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SPONTANEOUS TRANSITIONS IN DETERMINISTIC NETWORKS

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The neural assemblies undergo spontaneous changes between various dynamical states characterized usually by spiking or bursting at a single neuron level. These microscopic states contribute to a global neural dynamics that may be measured in a form of electric signal referred to as a local field potential. Here, we present a model neural network composed with nodes exhibiting autonomous spiking dynamics. We show that under a particular coupling configuration and slight mismatches between the nodes, the neural network exhibits deterministic transitions between two possible configurations of clusters. The clusters, composed of two neurons each, differ in internal (always chaotic) dynamics as well as in synchronization properties. Such clusters features may contribute to a temporal increase or decrease of local field potential in the neural network, and thus give an insight into the possible mechanisms of the spontaneous brain transitions. We consider two different models for nodes, namely, forced FitzHugh–Nagumo equations and Rulkov map, and show that the presented results are node-type independent. Finally, we propose a mechanism explaining the origin of these transitions.

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1. Introduction

The study of spontaneous or induced brain activity [1] is accessed experimentally mainly through measurements of electric activity. The sleep/wake state is a spontaneous circadian rhythm, following the endogenous clock that adjust to the external environment. According to the experimental observations [2], the dynamics of neural assemblies undergoes a transition from irregular unsynchronized spiking to synchronized bursting and *vice versa*. While the underlying mechanism of such transitions is still not well understood, there are many mathematical models proposed to explain this phenomenon (*e.g.* Refs. [3, 4]). In particular, they highlight the crucial role of the spike time dependent plasticity in the transitions.

On the other hand, there are also indirect strategies used to study of how the brains work, namely, through psychophysical experiments. A representative case is perceptual bistability, a phenomenon appearing in response to exposure to ambiguous figures like *Necker cube* [5] or to non-stationary ambiguous motion displays [6]. It consists of an alternate perception over time between two possible percepts in an irregular manner. Various mathematical models have been proposed to explain this phenomenon. The simplest model considers neuronal populations in which the alternations are generated with perfect periodicity due to a competition process [7]. However, the data obtained in psychophysical experiments show that these alternations have rather irregular durations [8]. The first hypothesis that the alternations may be mediated by noise was provided by Haken [9]. Many models based on this idea have been proposed, where the switching processes between the two attractors present in a network were elicited by noisy fluctuations [10]. It was demonstrated that the noise-based models reproduce well the main characteristics observed in experiments. On the other side, the existence of a high-dimensional dynamics in neural activity gives a cue that the deterministic processes alone could contribute as well to the emergence of this phenomenon. In fact, there are many experimental data that confirm the existence of complex deterministic dynamics in the brain [11].

In this paper, we propose possible dynamical mechanisms for autonomous switching during mental processes, based on chaos-generating systems coupled in a small network. The network includes direct couplings, both inhibitory and excitatory. We show that in a certain range of the coupling parameters, such a network undergoes bistable behaviour. In particular, we observe the emergence of two clusters exhibiting independent chaotic dynamics, which differ in the mean frequency and degree of correlation. These differences in the microscopic dynamics of clusters contribute to various local field potentials (LFPs). In the case of identical systems, the transitions between the bistable states may be elicited by applying an external stimulus [12]. Here, we show that introducing slight parameter mismatches between the nodes, the transition may emerge spontaneously without any external influences from deterministic or stochastic sources. We consider two types of nodes, FitzHugh–Nagumo systems driven with periodic forcing [13] and Rulkov map [14] showing that the phenomenology does not depend on the node type.

2. Model of a network

The network is composed with two inhibitory and two excitatory neurons coupled in a ring configuration (see Fig. 1 (a)). The coupling term Δx_i for the nodes $i = 1, 2, 3, 4$ is defined as follows:

$$\begin{aligned} \Delta x_{1,3} &= x_4 - x_2 \,, \\ \Delta x_{2,4} &= x_1 - x_3 \,. \end{aligned} \tag{1}$$

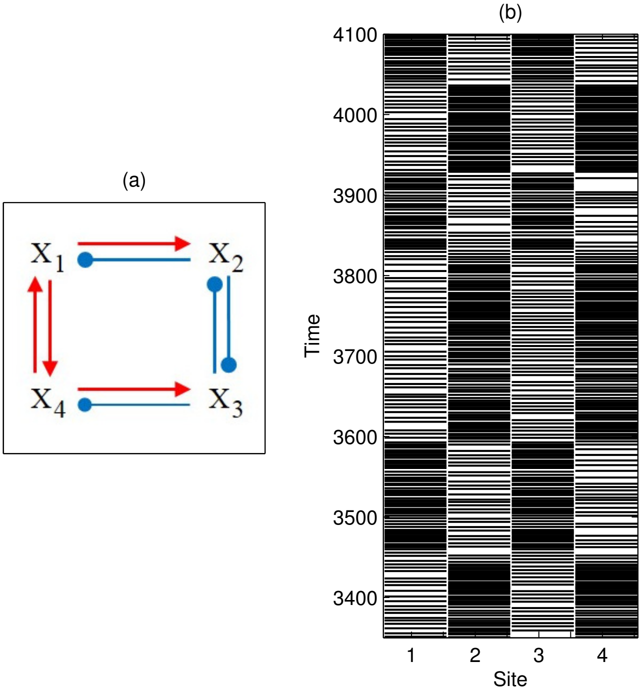

Fig. 1. (a) Scheme of the neural network with inhibitory (circles) and excitatory (arrows) connections in a ring configuration. (b) The raster plot for four coupled FHN systems obtained from the numerical simulations for $K = 2.1$ and $\Delta a = 2 \times 10^{-4}$. Each horizontal line marks the appearance of a spike in time at each site.

We consider two different kinds of systems as nodes. First, we concentrate on driven FHN system [15] that is ruled by the following equations:

$$\begin{aligned} \dot{x}_i &= x_i - x_i^3/3 - y_i + F + K\Delta x_i \,, \\ \dot{y}_i &= \beta(a + \Delta a_i - b y_i + x_i) \,, \end{aligned} \tag{2}$$

where x_i is the fast variable, y_i is the recovery variable, for $i = 1, 2, 3, 4$. $F = A \sin(2\pi\nu t)$ is an external driving term with amplitude A and frequency $\nu = 1/T$, K is the coupling strength. The coupling term $K\Delta x_i$ is composed of excitatory ($+Kx_j$) and inhibitory ($-Kx_k$) links (see Eq. (1)), that are distinguishable by a sign. The external forcing allows to obtain chaotic spiking dynamics, since by introducing a new variable $z_i = 2\pi\nu t = \omega t$, Eq. (2) may be transformed to a three-variable set of equations. We consider fixed parameters $\beta = 0.08$, $b = 0.8$ and $a = 0.7$. The mismatches are introduced through the mismatch parameter Δa_i. The second type of the systems we consider is a Rulkov map [14], defined as follows:

$$\begin{aligned} x^i_{n+1} &= f\left(x^i_n, y^i_n\right) + K\Delta x^i_n\,, \\ y^i_{n+1} &= y^i_n - \mu\left(x^i_n + 1\right) + \mu\sigma_n\,, \end{aligned} \tag{3}$$

where n marks a discrete time and $i = 1, 2, 3, 4$ marks the i^{th} system. Function f is the following:

$$f(x, y) = \left\{ \begin{array}{ll} \alpha_i/(1-x) + y\,, & x \leq 0 \\ \alpha_i + y\,, & 0 < x < \alpha_i + y \\ -1\,, & x \geq \alpha_i + y \end{array} \right\}. \tag{4}$$

We use the parameters $\mu = 10^{-3}$ and $\sigma = 0.16$. The parameter $\alpha_i = \alpha + \Delta\alpha_i$, where $\alpha = 4.6$ and $\Delta\alpha_i$ is a control parameter used to introduce the mismatches into the network. Parameter K, also in this case, is the coupling strength.

Both selected systems, in a certain range of control parameters, exhibit chaotic spiking dynamics, *e.g.* they elicit high amplitude spikes separated by irregular (chaotic) time intervals.

3. Results

The formation of synchronized clusters is observed when the proper coupling strength and mismatches are selected. The bistable switching occurs for the pairs of exhibitory–inhibitory neurons, which we call *clusters*. In Fig. 1 (b) we show the raster plots for the FHN system in the regime of autonomous switching between the two clusters. We estimate the correlation between the sites by using the Kuramoto order parameter [16]

$$R(t) = \frac{1}{M}\sum_{k=1}^{M} \left|\exp(i\theta_k(t))\right|\,, \tag{5}$$

where M is the number of nodes and $\theta(t)$ is the phase of each node at time t calculated from the following formula:

$$\theta(t) = 2\pi(t - s_{\mathrm{prev}})/(s - s_{\mathrm{prev}})\,, \tag{6}$$

where s is the time of spike occurrence and s_{prev} is the time of spike occurrence previous to s.

The parameter R varies from 0 to 1, indicating low and high correlations, respectively. We consider the mean of R in time, calculated over properly selected time intervals much smaller than the magnitude of the dominance times (residence times in the $R = 1$ state). Then, we introduce a threshold in order to separate temporal intervals for the full synchronization (at $R = 1$) and for uncorrelated dynamics (for $R < 1$). In Fig. 2 (a) we show the time evolution of $\langle R \rangle$ and in Fig. 2 (b) the corresponding rates for spiking. We observe that the crucial difference between the two formed clusters is not only in the degree of synchronization, but also in the mean frequency of chaotic spiking. As shown in Fig. 2 (a)–(b), one cluster dominates temporally over the other through the higher mean frequency and correlations, and thus contributes to higher values of LFP. We define LFP as a sum of spikes (action potentials) which appear during a certain period of time Δt. If there are no spikes during this period then LFP $= 0$, otherwise, if all neurons fire, the value of LFP reaches its maximum.

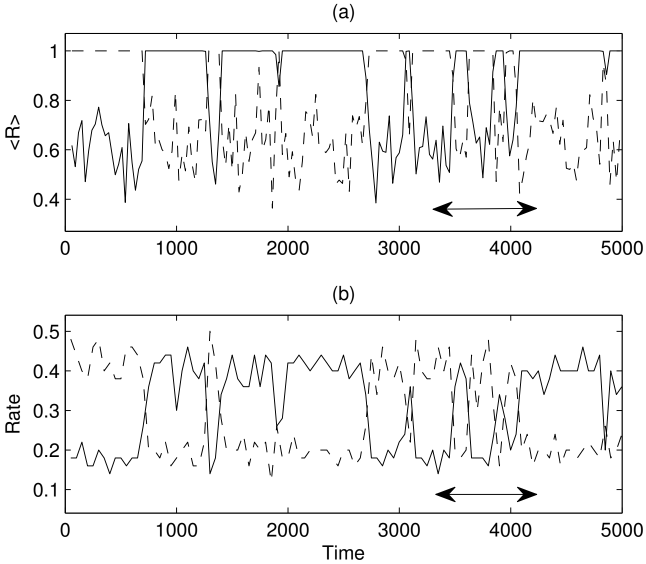

Fig. 2. (a) Mean Kuramoto parameter $\langle R \rangle$ *versus* time. (b) Rate *versus* time. The solid and dashed lines are for the two clusters, respectively. The horizontal double arrows show the temporal period for which the raster plot from Fig. 1 has been plotted.

We initially choose the mismatches between the systems such that $\Delta a_1 = \Delta a_4 = \Delta a$ and $\Delta a_2 = \Delta a_3 = 0$, thus the mismatches are introduced symmetrically into the network. In such a case, the distributions of dominance durations for given cluster are exponential (see Fig. 3 (a)) and so we consider the curve defined as $p \sim \exp(-\gamma T_{\rm d})$, where $T_{\rm d}$ stands for dominance duration. The exponents γ are calculated using numerical data fitted to the curve $\log(p) \sim -\gamma T_{\rm d}$ with the method of ordinary least squares. The exponents in the case of both formed clusters are the same. As the value of mismatch Δa increases, the value of γ decreases. It means that larger mismatches contribute to the faster exchange in clusters domination. Also, the synchronization without switching between the states is reached at the lower value of K in the case of smaller γ. In Fig. 3 (b) it can be noticed that there is a range of coupling strengths K in between the de-synchronization and synchronization regimes, where the autonomous switching is possible.

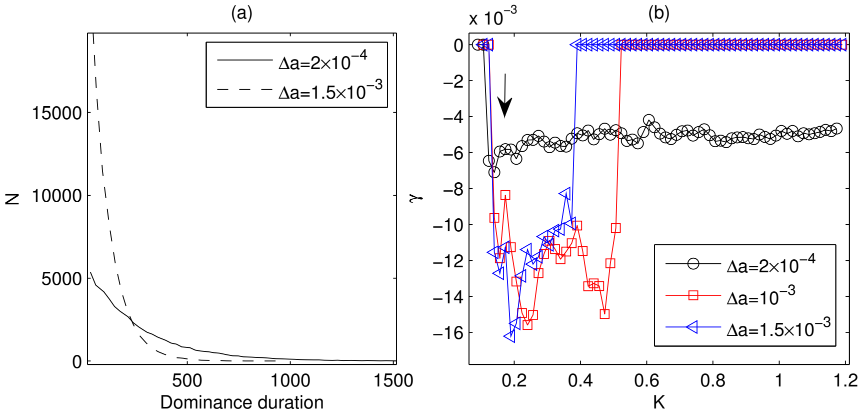

Fig. 3. (a) Selected distributions for dominance durations. (b) Exponents γ for varying coupling parameter K at fixed mismatches Δa. The vertical arrow marks the coupling strength at which the distributions in (a) are plotted. In the case of $\Delta a = 2 \times 10^{-4}$, the synchronization is reached approximately at $K = 2.1$.

Introducing more mismatches between the sites such that all $\Delta a_i \neq 0$, the exponents for given clusters differ from each other (see Fig. 4 (a)–(b)), *e.g.* $\gamma_1 < \gamma_2$ (or $\gamma_1 > \gamma_2$ — depending on the mismatch values). It means that during the transitions, one cluster dominates stronger over the other cluster, maintaining longer residence times in a synchronized state. This appear again, in a certain range of coupling parameter K, before the transition to synchronization occurs.

Finally, we replace the FHN nodes with the Rulkov map set in a chaotic spiking regime (see Fig. 5 (a)). In this case, chaotic spiking has a bursting form that is conserved during the transitions. However, the mean spike frequency inside the bursts increases significantly during the synchronized states. Also in this case, we observe similar phenomenology regarding the spontaneous transitions and the behaviour of the Kuramoto parameter in time (see Fig. 5 (b)).

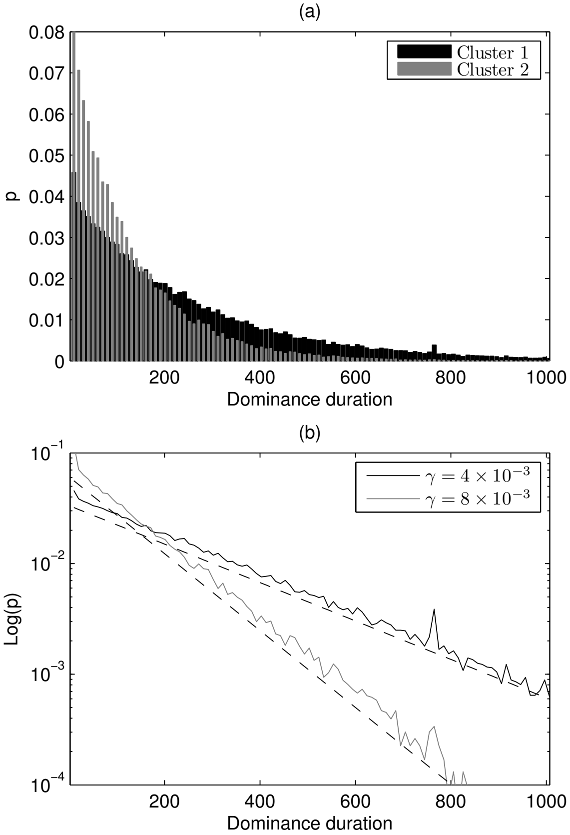

Fig. 4. An example of the network with mismatches such that all $\Delta a_i \neq 0$. (a) Distribution of dominance durations for the two formed clusters. (b) Estimation of the exponent γ.

The mechanism of the spontaneous transitions is related to the casual synchronization of spikes in the de-synchronized state. In other words, single coincidences may generate the avalanches of synchronously firing spikes. When the spikes at two sites, say x_1 and x_3, casually coincide, this induces a lowering of the coupling term feeding x_2 and x_4, thus causing their temporal de-synchronization. Consequently, x_1 and x_3 experience an increase of the coupling term and thus of the switching probability. The casuality of coincidences explains the exponential nature of the transitions.

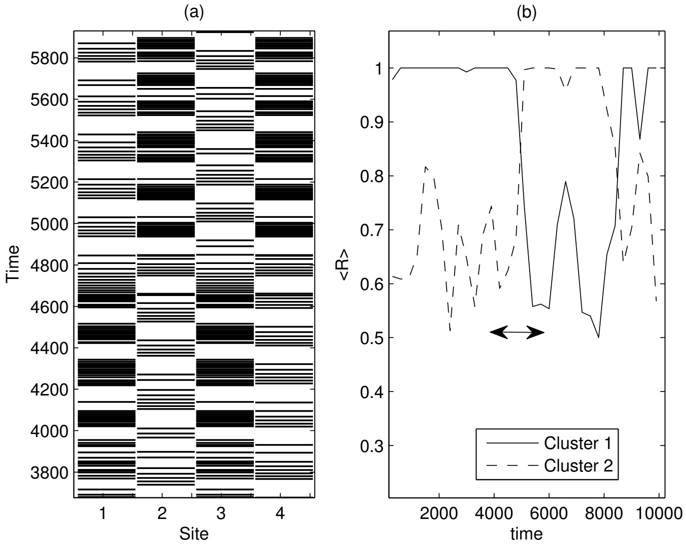

Fig. 5. (a) Raster plot for the network components composed with chaotic Rulkov map. (b) Mean R *versus* time. The horizontal double arrow shows the temporal period for which the raster plot in (a) has been plotted.

4. Conclusions

We proposed a simple network composed of chaos generating systems coupled through inhibitory and excitatory connections. The autonomous transitions are due to the existence of bistability in a network and due to the introduction of mismatches between the nodes. We observed the emergence of two clusters that undergo independent chaotic dynamics characterized by different mean spiking frequency and different degree of correlation. Only one cluster is synchronized at a given time. Due to these differences one cluster dominates over the other and contributes stronger to LFP, leading consequently to the strengthening of a given state. This observation from our simple toy model could give some insights into the mechanisms governing the autonomous transitions between mental states in the brain. In fact, many physical models describe the synchronization of high frequency synchronized neuronal activity as the coordinating mechanism for feature binding [17], whereby spatially segregated processing areas are bounded together to provide a coherent percept. Finally, we proposed the casual appearance of coincident spikes during de-synchronized state as a mechanism of the dynamical transition between the states.

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