact form to be found in Nyborg's work [33].

The amplitude for the process  $|M|^2$  contains the FSI enhancement factor defined in Eq. (2) with additional factor expressing the  $K^+K^-$  interaction. The  $pp$ ,  $pK^-$  and  $K^+K^-$  interactions were parametrized according to Eq. (3), Eq. (4) and Eq. (5), respectively. The second term of Eq. (6) corresponds to the Poisson likelihood chi-square value [32] describing the fit to the Goldhaber plots.  $N_{jk}^e$  denotes the number of events in the  $k^{\text{th}}$  bin of the  $j^{\text{th}}$  experimental Goldhaber plot, and  $N_{jk}^s$  stands for the content of the same bin in the simulated distributions.  $\beta_j$  is a normalization factor which is fixed by values of the fit parameters, and which is defined for the  $j^{\text{th}}$  excess energy as the ratio of the total number of events expected from

the calculated total cross section  $\sigma_j^m$  and the total luminosity  $L_j$  [10], to the total number of simulated  $pp \rightarrow ppK^+K^-$  events  $N_j^{\text{gen}}$

$$\beta_j = \frac{L_j \alpha \sigma_j^m}{N_j^{\text{gen}}}.$$

The  $\chi^2$  distributions (after subtraction of the minimum value) are presented as a function of the real and imaginary parts of  $a_{K^+K^-}$  and  $b_{K^+K^-}$  in Fig. 3.

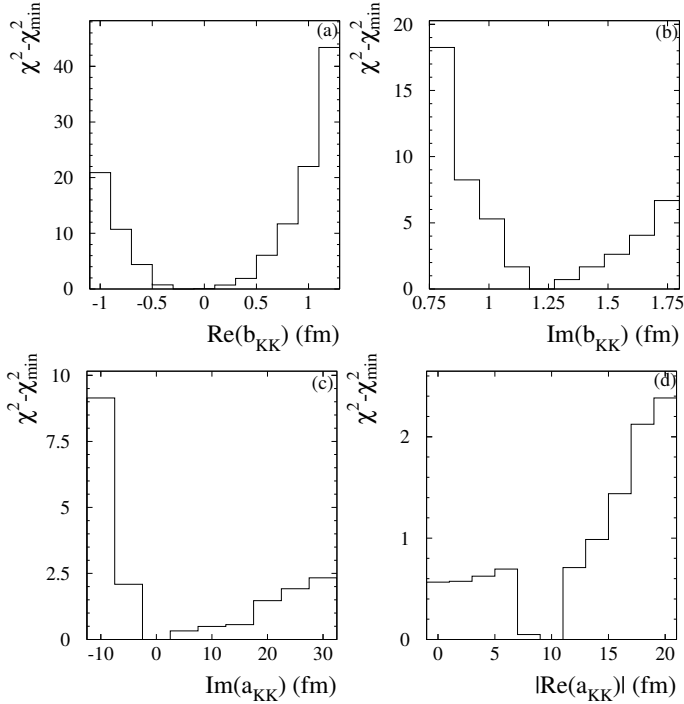


Fig. 3.  $\chi^2 - \chi_{\min}^2$  distribution as a function of: (a)  $\text{Re}(b_{K^+K^-})$ , (b)  $\text{Im}(b_{K^+K^-})$ , (c)  $\text{Im}(a_{K^+K^-})$  and (d)  $|\text{Re}(a_{K^+K^-})|$ .  $\chi_{\min}^2$  denotes the absolute minimum with respect to parameters  $\alpha$ ,  $\text{Re}(b_{K^+K^-})$ ,  $\text{Im}(b_{K^+K^-})$ ,  $|\text{Re}(a_{K^+K^-})|$ , and  $\text{Im}(a_{K^+K^-})$ .

The best fit to the experimental data corresponds to:

$$\begin{aligned} \text{Re}(b_{K^+K^-}) &= -0.2_{-0.6_{\text{stat}}-0.4_{\text{sys}}}^{+0.8_{\text{stat}}+0.4_{\text{sys}}} \text{ fm}, \\ \text{Im}(b_{K^+K^-}) &= 1.2_{-0.3_{\text{stat}}-0.3_{\text{sys}}}^{+0.5_{\text{stat}}+0.3_{\text{sys}}} \text{ fm}, \\ |\text{Re}(a_{K^+K^-})| &= 10_{-10_{\text{stat}}}^{+17_{\text{stat}}} \text{ fm}, \\ \text{Im}(a_{K^+K^-}) &= 0_{-10_{\text{stat}}}^{+37_{\text{stat}}} \text{ fm}, \end{aligned}$$



with a  $\chi^2$  per degree of freedom of:  $\chi^2/\text{ndof} = 1.70$ . The statistical uncertainties in this case were determined at the 70% confidence level taking into account that we have varied five parameters [34]. As in the previous analysis, we have estimated also systematic errors due to the assumed  $pK^-$  scattering length by repeating the analysis for every  $a_{pK^-}$  value quoted in Ref. [27]. Due to the fact that in the case of scattering length the obtained systematic uncertainties are much smaller than the statistical ones, we neglect them in the final result. As one can see in Fig. 4 calculations taking into account  $pp$ ,  $pK^-$ , and  $K^+K^-$  interactions with the scattering length  $a_{K^+K^-}$  and effective range  $b_{K^+K^-}$  obtained from the fit describe the experimental data quite well over the whole energy range.

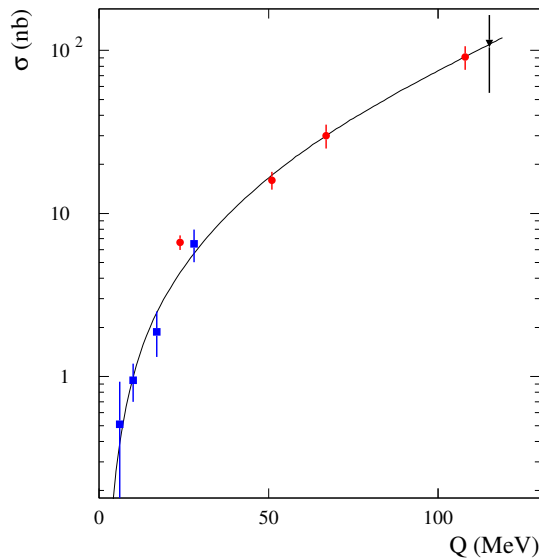


Fig. 4. Excitation function for the  $pp \rightarrow ppK^+K^-$  reaction. Triangle and circles represent the DISTO and ANKE measurements, respectively [15, 17, 28]. The squares are results of the COSY-11 [11, 12, 14] measurements. The solid curve corresponds to the result of calculations obtained taking into account  $pp$ ,  $pK^-$ , and  $K^+K^-$  interactions using the scattering length  $a_{K^+K^-}$  and effective range  $b_{K^+K^-}$  obtained in the fit taking into account updated COSY-11 cross sections and the latest ANKE measurement.

## 5. Summary and outlook

The new analysis of the  $K^+K^-$  final state interaction performed with updated COSY-11 cross sections and taking into account the latest ANKE measurement resulted in the new estimates of the  $K^+K^-$  scattering length

and effective range. As in the previous analysis, the fit is in principle sensitive to the effective range and with the available low statistics the sensitivity to the scattering length is very weak.

The latest ANKE results obtained at  $Q = 24$  MeV suggest however, that for the more accurate description of the interaction in the  $ppK^+K^-$  system a much more sophisticated model than the factorization ansatz used so far is needed [17]. Thus, the results of analysis quoted in this article should be considered rather as effective parameters.

The author is grateful to P. Moskal and E. Czerwiński for their valuable comments and corrections and for providing updated values of the COSY-11 luminosities for measurements at  $Q = 6$  MeV and  $Q = 17$  MeV. This research was supported by the FFE grants of the Research Center Jülich, by the Polish National Science Center and by the Foundation for Polish Science.

## REFERENCES

- [1] D. Lohse *et al.*, *Nucl. Phys.* **A516**, 513 (1990).
- [2] J.D. Weinstein, N. Isgur, *Phys. Rev.* **D41**, 2236 (1990).
- [3] K. Piscicchia *et al.*, *Eur. Phys. J. Web Conf.* **37**, 07002 (2012).
- [4] C. Curceanu *et al.*, *Nucl. Phys.* **A914**, 251 (2013).
- [5] D. Prasuhn *et al.*, *Nucl. Instrum. Methods Phys. Res.* **A441**, 167 (2000).
- [6] D. Morgan, M. R. Pennington, *Phys. Rev.* **D48**, 1185 (1993).
- [7] R.L. Jaffe, *Phys. Rev.* **D15**, 267 (1977).
- [8] E. Van Beveren, *et al.*, *Z. Phys.* **C30**, 615 (1986).
- [9] R.L. Jaffe, K. Johnson, *Phys. Lett.* **B60**, 201 (1976).
- [10] P. Winter *et al.*, *Phys. Lett.* **B635**, 23 (2006).
- [11] M. Wolke, IKP Jül-3532 (1997).
- [12] C. Quentmeier *et al.*, *Phys. Lett.* **B515**, 276 (2001).
- [13] P. Moskal *et al.*, *J. Phys. G* **29**, 2235 (2003).
- [14] M. Silarski *et al.*, *Phys. Rev.* **C80**, 045202 (2009).
- [15] Y. Maeda *et al.*, *Phys. Rev.* **C77**, 015204 (2008).
- [16] Q.J. Ye *et al.*, *Phys. Rev.* **C85**, 035211 (2012).
- [17] Q.J. Ye *et al.*, *Phys. Rev.* **C87**, 065203 (2013).
- [18] P. Moskal, *Hadronic Interaction of  $\eta$  and  $\eta'$  Mesons with Protons*, Jagiellonian University Press, Kraków, 2004, [arXiv:hep-ph/0408162](#).
- [19] C. Wilkin, *Acta Phys. Pol. B Proc. Suppl.* **2**, 89 (2009).
- [20] P. Moskal *et al.*, *Phys. Lett.* **B482**, 356 (2000).
- [21] H.P. Noyes, H.M. Lipinski, *Phys. Rev.* **C4**, 995 (1971).

- [22] H.P. Noyes, *Annu. Rev. Nucl. Part. Sci.* **22**, 465 (1972).
- [23] J.P. Naisse, *Nucl. Phys.* **A278**, 506 (1977).
- [24] A. Dzyuba, *Phys. Lett.* **B668**, 315 (2008).
- [25] M. Silarski, *Acta Phys. Pol. B Proc. Suppl.* **6**, 865 (2013).
- [26] M. Silarski, P. Moskal, *Phys. Rev.* **C88**, 025205 (2013).
- [27] Y. Yan, [arXiv:0905.4818\[nucl-th\]](#).
- [28] F. Balestra *et al.*, *Phys. Lett.* **B468**, 7 (1999).
- [29] D. Albers *et al.* [EDDA Collaboration], *Phys. Rev. Lett.* **78**, 1652 (1997).
- [30] D. Albers *et al.* [EDDA Collaboration], *Eur. Phys. J.* **A22**, 125 (2004).
- [31] E. Czerwiński, P. Moskal, M. Silarski, *Acta Phys. Pol. B* **45**, 739 (2014), this issue.
- [32] S. Baker, R.D. Cousins, *Nucl. Instrum. Methods Phys. Res.* **221**, 437 (1984).
- [33] P. Nyborg *et al.*, *Phys. Rev.* **140**, 914 (1965).
- [34] F. James, *Comput. Phys. Commun.* **20**, 29 (1980).