

# THE STANDARD MODEL AS A LOW-ENERGY EFFECTIVE THEORY: WHAT IS TRIGGERING THE HIGGS MECHANISM?\*

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*To the memory of my longtime friend and colleague, Prof. Dr. Jochem Fleischer, who recently passed away. The one-loop on-shell versus  $\overline{\text{MS}}$  matching conditions used in the present work we have worked out together more than 30 years ago.*

The discovery of the Higgs by ATLAS and CMS at the LHC not only provided the last missing building block of the electroweak Standard Model, the mass of the Higgs has been found to have a very peculiar value, about 126 GeV, which is such that vacuum stability may be extending up to the Planck scale. We emphasize the consequences for the running masses and we reconsider the role of quadratic divergences. A change of sign of the coefficient of the quadratically divergent terms, showing up at about  $\mu_0 \sim 1.4 \times 10^{16}$  GeV, may be understood as a first order phase transition restoring the symmetric phase in the early universe, while its large negative values at lower scales trigger the Higgs mechanism. Running parameters evolve in such a way that the symmetry is restored two orders of magnitude below the Planck scale. As a consequence, the electroweak phase transition takes place near the scale  $\mu_0$  much closer to the Planck scale than anticipated so far. The SM Higgs system and its phase transition plays a key role for the inflation of the early universe. Dark energy triggering inflation is provided by the huge bare Higgs mass term and a Higgs induced vacuum density in the symmetric phase at times before the electroweak phase transition takes place.

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## 1. Introduction

Evidence strengthens more and more that the new particle discovered by ATLAS [1] and CMS [2] at the LHC at CERN is the last missing state required by the Standard Model (SM) of particle physics [3, 4], the Higgs boson [5]. For the first time the complete SM spectrum is known now. With its discovery, the mass of the Higgs boson has been established within a narrow range such that all SM parameters (except for the neutrino ones) for the first time are known with remarkable accuracy. One of the interesting consequences is that now we can answer quite reliably the long standing question where the effective SM parameters evolve when going to highest energies. It may be no accident that the observed Higgs mass turned out to match expectations from considerations of SM Higgs vacuum stability bounds, addressed long ago in Ref. [6], for example, and more recently in Refs. [7–19].

Knowing the Higgs mass allows us to say more about the phase structure of the SM. Commonly, quadratic divergences are considered to bring the SM into trouble: the hierarchy, fine tuning or naturalness problem. If one understands the SM as the renormalizable [20] low-energy effective tail of a system existing at the Planck scale, which exhibits the inverse Planck length  $\Lambda_{\text{Pl}} = (G)^{-1/2} \simeq 1.22 \times 10^{19}$  GeV ( $G$  Newton's gravitational constant) as a fundamental cutoff<sup>1</sup>, the relation between bare and renormalized parameters acquires a direct physical meaning. The low-energy expansion in the small parameter  $x = E/\Lambda_{\text{Pl}}$  suggests that only operators of dimension 4 or less are seen at low energies, which means that the low-energy tail is a local renormalizable effective Lagrangian Quantum Field Theory (QFT). As energies increase, at some point the first non-renormalizable effective interactions show up: operators of dimension 5 involving fermion fields and operators of dimension 6 which can be build from bosonic fields or by four-fermion structures. Dimension 5 operators yield typically a 0.1% effect, a typical accuracy achieved in many particle physics experiments, at what is typical for Grand Unified Theory (GUT) scales, namely  $10^{16}$  GeV. Dimension 6 operators would yield an effect of similar size at  $\sim 3.6 \times 10^{18}$  GeV. So we can expect local renormalizable QFT structure to apply up to about two orders of magnitude below the Planck scale, because the infinite tower of non-renormalizable operators scaling like  $x^n$  with  $n = 1, 2, 3, \dots$  are irrelevant, *i.e.* they scale down with increasing powers of the inverse cutoff. The trouble makers are the relevant operators, those which have positive mass dimension: the mass terms in particular. The latter scale like  $\Lambda_{\text{Pl}}/m_f$  for fermions and like  $\Lambda_{\text{Pl}}^2/M_b^2$  for bosons. As relevant operators, they have to be tuned in order not to freeze out by acquiring effective masses scaled up by one or two power in the cutoff for fermions or bosons, respectively. In

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<sup>1</sup> We will used Planck mass and Planck cutoff synonymous *i.e.*  $M_{\text{Pl}} = \Lambda_{\text{Pl}} \approx 10^{19}$  GeV.

condensed matter physics one would tune, as a typical relevant parameter, the temperature  $T$  to its critical value  $T_c$ , in order to let the system built up long range correlations, known as critical phenomena (see *e.g.* Ref. [21]). In particle physics the role of the reduced temperature  $(T - T_c)/T_c$  is taken by the renormalized particle mass, which has to remain small enough in order the particle is seen in the low-energy spectrum. What is tuning particle masses in the low-energy effective theory? Symmetries as we know! Chiral symmetry protects the fermion masses, local gauge symmetry protects the gauge boson masses, their non-vanishing being a consequence of spontaneous symmetry breaking. The one exception are scalar masses, which only can be protected by doubling the states by pairing all SM particles, supplemented by an additional Higgs doublet, into a supersymmetric extension. Alternatively, a conformal conspiracy could be at work when the entire particle content of the SM or an extension of it is such that the fermionic and bosonic degrees of freedom compensate each other collectively. The well known example is Veltman's "Infrared-Ultraviolet Connection" proposed in Ref. [22] (see also Refs. [23–25]), which noted that the coefficient of the quadratic divergences could vanish if the sum of properly weighted Higgs,  $W$  and  $Z$  boson mass-squares would cancel the top quark mass-square contribution (see below).

One of the key indications that the SM is a low-energy effective theory is the occurrence of local gauge symmetries. In particular, the non-Abelian local symmetries are not symmetries in the usual sense, like global symmetries. They rather represent a dynamical principle (like the equivalence principle in gravity) implying a special form of the dynamics. One could call them "quantum symmetries" as they determine a form of quantum interference known as gauge cancellations. The latter are well known from processes like  $W$ -pair production in  $e^+e^-$ -annihilation, where three Born level diagrams conspire to produce large cancellations of terms growing badly with energy and as a result yield the tamed observable cross section (see *e.g.* Ref. [26] and references therein). In fact, a non-Abelian gauge structure is an automatic consequence of a low-energy expansion: it is the only possible residual interaction structure, which is not suppressed by the cutoff (often misleadingly referred to as "tree unitarity"<sup>2</sup> constraint) [27–31]. Note that spin 1 fields at long distances appear in a natural way via multipole excitations in the Planck medium [32]. Also anomaly cancellations may be understood as low-energy conspiracies, the otherwise non-renormalizable terms are suppressed

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<sup>2</sup> Terms in tree level amplitudes which grow faster with energy than those present in the renormalizable spontaneously broken Yang–Mills theory are required to be absent, since, formally, they seem to violate unitarity. In the low energy expansion, these terms are not absent but suppressed by large factors  $E/\Lambda$ , which are not seen because the cutoff is very large.

by inverse powers of the cutoff. The grouping of the SM fermions into families is a consequence of this. For a more general view on the emergence of the SM, see Refs. [32, 33]. The general set up for the construction of a long range effective theory is Wilson’s Renormalization Group (RG) [34–37] of integrating out short distance fluctuations while keeping the infrared tail. What emerges from Wilson’s RG in the infrared is equivalent to a continuum quantum field theory RG as we know it from the SM or elsewhere.

While the RG evolution equations in the symmetric phase of the SM have been known for a long time to two loops, recently also the three loop results have been calculated in Refs. [11, 12, 14–17, 19] in the  $\overline{\text{MS}}$  scheme. The latter is most suitable for investigating the high-energy behavior of the SM, which is expected to be represented by the symmetric phase<sup>3</sup>. The more critical point is the experimental values of the  $\overline{\text{MS}}$  parameters at the  $Z$  boson mass or at the electroweak scale  $v = 246.22$  GeV. Most parameters are known from “low-energy” experiments obtained in the real world in the broken phase of the SM, typically in the on-shell renormalization scheme. The transcription of data from the on-shell to the  $\overline{\text{MS}}$  scheme is non-trivial within the SM because of non-decoupling effects in the weak sector of the SM at low energies (see *e.g.* Ref. [38] for a discussion in our context).

Another, maybe more serious, issue which is very different for the electroweak (EW) sector in comparison to massless QCD, is the appearance of quadratic divergences. They are absent in massless QCD where in the chiral limit only logarithmic divergences show up. In the electroweak part of the SM, by the fact that spontaneous breaking of the symmetry does not affect the ultraviolet (UV) properties of the theory, quadratic divergences show up in the renormalization of the mass parameter  $m^2$  of the scalar potential, which in the symmetric phase is given by  $V = \frac{m^2}{2}\phi^2 + \frac{\lambda}{24}\phi^4$ , where  $\phi$  denotes the real scalar Higgs field. The limit  $m = 0$  in the SM is not protected by any symmetry, the famous naturalness or hierarchy problem. A non-zero quadratically UV divergent  $m^2$ -term in the Lagrangian in any case is induced by renormalization. Besides  $m^2$ , the  $\text{U}(1)_Y$  and  $\text{SU}(2)_L$  gauge couplings  $g'$  and  $g$ , respectively, the Yukawa couplings  $y_f$  and the Higgs self-coupling  $\lambda$  are logarithmically divergent only and their running is governed by the standard RG equations for dimensionless parameters. This carries over to the broken phase which represents the low-energy structure of the SM. The dimensionful parameter  $m^2$  transmutes to the Higgs mass  $M_H^2 = \frac{1}{3}\lambda v^2 \hat{=} 2m^2$  and since  $\lambda$  satisfies a normal RG equation governed by

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<sup>3</sup> Note that in the symmetric phase where all fields but the Higgses are massless an S-matrix does not exist, at least in perturbation theory, and correspondingly an on-shell scheme is not well-defined, because of the “infrared catastrophe”. The  $\overline{\text{MS}}$  parametrization is then a natural parametrization at hand, most tightly related to the bare parameters.

logarithmic divergences only, all quadratic divergences must be exhibited by the Higgs bare vacuum expectation value  $v_0$ , or, equivalently, in the bare Fermi constant  $G_{F0} = 1/(\sqrt{2}v_0^2)$ . Since all masses are proportional to  $v$ , all masses are affected by the issue of quadratic divergences. In the broken phase, the quadratic divergences show up in the tadpole contributions. Renormalizability guarantees that no other types of UV singularities are induced by renormalization, in other words a renormalizable theory is closed with respect to dimension  $d \leq 4$  operators (assuming dimensional counting within a renormalizable gauge).

If we do not take into consideration supersymmetric extensions of the SM, which is a possible solution of the naturalness problem, an alternative possibility within the framework of the SM could be a “conformal conspiracy”<sup>4</sup> collectively between SM particles: the quadratic divergences can be absent if SM fermion contributions balance against the bosonic ones [22]. Only the heavier states are relevant numerically. At the one loop level the quadratic divergences, which in dimensional regularization (DR) show up as poles<sup>5</sup> at  $D = 2$ , are known to be given by

$$\delta m_H^2 = \frac{\Lambda^2}{16\pi^2} C_1, \quad C_1 = \frac{6}{v^2} (M_H^2 + M_Z^2 + 2M_W^2 - 4M_t^2) \quad (1)$$

modulo small lighter fermion contributions. The condition for the absence of the quadratic divergences  $C_1 \simeq 0$  for the given top-quark mass would require a Higgs mass  $M_H \simeq 314.92$  GeV in the one-loop approximation. The two-loop corrections have been calculated in Refs. [40–42] with the results

$$C_2 = C_1 - 2 \frac{\ln(2^6/3^3)}{16\pi^2} [(-36M_t^4 + 18M_H^2 M_t^2 + 3M_H^4 + 14/3 M_Z^2 M_t^2 - 6M_Z^2 M_H^2 - 87M_Z^4 - 68/3 M_W^2 M_t^2 - 12M_W^2 M_H^2 + 144M_W^2 M_Z^2 - 120M_W^4)/v^4 + 32g_3^2 M_t^2/v^2] . \quad (2)$$

<sup>4</sup> As in the theory of critical phenomena, long distance (low-energy) effective theories are systematically constructable by applying Wilson’s renormalization group approach, and mass parameters similar to the temperature in condensed matter physics have to be tuned to the critical surface in parameter space [32, 39]. The idea is that the statistical fluctuations at the Planck scale conspire to select modes which are able to survive as long range correlations (light particles). Natural are conspiracies involving few fields: singlets, doublets, triplets as they actually appear in the SM. Note that GUTs are unnatural in such a low energy effective scenario, where symmetries show up because we do not see the details of the underlying model. GUT scenarios assume a specific large symmetry group to exist at the high scale and that symmetries are broken spontaneously at most. Renormalizability is imposed to hold at the high scale.

<sup>5</sup> The massive scalar tadpole in  $D \sim 2$  is independent of  $m$  given by  $A_0(m) \stackrel{D \sim 2}{=} \frac{1}{D-2} \frac{\mu^2}{2\pi}$ , while for  $D \sim 4$  we obtain  $A_0(m) = \frac{\Lambda^2}{16\pi^2}$  when regularized with an UV cutoff  $\Lambda$ .

It turns out that the two-loop correction is moderate. If we require  $C_2 \simeq 0$ , we get the solution  $M_H \simeq 253.77$  GeV closer but still far away from its experimentally established value. Therefore, such a possible scenario is definitely ruled out by the data, after the Higgs mass has been determined by ATLAS and CMS.

In this paper, I advocate that quadratic divergences actually could play an important role in a different way. In fact, the coefficient of the quadratic divergence is scale dependent and exhibits a zero as emphasized recently by Hamada, Kawai and Oda in Ref. [41]. While Hamada, Kawai and Oda find the zero to lie above the Planck scale, in our analysis, relying on matching conditions for the top-quark mass analyzed in Ref. [38]<sup>6</sup>, we find the zero not far below the Planck scale. The difference originates from a different estimate of the  $\overline{\text{MS}}$  top-quark Yukawa coupling at the  $Z$  mass scale, which also implies that the Higgs potential remains stable up to the Planck scale. For our discussion here, it is important that a zero exists below the Planck scale, where it has a simple physical interpretation. The corresponding change in sign seems to provide a natural explanation for the Higgs mechanism in the SM. In the very early universe, the quadratically enhanced bare Higgs mass term provides a large dark energy density, which triggers inflation. In this scenario, the hierarchy problem is not a problem but the solution which explains inflation in the evolution of the early universe as a natural phenomenon within the SM. As the universe is cooling down, the bare Higgs mass changes sign and thus triggers the Higgs mechanism, stops inflation and the negative  $m^2$  term falls into competition with the finite temperature term and allows for the EW phase transition. In our Low Energy Effective SM (LEESM) scenario the EW phase transition is closely correlated to the Higgs mechanism as we will see.

In the next section, we remind the reader about the emergence of a local renormalizable QFT in a low energy expansion from a system exhibiting a physical UV cutoff at the microscopic level. In Section 3 we discuss the matching conditions which determine the  $\overline{\text{MS}}$  parameters from their physical on-shell counterparts. We emphasize the failure of “decoupling by hand” prescriptions in the weak sector of the SM. The evolution of the SM running parameters up to the Planck scale is presented in Section 4 for couplings, masses and the Higgs vacuum expectation value (VEV). Section 5 is devoted to a discussion of the scale dependence of the quadratic divergences

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<sup>6</sup> Two issues which can cause different results (different parameterizations) are the inclusion of tadpole contributions in the EW corrections and the non-decoupling of heavy particles. It should be noted that, unlike in a calculation where we can drop tadpoles by hand, any measurement of a physical on-shell observable automatically includes tadpole contributions.

and the observed first order phase transition, which triggers the Higgs mechanism. The impact of the results on inflation scenarios is briefly addressed in Section 6. A summary and outlook follows in Section 7.

## 2. Low energy effective QFT of a cutoff system

If we say that the SM is a low-energy effective theory, we mean that there must exist a more fundamental system exhibiting a physical cutoff, as typical for condensed matter systems. Such a system we expect to reside at the Planck scale, and the SM is expected to be the renormalizable tail at long distances relative to the Planck length. The Planck energy scale being beyond any direct experimental access, so far we only know its long range structure and that the underlying fundamental system must be in the universality class of the SM. Let us be more specific and sketch the construction of a low-energy effective QFT by looking at the cutoff version of the Higgs system only, for simplicity

$$\mathcal{L} = \mathcal{L}_0 + \mathcal{L}_{\text{int}} = \frac{1}{2} \partial^\mu \phi(x) (1 + \square/\Lambda^2) \partial_\mu \phi(x) - \frac{1}{2} m_0^2 \phi(x)^2 - \frac{\lambda_0 \Lambda^\varepsilon}{4!} \phi^4(x). \quad (3)$$

The regularization is chosen here as a Pais–Uhlenbeck higher-derivative kinetic term [43], which is equivalent to a Pauli–Villars cutoff [44]. We are interested in the model for  $D = 4$  space-time dimensions but may consider the more general case in  $D = 4 - \varepsilon$  dimensions with  $2 \leq D \leq 4$  in order to make comparisons with the same model in dimensional regularization. It is characterized by its vertex functions (connected amputated one-particle irreducible diagrams) of  $N$  scalar fields  $\Gamma_{A,b}^{(N)}(p; m_0, \lambda_0) = \langle \tilde{\phi}(p_1) \dots \tilde{\phi}(p_{N-1}) \phi(0) \rangle^{\text{prop}}$  as a function of the set of independent momenta, which we denote by  $p \hat{=} \{p_i\}$  ( $i = 1, \dots, N-1$ ). The bare functions are related to the renormalized ones by (for specific renormalization conditions, see Ref. [45]) reparameterizing parameters and fields

$$\Gamma_{A,r}^{(N)}(p; m, \lambda) = Z^{N/2} (\Lambda/m, \lambda) \Gamma_{A,b}^{(N)}(p; \Delta m_0(\Lambda, m, \lambda), \lambda_0(\Lambda/m, \lambda)). \quad (4)$$

They satisfy a RG equation for the response to a change of the cutoff  $\Lambda$  for fixed renormalized parameters  $\Lambda \frac{\partial}{\partial \Lambda} \Gamma(N)_{A,b} \big|_{m,\lambda}$ , which by applying the chain rule of differentiation reads

$$\begin{aligned} & \left( \Lambda \frac{\partial}{\partial \Lambda} + \beta_0 \frac{\partial}{\partial \lambda} - N \gamma_0 + \delta_0 \Delta m_0^2 \frac{\partial}{\partial \Delta m_0^2} \right) \Gamma_{A,b}^{(N)}(p; m_0, \lambda_0) \\ &= Z^{-N/2} \Lambda \frac{\partial}{\partial \Lambda} \Gamma_{A,r}^{(N)}(p; m, \lambda). \end{aligned} \quad (5)$$

$m_{0c}^2$  is the “critical value” of the bare mass for which the renormalized mass is zero, *i.e.*  $\Gamma_{Ab}^{(2)}\big|_{p=0} = 0$ , and  $\Delta m_0^2 = m_0^2 - m_{0c}^2$  corresponds to the renormalized mass parameter. Since the renormalized vertex functions have a regular limit as  $\Lambda \rightarrow \infty$ , to all orders in perturbation theory the inhomogeneous part behaves as

$$Z^{N/2} \Lambda \frac{\partial}{\partial \Lambda} \Gamma_{Ar}^{(N)}(p; m, \lambda) = O\left(\Lambda^{-2}(\ln \Lambda)^l\right), \quad (6)$$

*i.e.*, the inhomogeneous part, representing a cutoff insertion, falls off faster than the l.h.s. of Eq. (5) by two powers in the cutoff for large cutoffs. This is easy to understand given the fact that the cutoff enters  $\mathcal{L}$  as a term proportional to  $\Lambda^{-2}$ . Beyond perturbation theory one would have to require the condition

$$Z^{N/2} \Lambda \frac{\partial}{\partial \Lambda} \Gamma_{Ar}^{(N)}(p; m, \lambda) / \Gamma_{Ab}^{(N)} = O\left(\Lambda^{-\eta}\right), \quad (7)$$

for some positive  $\eta$ . In addition, also all the RG equation coefficients exist as non-trivial functions in the limit of infinite cutoff

$$\lim_{\Lambda \rightarrow \infty} \alpha_0(\Lambda/m, \lambda) = \alpha(\lambda), \quad \alpha = \beta, \gamma, \delta, \quad (8)$$

for dimensions  $2 \leq D \leq 4$ . In  $D = 4 - \varepsilon$  dimensions the proper vertex-functions have a large cutoff  $\Lambda$ -expansion (see Ref. [46])

$$\Gamma_{Ab}^{(N)}(p; \Delta m_0, \lambda_0) = \sum_{j,k,l \geq 0} \Lambda^{-2j-\varepsilon k} (\ln \Lambda)^l f_{jkl}^{(N)}(p, \Delta m_0, \lambda_0 \Lambda^\varepsilon), \quad (9)$$

and for large  $\Lambda$ , we obtain the preasymptote of  $\Gamma_{Ab}^{(N)}$

$$\Gamma_{Aas}^{(N)}(p; \Delta m_0, \lambda_0) = \sum_{k,l \geq 0} \Lambda^{-\varepsilon k} (\ln \Lambda)^l f_{0kl}^{(N)}(p, \Delta m_0, \lambda_0 \Lambda^\varepsilon), \quad (10)$$

collecting the leading terms and satisfying the bound

$$\left| \Gamma_{Ab}^{(N)}(p; \Delta m_0, \lambda_0) - \Gamma_{Aas}^{(N)}(p; \Delta m_0, \lambda_0) \right| = O\left(\Lambda^{-2} \left(\ln \Lambda^{l_x}\right)\right). \quad (11)$$

The index  $l_x$  is bounded to all orders in the perturbation expansion. The key point is that the still cutoff dependent preasymptote satisfies a homogeneous RG equation, a special property of the long range tail of the bare theory

$$\begin{aligned} & \left( \Lambda \frac{\partial}{\partial \Lambda} + \beta_{as}(\Lambda/\Delta m_0, \lambda_0) \frac{\partial}{\partial \lambda_0} - N \gamma_{as}(\Lambda/\Delta m_0, \lambda_0) \right. \\ & \left. + \delta_{as}(\Lambda/\Delta m_0, \lambda_0) \Delta m_0^2 \frac{\partial}{\partial \Delta m_0^2} \right) \Gamma_{Aas}^{(N)}(p; \Delta m_0, \lambda_0) = 0. \end{aligned} \quad (12)$$



For more details, see Refs. [45–47]. The homogeneity tells us that  $\Lambda$  has lost its function as a cutoff and takes the role of a renormalization scale, *i.e.*, (12) represents the response of a rescaling of the system: a change in  $\Lambda$  is compensated by a finite renormalization of the fields, the couplings and the masses<sup>7</sup>. By a finite renormalization we may reparametrize the preasymptote by imposing appropriate renormalization conditions. Then there exists a rescaling  $\Lambda = \kappa\mu$  such that we obtain the usual RG in the renormalization scale  $\mu$  of a non-trivial continuum QFT. This provides a precise interrelation between preasymptotic and  $\overline{\text{MS}}$  renormalized quantities, and hence between the bare system seen from a long distance and the familiar renormalized QFT physics. Thus, what we observe as the SM is a physical reparameterization (renormalization) of the preasymptotic bare theory. In the language of critical phenomena the “bare world” at the Planck scale has to be in the universality class of the SM. As we only observe the tail, details of the bare world remain largely unknown. One of the impacts of the very high Planck scale is that the local renormalizable QFT structure of the SM is presumably valid up to what is a typical GUT scale. It has nothing to do with grand unification though. This also justifies the application of the SM RG up to high scales. The tuning “to criticality” of the bare mass to the critical mass  $m_{0c}$  corresponds to what is known as the hierarchy or naturalness problem in the SM. This naturalness problem is asking for an answer to the question “who is tuning the knob of the thermostat to adjust the temperature to its critical value (which is determined by the underlying atomic structure of the condensed matter system)”. In the symmetric phase of the SM, where masses of fermions and gauge bosons are forbidden by the known chiral and gauge symmetries, respectively, there is only one mass, common for all four fields in the complex Higgs doublet, to be renormalized.

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<sup>7</sup> This is similar to the well known response of the on-shell renormalized theory to a change in the mass, now considered in the continuum limit  $\Lambda \rightarrow \infty$  renormalized QFT. It is given by the Callan–Symanzik equation [48, 49]

$$\left( m \frac{\partial}{\partial m} + \beta(\lambda) \frac{\partial}{\partial \lambda} - N \gamma(\lambda) \right) \Gamma_r^{(N)}(p; m, \lambda) = -m^2 (2 - \delta(\lambda)) \Delta_0 \Gamma_r^{(N)}(p; m, \lambda),$$

where  $\Delta_0$  is the integrated mass operator insertion. For large momenta the r.h.s. is suppressed  $O(m^2 \ln(m)^l)$  by the small mass-square  $m^2 \ll p^2$  up to logarithms, such that for large momenta asymptotically

$$\left( m \frac{\partial}{\partial m} + \beta(\lambda) \frac{\partial}{\partial \lambda} - N \gamma(\lambda) \right) \Gamma_{\text{ras}}^{(N)}(p; m, \lambda) = 0.$$

The mass asymptotically only plays the role of a renormalization scale,  $\Gamma_{\text{ras}}^{(N)}(p; m, \lambda)$  are vertex functions of an effectively massless theory. Up to appropriate finite reparameterization and a rescaling  $m = \kappa\mu$ , the homogeneous CS equations are nothing but the standard RG equation in the  $\overline{\text{MS}}$  scheme.

Here, we encounter the fine tuning relation of the form

$$m_0^2 = m^2 + \delta m^2, \quad \delta m^2 = \frac{\Lambda^2}{32\pi^2} C \quad (13)$$

with a coefficient typically  $C = O(1)$ . To keep the renormalized mass  $m$  at some small value, which can be seen at low energy,  $m_0^2$  has to be adjusted to compensate the huge number  $\delta m^2$  such that about 35 digits must be adjusted in order to get the observed value around the electroweak scale. This is a problem only in cases where we have to take the relation between bare and renormalized theory serious, like in a condensed matter system or here in the LEESM scenario. The difference is, of course, that in particle physics we never will be able to directly access experimentally the bare system sitting at the Planck scale. Furthermore, we do not know what should be the renormalized  $m^2$  in the symmetric phase where all physics is different anyway. The hierarchy problem thus can be reformulated as “why is  $m^2$  in the symmetric phase so much larger than  $M_H^2$  in the broken phase?” The answer is:  $m^2$  is naturally large because of the quadratic divergences, while  $M_H^2 = \frac{1}{3} \lambda v^2$  is small because the order parameter  $v$ , which sets the scale for the low energy mass spectrum, is naturally a long range (low-energy) quantity (similar to the magnetization in a ferromagnetic system). What would it mean if  $v = O(M_{\text{Pl}})$ ? It would mean that spontaneous symmetry breaking would not break the symmetry only via an asymmetric ground state, but actually breaks the symmetry at the high energy scale, *i.e.*, the symmetry would not be recovered at high energies. This would contradict all basic knowledge about spontaneous symmetry breaking in physical systems.

Of course, the question we would like to answer is why  $v/M_{\text{Pl}} \sim 2 \times 10^{-17}$  is that small. In a ferromagnetic system, it would mean that the magnetization  $M$  in units of the lattice spacing  $a$  given by  $Ma$  is very small. The magnetization is a function of the reduced temperature  $t = (T - T_c)/T_c$  and goes to zero as  $t \rightarrow -0$ , so to have  $Ma$  very small means that we are close to the critical temperature from below. The quasi-criticality is not unnatural in our context as the system seems to be self-tuning for its emergence at long distances. Thus, in principle, having  $v$  small is not necessarily a mystery. This question, in principle, can be answered by simulating the lattice SM in the unitary gauge, where  $v$  is a decent  $Z_2$  order parameter (spontaneous breaking of the symmetry  $H \leftrightarrow -H$ ), and can be calculated by non-perturbative means. In order to understand the  $v$  *vs.*  $M_{\text{Pl}}$  hierarchy more quantitatively, it would suffice to investigate this question in a QCD, top-Yukawa, Higgs system, where couplings must be such that all three couplings remain asymptotically free and the Higgs vacuum stays stable up to the cutoff (for related attempts in a different direction, see *e.g.* [50] and references therein). In my opinion, the misunderstanding in arguments con-

cerning fine-tuning problems is that a moderately large physical number is considered to be the difference of two large uncorrelated numbers. In fact, the structure of most fine tuning problems is different: a very large number, like  $\frac{\Lambda_{\text{Pl}}^2}{32\pi^2}$  in our case, may be multiplied by an  $O(1)$  size function which depends on some parameters and which exhibits a zero for particular values of the parameters. The magnetization as a function of temperature is a typical well known example of this, namely, at the critical point the magnetization necessarily gets zero, and it is naturally small if we are near the phase transition point.

In the following, we consider the SM as a strictly renormalizable theory, regularized as usual by dimensional regularization [51] in  $D = 4 - \varepsilon$  space-time dimensions, such that the  $\overline{\text{MS}}$  parametrization and the corresponding RG can be used in the well known form [52]. Some care is necessary in applying DR when dealing with the quadratic divergences as noted in Refs. [22, 24, 40]. For our LEESM scenario it is the cutoff structure of the  $D = 4$  world which is relevant. It should be noted that in DR, as applied to  $D = 4$  theories with non-trivial spin structure, the latter is always taken to reside in  $D = 4$  space-time (see *e.g.* Sect. 2.4.2 of Ref. [53] for a short outline) and in this sense is a hybrid “analytic continuation” designed to provide a gauge-symmetry preserving regularization of the  $D = 4$  dimensional gauge theory. The DR as designed in Ref. [51] is not thought to deal with the interrelation between true  $D$ -dimensional theories. Thus standard DR singles out Veltman’s relation (1) as the relevant one against others in our case.

In order to avoid misunderstandings, the  $\overline{\text{MS}}$  scale parameter  $\mu$  in our analysis is to be interpreted as the energy scale of physical processes taking place at that scale, in the sense we know the effective strong coupling  $\alpha_s(M_\tau^2)$  at the  $\tau$  mass scale (*e.g.* hadronic  $\tau$ -decays) and its value  $\alpha_s(M_Z^2)$  at the  $Z$  boson mass scale (*e.g.* hadronic  $Z$ -decays). What we are interested in is how the effective theory looks like at energies beyond present accelerator energies. The vertex functions with scaled up momenta for fixed parameters follows from a solution of the RG equation. To remind the Reader: a vertex function of  $n_B$  boson fields and  $n_\psi$  conjugate pairs of Fermi fields in the Landau gauge satisfies the RG equation

$$\left\{ \mu \frac{\partial}{\partial \mu} + \beta \frac{\partial}{\partial g} + \gamma_m m \frac{\partial}{\partial m} - n_A \gamma_A - 2n_\psi \gamma_\psi \right\} \Gamma(\{p\}, g, m, \mu) = 0, \quad (14)$$

and is a homogeneous function of canonical dimension  $\dim \Gamma = 4 - n_B - 3n_\psi$  under rescaling of all dimensionful quantities including momenta, masses **and** the renormalization scale  $\mu = \kappa \mu_0$ . The RG solution then may be

written in the form

$$\begin{aligned} \Gamma(\{\kappa p\}, g, m, \mu_0) &= \kappa^{\dim \Gamma} z_B(g, \kappa)^{-n_B} z_\psi(g, \kappa)^{-2n_\psi} \\ &\times \Gamma\left(\{p\}, g(\kappa), \frac{m(\kappa)}{\kappa}, \mu_0\right). \end{aligned} \quad (15)$$

The  $z$ -factors include the anomalous dimensions of the fields (for details, see *e.g.* Sect. 2.6.5 of Ref. [53]). Thus, the vertex functions at higher momenta  $\{\kappa p\}$ , up to an overall factor are given by the vertex functions at the reference momenta  $\{p\}$  and reference scale  $\mu_0$ , *e.g.*  $\mu_0 = M_Z$ , with effective coupling  $g(\kappa)$  and effective mass  $m(\kappa)/\kappa$ . This is the basic type of relation for a discussion of the high energy asymptotic behavior<sup>8</sup>.

Let me summarize the advantage of taking seriously the idea that the SM is a low energy effective theory of a cutoff system residing at the Planck scale. Many structural elements usually derived from phenomenology naturally emerge in the low energy regime we are living in. One is the simplicity of the SM as a result of our blindness to details, which implies more symmetries. Yang–Mills structure (gauge cancellations) with small groups: doublets, triplets besides singlets, Lorentz invariance<sup>9</sup>, anomaly cancellation and family structure, triviality for space-time dimensions  $D > 4$  are emergent properties.  $D = 4$  is the border case for an interacting world at long distances, extra dimensions just trivialize by themselves and have nothing

<sup>8</sup> A very different well known role played by the  $\overline{\text{MS}}$  parameter  $\mu$  is the following: predictions of observables (S-matrix elements and related cross-sections) in fixed order perturbation expansion are renormalization scheme dependent because of truncation errors (missing higher order contributions, which depend on the order of the perturbative expansion and on the parametrization chosen). In the  $\overline{\text{MS}}$  scheme, the scheme dependence particularly is manifest in the unphysical  $\mu$  dependence of the prediction of the physical quantity which, in general, gets weaker the more terms are included.

<sup>9</sup> It emerges in a similar way as rotational invariance in condensed matter systems. Take as an example the Planck medium to be a  $d$  dimensional Euclidean lattice system. Rotational invariance is emerging as follows: expand the hyper-cubic lattice propagator on the Brillouin zone

$$G_0^{-1}(\vec{q}) = m_0^2 + 4a^{-2} \sum_{i=1}^d \sin^2 \frac{aq_i}{2} \rightarrow m_0^2 + q^2 + \Lambda^{-2} q^4, \quad q^2 = \vec{q}^2, \quad \Lambda = \pi/a$$

for small  $\vec{q}$  and replace the cutoff box by a sphere of radius  $\Lambda$

$$\int_{-\pi/a}^{+\pi/a} d^d q \dots \rightarrow \int_{|\vec{q}| \leq \Lambda} d^d q \dots,$$

up to field renormalization which does not affect the long range properties of the original system. For  $\Lambda$  large, resulting correlation functions are identical with those of a rotational invariant Euclidean QFT with a cutoff.

to do with compactification *etc.* Last but not least, the low energy tail is a non-trivial renormalizable QFT. The high cutoff implies the reliability of the LEESM scenario up to close to the Planck scale. **This scenario does not imply that no new physics is expected to show up even at close-by or intermediate energy scales, but we expect it to be constrained by its natural emergence in a low energy expansion.** Remember that the hot Planck medium residing at the Planck scale is expected to exhibit a whole spectrum of modes, a “chaos” so to say, from which long range properties emerge as a self-organizing system. We should also note that emergent low energy symmetries are all violated near the Planck scale, which could be important for quantities like baryon or lepton number conservation. It is unlikely that going to higher energies what we see as the SM will not be decorated by yet unseen physics, which still would naturally appear as a renormalizable extension of the SM. An example could be particle quartet conspiracies forming a low energy effective SU(4) in addition to the SM gauge group.

### 3. Matching conditions

When studying the scale dependence of a theory at very high energies, where the theory is effectively massless and hence practically in the symmetric phase, the  $\overline{\text{MS}}$  renormalization scheme is the favorite choice to study the scale dependence of the theory. On the other hand, the physical values of parameters are determined by physical processes described by on-shell matrix elements and thus usually are available in the on-shell renormalization scheme primarily. The transition from one scheme to the other is defined by appropriate matching conditions. For the physical masses, they are given by the mass counterterms relating the bare and the renormalized masses as  $m_{b0}^2 = M_b^2 + \delta M_b^2$  for bosons and  $m_{f0} = M_f + \delta M_f$  for fermions, respectively. By  $m_{i0}$  we denoted the bare, by  $m_i$  the  $\overline{\text{MS}}$  and by  $M_i$  the on-shell masses.  $\text{Reg} = \frac{2}{\epsilon} - \gamma + \ln 4\pi + \ln \mu_0^2$  is the UV regulator term to be set equal to  $\ln \mu^2$  where  $\mu_0$  is the bare  $\mu$ -parameter while  $\mu$  denotes the renormalized one. The substitution defines the UV finite  $\overline{\text{MS}}$  parametrization. By identifying  $m_b^2 = M_b^2 + \delta M_b^2|_{\text{Reg}=\ln \mu^2}$  and  $m_f = M_f + \delta M_f|_{\text{Reg}=\ln \mu^2}$ , respectively, we then obtain the  $\overline{\text{MS}}$  masses in terms of the on-shell masses. More precisely, this follows from the following relations valid for bosons

$$m_{b0}^2 = M_b^2 + \delta M_b^2|_{\text{OS}} = m_b^2 + \delta M_b^2|_{\overline{\text{MS}}} , \quad (16)$$

where

$$\delta M_b^2|_{\overline{\text{MS}}} = (\delta M_b^2|_{\text{OS}})_{\text{UV sing}} \quad (17)$$

which means that only the UV singular Reg terms are kept as  $\overline{\text{MS}}$  counterterms. Thus

$$m_b^2(\mu^2) = M_b^2 + \delta M_b^2|_{\text{OS}} - \delta M_b^2|_{\overline{\text{MS}}} = M_b^2 + (\delta M_b^2|_{\text{OS}})_{\text{Reg}=\ln \mu^2} . \quad (18)$$

Corresponding linear relations hold for the fermion masses. Similar relations apply for the coupling constants  $g$ ,  $g'$ ,  $\lambda$  and  $y_f$ , which, however, usually are fixed using the mass-coupling relations in terms of the masses and the Higgs VEV, which is determined by the Fermi constant as  $v = (\sqrt{2}G_\mu)^{-1/2}$ . Here,  $G_\mu$  is the muon decay constant, which represents the Fermi constant in the on-shell scheme. The  $\overline{\text{MS}}$  version of the Fermi constant we denote by  $G_F^{\overline{\text{MS}}}$  or simply by  $G_F$ . The matching condition for the Higgs VEV may be represented in terms of the matching condition for the muon decay constant

$$G_F^{\overline{\text{MS}}}(\mu^2) = G_\mu + (\delta G_\mu|_{\text{OS}})_{\text{Reg}=\ln \mu^2} , \quad (19)$$

where  $\frac{\delta G_\mu}{G_\mu}|_{\text{OS}} = 2 \frac{\delta v^{-1}}{v^{-1}}$ , which at one-loop is given in the Appendix. For the relevant two-loop counterterms, see Ref. [54, 55]. Then the  $\overline{\text{MS}}$  top quark Yukawa coupling is given by

$$y_t^{\overline{\text{MS}}}(\mu^2) = \sqrt{2} \frac{m_t(M_t^2)}{v^{\overline{\text{MS}}}(M_t^2)} , \quad v^{\overline{\text{MS}}}(\mu^2) = \left( \sqrt{2} G_F^{\overline{\text{MS}}} \right)^{-1/2}(\mu^2) , \quad (20)$$

and the other  $\overline{\text{MS}}$  mass-coupling relations correspondingly.

In the mass relations just presented, tadpole contributions have to be included in order to get a gauge invariant relationship between on-shell and  $\overline{\text{MS}}$  masses as well as in order to preserve the UV singularity structure and hence the RG equations. Tadpoles show up as renormalization counterterms of the Higgs VEV  $v$  and quantities which depend on it, in particular, the masses which are generated by the Higgs mechanism. It is important to note that measured on-shell observables always **include** tadpole terms. Unlike in theory, experiments cannot switch off or omit subsets of diagrams. Even measured on-shell values of dimensionless couplings are affected by tadpoles via the on-mass-shell condition.

The proper expressions including the relevant tadpole terms for the SM counterterms at one-loop have been given in Ref. [56] and may be found in the Appendix. For the Higgs mass, such a relation has been elaborated in Ref. [57] as a relation between  $\lambda$  and  $\lambda_{\overline{\text{MS}}}$  under the proviso that  $G_F^{\overline{\text{MS}}} = G_\mu$ , which is not generally true, because, in general,  $G_F^{\overline{\text{MS}}}$  is expected to be a running parameter as well. Interpreted as a relation between  $m_H$  and  $M_H$ , the relation is identical to what is obtained from the relation  $m_H^2 = M_H^2 +$

$(\delta M_H^2|_{\text{OS}})_{\text{Reg}=\ln \mu^2}$ . Note that the only information we have on  $\lambda$  is from the experimental results on  $M_H$  via  $\lambda = 3\sqrt{2}G_\mu M_H^2$ . For the top-quark mass, the full SM relation between the pole mass and the  $\overline{\text{MS}}$  mass has been evaluated recently in Ref. [38], evaluating known results from Refs. [58–68] (see also Refs. [69, 70] and comments in Ref. [38]) in the relation

$$M_t - m_t(\mu^2) = m_t(\mu^2) \sum_{j=1} \left( \frac{\alpha_s(\mu^2)}{\pi} \right)^j \rho_j + m_t(\mu^2) \sum_{i=1; j=0} \left( \frac{\alpha(\mu^2)}{\pi} \right)^i \left( \frac{\alpha_s(\mu^2)}{\pi} \right)^j r_{ij}. \quad (21)$$

There is an almost perfect cancellation between the QCD and EW effects for the now known value of the Higgs boson mass. While  $[m_t(M_t^2) - M_t]_{\text{QCD}} = -10.38 \text{ GeV}$ , one finds [38]  $[m_t(M_t^2) - M_t]_{\text{SM}} = 1.14 \text{ GeV}$  for  $M_H = 125 \text{ GeV}$ .

As elaborated in Ref. [38], some care is required in the evaluation of the matching conditions. It is important to remind that the Appelquist–Carazzone theorem [71] does not apply to the weak sector of the SM, *i.e.* we cannot parametrize and match together effective theories by switching off fields of mass  $M > \mu$  at a given scale  $\mu$ . As we know, the theorem applies to QCD and QED, and in these cases provides the basis for the “decoupling by hand” prescription usually used in conjunction with the  $\overline{\text{MS}}$  parametrization, the preferred parametrization in perturbative QCD. The non-decoupling in the weak sector of the SM is a consequence of the mass coupling relations, which follow if the masses are generated by the Higgs mechanism. An important question then is what role tadpoles play in implementing the matching conditions, since tadpoles, potentially, give large contributions. However, we may take advantage of the fact that tadpole contributions drop out from relations between physical (on-shell) parameters and amplitudes [72, 73], while they can produce large shifts in the relations between the “quasi-bare”  $\overline{\text{MS}}$  parameters and the on-shell ones. As mentioned before, potentially, the Higgs VEV  $v$ , which determines the Fermi constant via  $G_F = \frac{1}{\sqrt{2}v^2}$ , could be particularly affected. However, we may compare the low energy effective Fermi constant  $G_F$  given by  $G_\mu$ , which is determined by the muon lifetime observed in  $\mu$ -decay, with its “high energy” variant at the  $W$  boson mass scale, where it can be identified with  $\hat{G}_\mu = \frac{12\pi\Gamma_{W\ell\nu}}{\sqrt{2}M_W^3}$  in terms of the

leptonic  $W$ -decay rate. The fact that  $\hat{G}_\mu \approx G_\mu$  with good accuracy is not surprising because the tadpole corrections which potentially lead to substantial corrections are absent in relations between observable quantities as we know. To be precise: with the PDG values  $M_W = 80.385 \pm 0.015 \text{ GeV}$ ,

$\Gamma_W = 2.085 \pm 0.042$  GeV and the leptonic branching fraction  $B(W \rightarrow \ell \nu_\ell) = 10.80 \pm 0.09\%$  we obtain  $\hat{G}_\mu = 1.15564(55) \times 10^{-5} \text{ GeV}^{-2}$ , while  $G_\mu = 1.16637(1) \times 10^{-5} \text{ GeV}^{-2}$ , *i.e.*, the on-shell Fermi constant at scale  $M_Z$  appears reduced by 0.92% relative to  $G_\mu$ .

Therefore, a SM parametrization in terms of  $\alpha(M_Z), \alpha_s(M_Z), \hat{G}_\mu$  and  $M_Z$  (besides the other masses), provides a good parametrization of the observables extracted from experiments at the vector boson mass scale. The  $Z$  mass scale, thus is an ideal matching scale, to evaluate the  $\overline{\text{MS}}$  parameters in terms of corresponding on-shell values. Note that the running of  $G_F$  starts to be important once  $M_W, M_Z, M_H$  and  $M_t$  come into play. At higher scales, certainly, the  $\overline{\text{MS}}$  version of  $v(\mu^2)$  or equivalently  $G_F^{\overline{\text{MS}}}(\mu^2)$  must be running as required by the corresponding RG.

For numerical results presented in the following sections, we use values for the input parameters [74]

$$\begin{aligned} M_Z &= 91.1876(21) \text{ GeV}, & M_W &= 80.385(15) \text{ GeV}, \\ M_t &= 173.5(1.0) \text{ GeV}, \\ G_\mu &= 1.16637(1) \times 10^{-5} \text{ GeV}^{-2}, \\ \hat{G}_\mu &= G_\mu(M_Z) = 1.15564(55) \times 10^{-5} \text{ GeV}^{-2}, \\ \alpha^{-1} &= 137.035999, & \alpha^{-1}(M_Z^2) &= 127.944, & \alpha_s(M_Z^2) &= 0.1184(7). \end{aligned} \tag{22}$$

For the Higgs mass we adopt

$$M_H = 125.9 \pm 0.4 \text{ GeV}, \tag{23}$$

in accord with latest ATLAS and CMS reports. All light-fermion masses  $M_f$  ( $f \neq t$ ) give negligible effects and do not play any role in our consideration. The top-quark mass given above is taken to be the pole mass. It should be reminded that it is not precisely clear whether the value reported by experiments or by the PDG can be identified with the on-shell mass within the given accuracy. For a recent review on the subtleties in defining/measuring the top-quark mass, see *e.g.* Ref. [75] and references therein. The evaluated  $\overline{\text{MS}}$  parameters may be found in Table I below.

#### 4. The SM RG evolution to the Planck scale

The SM RG in the symmetric phase to two loops has been known for a long time [76–80]. More recently, important extensions to three loops have been presented in Refs. [11, 12, 14–17, 19]. Of special interest is the behavior of the Higgs self-coupling  $\lambda$ , which plays a key role for the possible stability or instability of the SM ground state. In fact, solutions depend crucially on



including all couplings contributing. For example, it makes a big difference whether one works in the so-called *gaugeless limit*<sup>10</sup> in the evolution of  $y_t$  and  $\lambda$  as in Ref. [12], for example, or is including also the gauge coupling contributions as far as they are known (see *e.g.* Ref. [13, 18] for a fairly complete set of known corrections). Some time ago RG equations to two loops for the SM masses as well as for the Higgs VEV in the broken phase have been calculated in Ref. [54, 55, 81], where it has been shown that the RG equations of the symmetric phase are correctly obtained from the ones in the broken phase. The inclusion of the tadpoles thereby is crucial.

The RG equation for  $v^2(\mu^2)$  follows from the RG equations for the masses and the dimensionless coupling constants using one of the relations

$$v^2(\mu^2) = 4 \frac{m_W^2(\mu^2)}{g^2(\mu^2)} = 4 \frac{m_Z^2(\mu^2) - m_W^2(\mu^2)}{g'^2(\mu^2)} = 2 \frac{m_f^2(\mu^2)}{y_f^2(\mu^2)} = 3 \frac{m_H^2(\mu^2)}{\lambda(\mu^2)}. \quad (24)$$

As a key relation, we will use Eq. (10) of Ref. [54]

$$\mu^2 \frac{d}{d\mu^2} v^2(\mu^2) = 3 \mu^2 \frac{d}{d\mu^2} \left[ \frac{m_H^2(\mu^2)}{\lambda(\mu^2)} \right] \equiv v^2(\mu^2) \left[ \gamma_{m^2} - \frac{\beta_\lambda}{\lambda} \right]. \quad (25)$$

We remind that all dimensionless couplings satisfy the same RG equations in the broken and in the unbroken phase.

As we know, the Higgs VEV  $v$  is a key parameter of the SM, which interrelates masses and couplings in a well defined way. As a consequence, the RG for mass parameters can be obtained not only by direct calculation in the broken phase, but also from the knowledge of the RG of the parameters in the symmetric phase together with the one for  $v(\mu^2)$  or  $v^2(\mu^2) = 1/\left(\sqrt{2} G_F^{\overline{\text{MS}}}(\mu^2)\right)$  as given in Eq. (25). The proper  $\overline{\text{MS}}$  definition of a running fermion mass is

$$m_f(\mu^2) = \frac{1}{\sqrt{2}} v(\mu^2) y_f(\mu^2). \quad (26)$$

Of particular interest in our context is the top-quark mass for which the RG equation reads

$$\mu^2 \frac{d}{d\mu^2} \ln m_t^2 = \gamma_t(\alpha_s, \alpha). \quad (27)$$

We split  $\gamma_t(\alpha_s, \alpha)$  into two parts  $\gamma_t(\alpha_s, \alpha) = \gamma_t^{\text{QCD}} + \gamma_t^{\text{EW}}$ , where  $\gamma_t^{\text{QCD}}$  is the QCD anomalous dimension, and  $\gamma_t^{\text{EW}}$  the corresponding electroweak

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<sup>10</sup> This term is often used for the approximation  $g' = g = 0$ . The QCD coupling  $g_3$  in any case has to be taken into account, besides the top Yukawa coupling  $y_t$  and the Higgs self-coupling  $\lambda$ .

one.  $\gamma_t^{\text{QCD}}$  includes all terms which are proportional to powers of  $\alpha_s$  only and  $\gamma_t^{\text{EW}}$  includes all other terms proportional to at least one power of  $\alpha$ , and beyond one-loop multiplied by further powers of  $\alpha$  and/or  $\alpha_s$ .

In Refs. [54, 55, 81] the electroweak contribution to the fermion mass anomalous dimension  $\gamma_f^{\text{EW}}$  has been calculated in terms of the RG functions of the parameters in the unbroken phase of the SM: the result is given by

$$\gamma_t^{\text{EW}} = \gamma_{y_t} + \frac{1}{2}\gamma_{m^2} - \frac{1}{2}\frac{\beta_\lambda}{\lambda}, \quad (28)$$

where  $\gamma_{m^2} \equiv \mu^2 \frac{d}{d\mu^2} \ln m^2$ ,  $\beta_\lambda \equiv \mu^2 \frac{d}{d\mu^2} \lambda$ , and  $\gamma_{y_q} \equiv \mu^2 \frac{d}{d\mu^2} \ln y_q$ , with  $y_q$  the quark Yukawa coupling.

In the following, we present the results for the running SM parameters in various plots. The RG equations for the gauge couplings  $g_3 = (4\pi\alpha_s)^{1/2}$ ,  $g_2 = g$  and  $g_1 = g'$ , for the Yukawa coupling  $y_t$  and for the Higgs potential parameters  $\lambda$  and  $\ln m^2$  have been solved in the  $\overline{\text{MS}}$  scheme with initial values obtained by evaluating the matching conditions between pole and running masses. For the case of the dimensionless couplings, we reproduce known results within uncertainties. The  $\overline{\text{MS}}$  Higgs VEV square is then obtained by  $v^2(\mu^2) = -\frac{6m^2(\mu^2)}{\lambda(\mu^2)}$  and the other masses by the relations (24).

Figure 1 shows the solutions of the RG equations and the  $\beta$ -functions up to  $\mu = M_{\text{Pl}}$ . The running masses and the solutions for the Higgs potential mass parameter  $m$  as well as  $v$  and the equivalent  $G_F$  are depicted in Fig. 2.

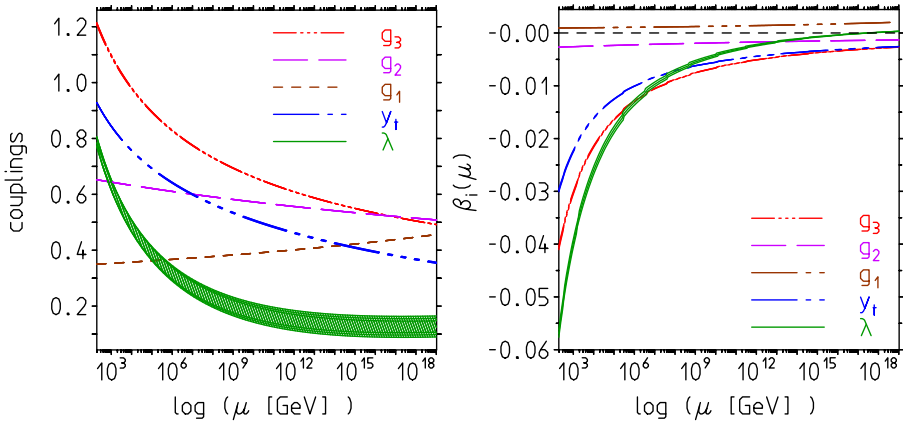


Fig. 1. Left: the SM dimensionless couplings in the  $\overline{\text{MS}}$  scheme as a function of the renormalization scale (see Refs. [8, 9, 13, 18, 41]). The input parameter uncertainties as given in Eqs. (22) and (23) are exhibited by the line thickness. The shaded/green band corresponds to Higgs masses in the range [124–127] GeV. Right: the  $\beta$ -functions for the couplings  $g_3$ ,  $g_2$ ,  $g_1$ ,  $y_t$  and  $\lambda$ . The uncertainties are represented by the line widths.

Remarkably, as previously found for the running couplings in Refs. [8, 9, 13, 18, 41], all parameters stay in bounded ranges up to the Planck scale if one adopts our matching conditions together with the so far calculated RG coefficients. With the input parameters evaluated in the previous section, we note that including all known terms no transition to a metastable state in the effective Higgs potential is observed, *i.e.* no change of sign in  $\lambda$  occurs. This is in contrast to a number of other evaluations (which however are not independent as they essentially use the same input parameters). The difference concerns the  $\overline{\text{MS}}$  input-value for the top-quark Yukawa coupling, which in our case is based on the analysis Ref. [38], and has been confirmed more recently in Ref. [19].

We observe that the various couplings evolve to values of similar magnitude at the Planck scale, within a factor of about 2 if we compare  $\sqrt{\lambda}$  with the others. While the gauge couplings are much closer than they are at low energies, there is no reason for perfect unification. The different types, the gauge boson-, the fermion- and the Higgs-couplings have no reason not to differ even if they emerge from one cutoff system. That the leading couplings are of the same order of magnitude, however, makes sense in such a kind of scenario. The emergence of the fermion mass hierarchy is a different issue.

We may understand the key point concerning the behavior of the effective parameters when we look at the leading terms of the  $\beta$ -functions. At the  $Z$  boson mass scale, the couplings are given by  $g_1 \simeq 0.350$ ,  $g_2 \simeq 0.653$ ,  $g_3 \simeq 1.220$ ,  $y_t \simeq 0.935$  and  $\lambda \simeq 0.807$ . While the gauge couplings behave as expected,  $g_1$  is infrared (IR) free,  $g_2$  and  $g_3$  are asymptotically (ultraviolet) free (AF), with leading coefficients exhibiting the related coupling only

$$\begin{aligned}\beta_1 &= \frac{41}{6} g_1^3 c \simeq 0.00185, & \beta_2 &= -\frac{19}{6} g_2^2 c \simeq -0.00558, \\ \beta_3 &= -7 g_3^3 c \simeq -0.08049,\end{aligned}$$

with  $c = \frac{1}{16\pi^2}$ , the leading top Yukawa  $\beta$ -function given by

$$\begin{aligned}\beta_{y_t} &= \left( \frac{9}{2} y_t^3 - \frac{17}{12} g_1^2 y_t - \frac{9}{4} g_2^2 y_t - 8 g_3^2 y_t \right) c \\ &\simeq 0.02327 - 0.00103 - 0.00568 - 0.07048 \\ &\simeq -0.05391\end{aligned}$$

not only depends on  $y_t$ , but also on mixed terms with the gauge couplings which have a negative sign. In fact, the QCD correction is the leading contribution and determines the behavior. Notice the critical balance between the dominant strong and the top Yukawa couplings: QCD dominance requires  $g_3 > \frac{3}{4} y_t$  in the gaugeless limit.

Similarly, the  $\beta$ -function of the Higgs self-coupling, given by

$$\begin{aligned}\beta_\lambda &= \left( 4\lambda^2 - 3g_1^2\lambda - 9\lambda g_2^2 + 12y_t^2\lambda + \frac{9}{4}g_1^4 + \frac{9}{2}g_1^2g_2^2 + \frac{27}{4}g_2^4 - 36y_t^4 \right) c \\ &\simeq 0.01650 - 0.00187 - 0.01961 + 0.05358 + 0.00021 + 0.00149 + 0.00777 \\ &\quad - 0.17401 \simeq -0.11595\end{aligned}$$

is dominated by the top Yukawa contribution and not by the  $\lambda$  coupling itself. At leading order it is not subject to QCD corrections. Here, the  $y_t$  dominance condition reads  $\lambda < \frac{3(\sqrt{5}-1)}{2}y_t^2$  in the gaugeless limit. The top Yukawa coupling is turned from an intrinsically IR free to an AF coupling by the QCD term and similarly the Higgs self-coupling is transmuted from IR free to AF by the dominating top Yukawa term. Including known higher order terms, except from  $\beta_\lambda$ , which exhibits a zero at about  $\mu_\lambda \sim 10^{17}$  GeV, all other  $\beta$ -functions do not exhibit a zero in the range from  $\mu = M_Z$  to  $\mu = M_{\text{Pl}}$ . So, apart from the  $U(1)_Y$  coupling  $g_1$ , which increases moderately only, all other couplings decrease and perturbation theory is in good condition. Actually, at  $\mu = M_{\text{Pl}}$  gauge couplings are all close to  $g_i \sim 0.5$ , while  $y_t \sim 0.35$  and  $\sqrt{\lambda} \sim 0.36$ .

As shown in Fig. 2, the masses stay bounded up to the transition point to the symmetric phase, discussed in the next section. In the broken phase the effective mass relevant for the high energy behavior, obtained by rescaling all the momenta of the process  $\{p_i\} \rightarrow \{\kappa p_i\}$   $\kappa \rightarrow \infty$ , up to an overall factor is  $m(\kappa)/\kappa \rightarrow 0$  (see Eq. (15)).

What is interesting is that the hierarchy of the effective masses gets mixed up. While the effective Higgs mass  $m_H$  and the related Higgs potential mass  $m$  are weakly scale dependent, the Higgs coupling  $\lambda$  drops pretty fast by a factor about 8, together this is causing the Higgs VEV  $v = \sqrt{3/\lambda}m_H$  to increase by a factor about 3.5. Note that, according to the mass-coupling relationships (24), what compares to the other couplings is  $\sqrt{\lambda}$  not  $\lambda$  itself. Given that  $m_H$  is weakly scale dependent, what determines the mass hierarchy are the relations (see Eq. (8) of Ref. [54])

$$\begin{aligned}\frac{m_W(\mu^2)}{m_H(\mu^2)} &= \sqrt{\frac{3}{4} \frac{g^2(\mu^2)}{\lambda(\mu^2)}}, & \frac{m_Z(\mu^2)}{m_H(\mu^2)} &= \sqrt{\frac{3}{4} \frac{g^2(\mu^2) + g'^2(\mu^2)}{\lambda(\mu^2)}}, \\ \frac{m_t(\mu^2)}{m_H(\mu^2)} &= \sqrt{\frac{3}{2} \frac{y_t^2(\mu^2)}{\lambda(\mu^2)}},\end{aligned}$$

which must hold in the broken phase. Since  $g$  is decreasing while  $g'$  is increasing, the  $Z$  boson mass grows most and exceeds  $m_H$  above about  $8 \times 10^4$  GeV and even  $m_t$  above about  $7 \times 10^{10}$  GeV. The  $W$  boson mass

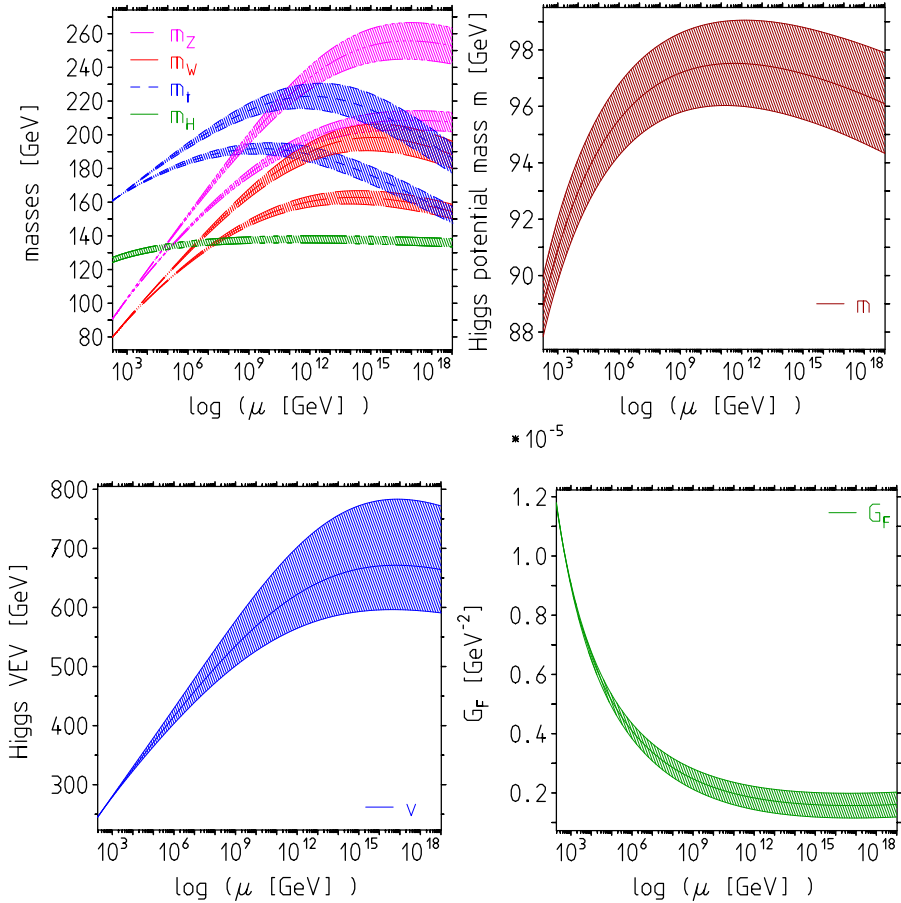


Fig. 2. Non-zero dimensional  $\overline{\text{MS}}$  running parameters. Top left: the running  $\overline{\text{MS}}$  masses. The shadowed regions show parameter uncertainties, mainly due to the uncertainty in  $\alpha_s$ , for a Higgs mass of 124 GeV, higher bands, and for 127 GeV, lower bands. The range also determines the lowest (green) band for the Higgs mass evolution. Top right: the  $\overline{\text{MS}}$  Higgs potential parameter  $m$ . Bottom:  $v = \sqrt{6/\lambda} m$  and  $G_F = 1/(\sqrt{2} v^2)$ . Error bands include SM parameter uncertainties and a Higgs mass range  $125.5 \pm 1.5$  GeV which essentially determines the widths of the bands.

exceeds  $m_H$  above about  $5 \times 10^6$  GeV. These crossings happen in the history of the universe some time after inflation, Higgs mechanism and EW phase transition, but long before processes like nucleosynthesis set in. The effective mass hierarchy is expected to play a role during the EW phase transition and in some temperature range just below it.

Table I lists  $\overline{\text{MS}}$  couplings at various scales, representing the central values for  $M_H = 126$  GeV, which we will use in the following. Other quark Yukawa couplings are given by  $y_s(M_t[M_{\text{Pl}}]) = 1.087[0.357] \times 10^{-3}$ ,  $y_d(M_t[M_{\text{Pl}}]) = 5.151[1.689] \times 10^{-5}$ .

For  $M_H = 126$  GeV the zero of  $C_1$  is at  $\mu_0 \simeq 1.4 \times 10^{16}$  GeV the one of  $C_2$  at  $\mu_0 \simeq 1.1 \times 10^{16}$  GeV. For the same Higgs mass the beta-function  $\beta_\lambda$  has a zero at  $1.3 \times 10^{17}$  GeV. Since the difference between  $C_1$  and  $C_2$  is small, we will adopt  $C_1$  and the corresponding value for  $\mu_0$ , in what follows.

TABLE I

$\overline{\text{MS}}$  parameters at various scales for  $M_H = 126$  GeV and  $\mu_0 \simeq 1.4 \times 10^{16}$  GeV.  $C_1$  and  $C_2$  are the one- and two-loop coefficients of the quadratic divergence Eqs. (29) and (30), respectively. Last two columns show corresponding results from Ref. [18].

Coupling/scale	$M_Z$	$M_t$	$\mu_0$	$M_{\text{Pl}}$	$M_t$ [18]	$M_{\text{Pl}}$ [18]
$g_3$	1.2200	1.1644	0.5271	0.4886	1.1644	0.4873
$g_2$	0.6530	0.6496	0.5249	0.5068	0.6483	0.5057
$g_1$	0.3497	0.3509	0.4333	0.4589	0.3587	0.4777
$y_t$	0.9347	0.9002	0.3872	0.3510	0.9399	0.3823
$y_b$	0.0238	0.0227	0.0082	0.0074		
$y_\tau$	0.0104	0.0104	0.0097	0.0094		
$\sqrt{\lambda}$	0.8983	0.8586	0.3732	0.3749	0.8733	<i>i</i> 0.1131
$\lambda$	0.8070	0.7373	0.1393	0.1405	0.7626	-0.0128
$C_1$	-6.768	-6.110	0	0.2741		
$C_2$	-6.672	-6.217	0	0.2845		
$m$ [GeV]	89.096	89.889	97.278	96.498	97.278	

## 5. The issue of quadratic divergences in the SM

A discussion of the large scale behavior of the SM is incomplete if we disregard the problem of quadratic divergences and the related hierarchy problem. In contrast to the dimensionless running couplings, all mass renormalizations (except the photon) are affected by quadratic ( $H$ ,  $W$  and  $Z$ ) or linear divergences (fermions), which are related universally to the renormalization of the Higgs potential parameter  $m^2$  or equivalently to the Higgs VEV  $v$  in the broken phase. Standard  $\overline{\text{MS}}$  mass RG equations usually take into account only the logarithmic singularities remaining after “throwing away”, by analytic continuation and subsequent  $\varepsilon$ -expansion, quadratic or linear divergences. *Per se*, the RG is a leading log, next-to-leading log, and so forth resummation tool. Note that this is possible in this way only in the purely perturbative  $\overline{\text{MS}}$  scheme, while with a more physical lattice regularization,

which applies beyond perturbation theory, the quadratic divergences cannot be eliminated in this way. In other words, the hierarchy problem is a real problem as reanalyzed recently in Ref. [41]. In terms of masses, the leading one-loop Higgs mass counterterm is given in Eq. (1), modulo small lighter fermion contributions [22] (see also Ref. [33]). The one-loop coefficient function  $C_1$  may be written as

$$C_1 = 2\lambda + \frac{3}{2}g'^2 + \frac{9}{2}g^2 - 12y_t^2 \quad (29)$$

and is uniquely determined by dimensionless couplings. Surprisingly, taking into account the running of the SM couplings, which are not affected by quadratic divergences such that standard RG equations apply, the coefficient of the quadratic divergences of the Higgs mass counterterm vanishes at about  $\mu_0 \sim 1.4 \times 10^{16}$  GeV given our set of  $\overline{\text{MS}}$  input parameters at the scale  $M_Z$ . As shown in Ref. [41], the next-order correction

$$C_2 = C_1 + \frac{\ln(2^6/3^3)}{16\pi^2} \left[ 18y_t^4 + y_t^2 \left( -\frac{7}{6}g'^2 + \frac{9}{2}g^2 - 32g_s^2 \right) - \frac{87}{8}g'^4 - \frac{63}{8}g^4 - \frac{15}{4}g^2g'^2 + \lambda \left( -6y_t^2 + g'^2 + 3g^2 \right) - \frac{2}{3}\lambda^2 \right], \quad (30)$$

calculated first in Ref. [40] (see also [42]), numerically does not change significantly the one-loop result. The same result applies for the Higgs potential parameter  $m^2$  which corresponds to  $m^2 \doteq \frac{1}{2} M_H^2$ . Thus

$$m_0^2 = m^2 + \delta m^2, \quad \delta m^2 = \frac{\Lambda^2}{32\pi^2} C. \quad (31)$$

The relevant parameters are entirely given in terms of SM parameters in the unbroken phase, which is physical at high energies, as well as at a different scale in the broken low energy phase, where parameters are directly accessible. It is important to note that the renormalized  $m^2$  in the symmetric phase is not known and not accessible directly to experiment, which means that it is not known whether there is a fine tuning problem in the symmetric phase. As we will see below, if  $m^2$  is not much smaller than the very large  $\delta m^2$  it would affect the inflation pattern and thus, in principle, is constrained by the observed properties of Cosmic Microwave Background (CMB) fluctuations [82].

For scales  $\mu < \mu_0$ , we have  $\delta m^2$  large negative, which is triggering spontaneous symmetry breaking by a negative bare mass  $m_0^2 = m^2 + \delta m^2$ , where  $m$  denotes a so-far unknown renormalized mass. With increasing energy scale, at  $\mu = \mu_0$ , the sign of  $\delta m^2$  flips and implies a phase transition to the symmetric phase, which persists up to the Planck scale. This means that

in the early universe, up to times about  $\sim 0.23 \times 10^{-38}$  to  $10^{-42}$  seconds after the Big Bang, the SM is in the unbroken phase. Finite temperature effects, to be discussed below, generally are accelerating the transition to the symmetric phase. This transition is relevant for inflation scenarios in the evolution of the universe. At  $\mu_0$  the Higgs VEV jumps to zero and SM gauge boson and fermion masses all vanish, at least provided the scalar self-coupling  $\lambda$  continues to be positive. Note that the phase transition scale  $\mu_0$  is close to the zero  $\mu_\lambda \sim 1.3 \times 10^{17}$  GeV of the  $\beta$ -function  $\beta_\lambda$ , where  $\beta_\lambda(\mu_\lambda) = 0$ . While  $\lambda$  is decreasing below  $\mu_\lambda$ , it starts to increase weakly above that scale.

The important point is that to all orders of perturbation theory as well as beyond perturbation theory there exists a solution  $C = 0$ , *i.e.* the relation (1) is expected to get corrections from higher-order effects which are shifting the location of the zero but do not affect its existence. Such relations are relations between the dimensionless gauge-, Yukawa- and Higgs-couplings and do not depend on dimensionful low-energy parameters like the Higgs potential mass  $m$  (in the symmetric phase) or the Higgs VEV  $v$  (in the broken phase). Of course  $m, v$  like  $\Lambda_{\text{Pl}}$  can/must show up as overall factors in dimensionful quantities. Higher order corrections may depend on ratios of these.

Given all masses of the SM, we note that  $4M_t^2 > M_H^2 + M_Z^2 + 2M_W^2$  which makes the Higgs mass counterterm  $\delta M_H^2 < 0$  and it is the heavy top quark which triggers spontaneous symmetry breaking in the SM as the bare mass square  $m_{H0}^2 = M_H^2 + \delta M_H^2$  is driven to negative values. Figure 3 shows how the coefficient  $C$  is dominated by the leading order term and as a function of the  $\overline{\text{MS}}$  running couplings vanishes and changes sign below the Planck scale. Interestingly, the top-quark mass and the Higgs mass, in conspiracy with the other relevant couplings, are such that the quadratic divergence vanishes precisely not far below the Planck scale, as illustrated in a different way in Fig. 4. The observation that, taking into account the scale dependence, there is a zero in the coefficient  $C_i$  ( $i = 1, 2$ ) of the quadratic divergence has been pointed out in Ref. [41]. Also in this case, the precise location of the zero is sensitive in particular to the top-quark Yukawa coupling and with their input, in Ref. [41], the zero was found to be located above the Planck scale, which in our LEESM scenario would not have a physical meaning.

Concerning the hierarchy problem, a zero at some scale does not eliminate the problem altogether of course, as illustrated by Fig. 5. Nevertheless, in the vicinity of the transition point, the system seems not to remember the short distance scale  $\Lambda_{\text{Pl}}$  and the VEV which develops at the close-by EW phase transition takes a value which need not be related to the short distance scale.



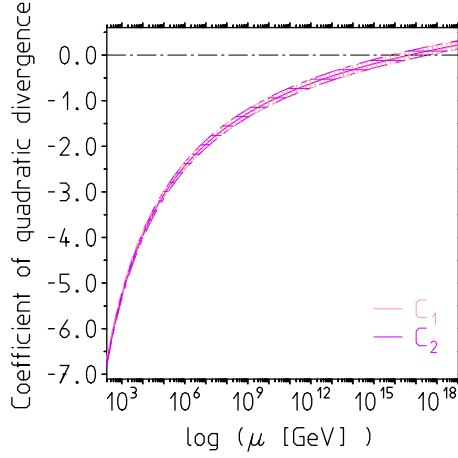


Fig. 3. The coefficient of the quadratic divergence term at one and two loops as a function of the renormalization scale. The one-loop result essentially determines the behavior. The coefficient exhibits a zero, for  $M_H = 126$  GeV at about  $\mu_0 \sim 1.4 \times 10^{16}$  GeV, not far below  $\mu = M_{\text{Pl}}$ . The shaded band shows the parameter uncertainties given in Eqs. (22), (23).

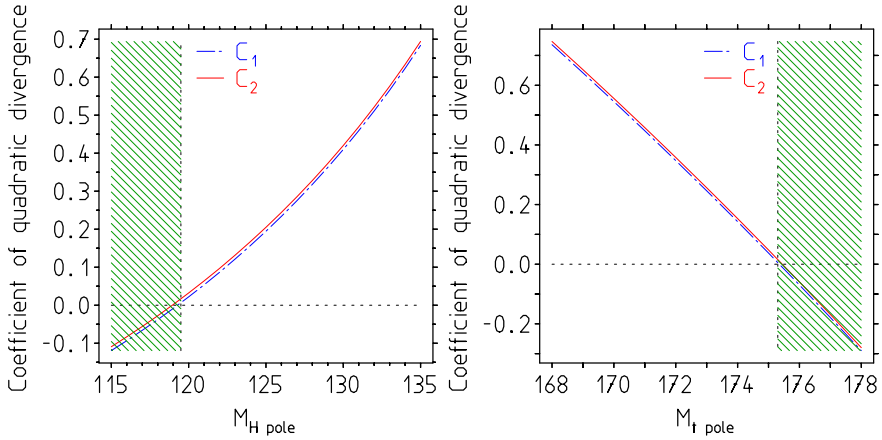


Fig. 4. Left: the coefficient of the quadratic divergence term at  $\mu = M_{\text{Pl}}$  as a function of  $M_H$  for  $M_t = 173.5$  GeV. Right: the same as a function of  $M_t$  for  $M_H = 125$  GeV. In the shaded region, the zero is above the Planck scale and thus unphysical in our LEESM context.

Here, we have another argument why  $v$  can have any value we want: let us consider the location of the minimum of the potential as a function of the Higgs potential parameters  $m_0^2$  and  $\lambda$ . For given positive  $\lambda$ , when we vary  $m_0^2$  from positive to negative values as it happens when changing the

energy scale  $\mu$  from above  $\mu_0$  to below it. One has  $v \equiv 0$  when  $m_0^2 \geq 0$  and  $v^2 = -\frac{6m_0^2}{\lambda}$  as soon as  $m_0^2 < 0$ . So  $v$  is a continuous function of  $m_0^2$  and can take any value. It is certainly not justified to assume that  $v$  is jumping from zero to  $M_{\text{Pl}}$ , suddenly.

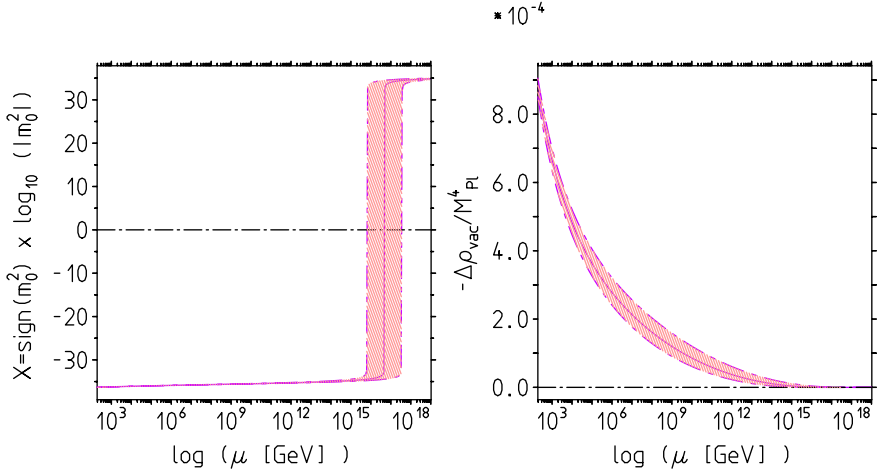


Fig. 5. The Higgs phase transition in the SM. Left: shown is  $X = \text{sign}(m_0^2) \times \log_{10}(|m_0^2|)$  which represents  $m_0^2 = \text{sign}(m_0^2) \times 10^X$ . In the broken phase  $m_0^2 = \frac{1}{2} m_{H0}^2$ . At the zero of the coefficient function shown in Fig. 3 the counterterm  $\delta m^2 = m_0^2 - m^2 = 0$  ( $m$  the  $\overline{\text{MS}}$  mass) vanishes and the bare mass changes sign. The band represents the parameter uncertainties Eqs. (22), (23). Right: the “jump”  $-\Delta\rho_{\text{vac}} = \frac{\lambda}{24} v_0^4$  in units of  $M_{\text{Pl}}^4$  as a function of the renormalization scale  $\mu$ . The jump, too small to be seen in this plot, agrees with the renormalized one.

Another point concerning the Higgs transition<sup>11</sup> and the meaning of the key relation (31). Before the Higgs mechanism has taken place in the cooling down of the universe we are at very high energy and we see the bare theory. At these scales a relation like (31) is observable, *i.e.* all three terms have a physical meaning and, in principle, are accessible to experiments. Below the Higgs transition, we are in the low energy regime characterized by the long range quantity  $v$ , which results from long range collective behavior of the system. In this case, the relation (31) provides a matching relation between renormalized and bare quantities in the broken phase, for  $\delta m^2 = 0$ . In the low energy phase, the bare  $m_0^2$  and the counterterm  $\delta m^2$  are not observable

<sup>11</sup> We use the term “Higgs transition point” for the point where the Higgs mechanism would take place in the zero temperature SM. The Higgs transition point lies above the EW phase transition point because of finite temperature effects which must be taken into account when considering the evolution of the hot early universe (see below).

any longer. The relation is not testable by low energy experiments, and if we try to test it by short distance experiments we are back testing the symmetric phase where the quantities have changed their values and meaning.

At the transition point  $\mu_0$ , we have  $v_0 = v(\mu_0^2)$ , where  $v(\mu^2)$  is the  $\overline{\text{MS}}$  renormalized VEV. Thus the contribution to the vacuum energy

$$\Delta\rho_{\text{vac}} = -\frac{\lambda(\mu_0^2)}{24} v^4(\mu_0^2), \quad (32)$$

is  $O(v^4)$  and **not**  $O(M_{\text{Pl}}^4)$ .

In any case, depending on finite temperature effects, near the zero of the coefficient function there is a phase transition, which corresponds to a restoration of the symmetry. Taking into account input parameter uncertainties, the transition is found to take place at a scale in the range of  $\mu \sim 10^{16}$  to  $10^{18}$  GeV, also depending on finite temperature effects. This is one to three orders of magnitude below the Planck scale. Now, in the symmetric phase, the positive quadratically enhanced bare mass term has the potential to trigger inflation. Note that at the zero of  $\beta_\lambda$  at about  $\mu_\lambda \sim 1.3 \times 10^{17}$  GeV  $> \mu_0$  the Higgs self-coupling  $\lambda$  although rather small is still positive and then starts slowly increasing up to  $M_{\text{Pl}}$ . The point  $\mu = \mu_\lambda$ , where  $\beta_\lambda(\mu_\lambda) = 0$ , corresponds to a phase transition from the antiscreening to the screening phase. It is not an RG fixed point though, because the  $\beta_\lambda(\mu)$ -function depends on other couplings which also change with the scale.

We also note that a zero of  $\lambda$  in the Higgs phase formally lets the Higgs VEV  $v$  explode:  $v^2(\mu^2) = -6m^2(\mu^2)/\lambda(\mu^2) \rightarrow \infty$  as  $\lambda(\mu^2) \rightarrow 0$ . Several analyses (see Ref. [18] and references therein), which find a somewhat higher  $\overline{\text{MS}}$  input value for  $y_t(M_t^2)$  find a zero of  $\lambda$  as low as  $\mu \sim 10^9$  GeV. Except for the Higgs mass  $m_H = \sqrt{2}m$  all masses would reach values  $O(M_{\text{Pl}})$ . In the LEESM scenario, of course, higher dimensional operators would save stability of the potential, which is assumed to be a given property of the Planck medium. It would mean that dimension 6 operators come into play at much lower scales than expected by naive  $E/\Lambda_{\text{Pl}}$  counting.

If we take renormalization as a physical process similar to what it is in condensed matter physics where both bare and renormalized (effective) quantities are physical and accessible to experiments, the key question is what happens to the effective Higgs potential  $V = \frac{m_0^2}{2} H^2 + \frac{\lambda}{24} H^4$ . When the  $m_0^2$ -term changes sign and  $\lambda$  stays positive, we know it is a first order phase transition. The latter term maybe is used somewhat sloppy here. Actually, when continuously lowering the temperature coming from the high energy side  $m_0^2(\mu)$  is a continuous function of  $\mu$ . If we would have  $m^2 = 0$ , *i.e.*  $m_0^2 = \delta m^2$ , then  $v_0(\mu)$  ( $v_0^2 = -\frac{6m_0^2}{\lambda}$  when  $m_0^2 < 0$ ) would be a continuous function of the scale as well, although a non-analytic one. When  $\mu \geq \mu_0$

$v_0(\mu) \equiv 0$  and for  $\mu < \mu_0$  we have  $v_0(\mu) > 0$  monotonically increasing as  $\mu$  decreases. Thus, the point  $\mu_0$  is the end point from below of a continuous family of first order transition points and hence itself represents a second order phase transition point. Actually, because the renormalized  $m^2 > 0$  is non-zero, more precisely, the Higgs transition point  $m_0^2 = 0$  is reached when  $\delta m^2 = -m^2$ , which is slightly below  $\mu_0$  at  $\mu_H$ . However, the “high energy meets low energy” and matching point is  $\delta m^2 = 0$  where  $m_0^2 = m^2$ . Here, the renormalized  $\overline{\text{MS}}$  mass square  $m^2 = m^2(\mu_0)$  is given, fixed via RG running and low energy  $\overline{\text{MS}}$  *vs.* on-shell matching condition by the experimental value of the Higgs mass  $M_H$ . Note that the initial condition  $m^2 = m^2(\mu_0)$  is a point still in the symmetric phase and when the Higgs transition takes place at  $\mu_H < \mu_0$  we have a finite  $v^2(\mu_H) = -\frac{6m^2(\mu_H)}{\lambda(\mu_H)}$ , *i.e.* factually we see a jump from  $v = 0$  to  $v \neq 0$  at the Higgs transition. In that sense, the Higgs transition represents a first order phase transition.

Since the bare Lagrangian is the true Lagrangian (renormalization is just reshuffling terms) the change in sign of the bare mass is what determines the phase<sup>12</sup>.

Above the transition point the number of massless degrees of freedom (radiation) of the SM consists of  $g_f = 90$  fermionic degrees of freedom and  $g_B = 24$  bosonic ones such that the effective number of degrees of freedom

$$g_*(T) = g_B(T) + \frac{7}{8} g_f(T) = 102.75 \quad (33)$$

(the factor  $\frac{7}{8}$  accounts for the Pauli exclusion principle which applies for the fermions). The four Higgses in the symmetric phase have equal masses, and are very heavy. If all SM modes would be massless we would have  $g_*(T) = 106.75$ , which effectively applies for temperatures large relative to about  $2 M_{\text{max}} < 500$  GeV, where  $M_{\text{max}}$  is the upper bound for all running SM masses in the range up to the transition point (see Fig. 2). Below the transition point, we know that the one remaining physical Higgs is as light as the other SM particles<sup>13</sup>. This shows that it need not be true that the higher the energy the more relativistic degrees of freedom must show up. The reason of course is that we crossed phase transition line.

Since the Higgs phase transition and inflation happen very early in the thermal history of the universe at times when the universe is very hot and dense (hot Big Bang), finite temperature effects must be included

<sup>12</sup> In the broken phase, we have the mass coupling relations (29), which also must hold for the bare parameters. These relations then tell us that linear (fermion masses) and quadratic (boson masses) divergences are absent at the transition point. Except for  $m$ , all other masses ideally vanish in the symmetric phase.

<sup>13</sup> Highly relativistic particles then contribute  $\rho_{\text{rad}}(T) = \frac{\pi^2}{30} g_*(T) T^4$  to the radiation density.

in a realistic treatment of the EW phase transition and inflation [83–86]. The leading modification caused by finite temperature effects enters the finite temperature effective potential  $V(\phi, T)$ : while at zero temperature  $V(\phi, T = 0) = -\frac{\mu^2}{2} \phi^2 + \frac{\lambda}{24} \phi^4$ , at finite temperature we have

$$V(\phi, T) = \frac{1}{2} (g_T T^2 - \mu^2) \phi^2 + \frac{\lambda}{24} \phi^4 + \dots \quad (34)$$

Usually, it is assumed that the Higgs is in the broken phase ( $\mu^2 > 0$ ) and that the EW phase transition is taking place when the universe is cooling down below the critical temperature  $T_c = \sqrt{\mu^2/g_T}$ . However, above the scale  $\mu_0$  we are in the symmetric phase with  $-\mu^2 \rightarrow m_0^2 = m^2 + \delta m^2 > 0$ . As claimed before, the phase transition is triggered by  $\delta m^2$  with  $m_0^2 \simeq 1.74 \times 10^{-3} M_{\text{Pl}}^2$ . In our case, we have  $T(\mu = \mu_0) \simeq 1.62 \times 10^{29} \text{ }^\circ\text{K}$  and  $T(\mu = M_{\text{Pl}}) \simeq 1.42 \times 10^{32} \text{ }^\circ\text{K}$  such that we expect the EW phase transition to be triggered by the bare Higgs mass in spite of the fact that the finite temperature term  $g_T T^2$  is very large in the early universe. The SM coefficient  $g_T$  is given by [86]

$$g_T = \frac{1}{4v^2} \left( 2m_W^2 + m_Z^2 + 2m_t^2 + \frac{1}{2} m_H^2 \right) = \frac{1}{16} \left[ 3g^2 + g'^2 + 4y_t^2 + \frac{2}{3} \lambda \right], \quad (35)$$

and we can calculate its value near  $M_{\text{Pl}}$  given the effective couplings at  $M_{\text{Pl}}$  listed in Table I. We estimate  $g_T \approx 0.0980$ . Therefore, near above the phase transition point, where  $m_0^2 = m^2(\mu_0^2)$ , the bare mass term is dominating. Up at the Planck scale, the temperature term is expected to dominate in general. However, this depends on the value of the renormalized  $m^2$ -term in the symmetric phase. Here and in the following, we **assume** that the bare mass is dominated by the quadratically enhanced mass counterterm, meaning we consider the renormalized  $m^2$  to satisfy  $m^2 \ll \delta m^2$ . Inflation as well as EW phase transition scenarios certainly depend on this assumption. As  $\delta m^2(\mu)$  is a running mass, which vanishes at  $\mu_0$ , the finite renormalized  $m^2$  could come into play at a late stage of inflation, and could be related to the value  $m \sim 10^{-6} M_{\text{Pl}}$  which has been extracted from observed inflation properties as a preferred scalar mass in standard inflation scenarios. Such a finite addition would not affect much the Higgs transition and the subsequent EW phase transition. In any case, the phase transition seems to be triggered quite generally by the sign flip of the bare mass term, as is illustrated in Fig. 6. The EW phase transition can take place only after the Higgs mass flip. Of course, our rough estimates are no substitute for a more careful reanalysis of the EW phase transition.

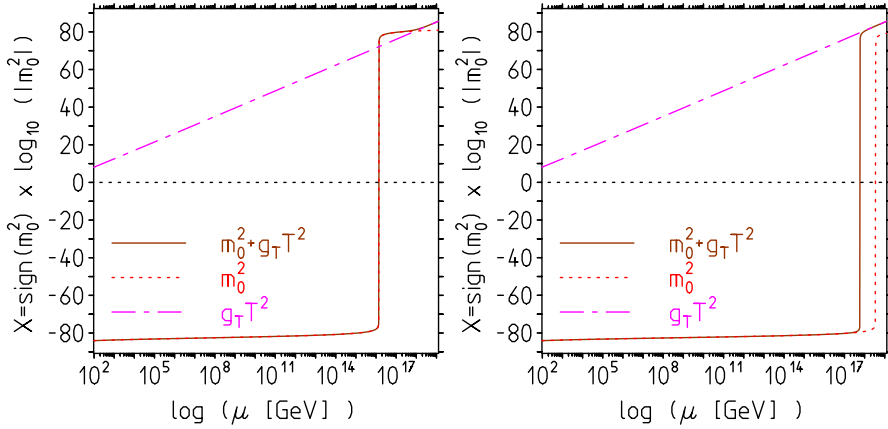


Fig. 6. The role of the effective bare mass in the finite temperature SM. Left: for  $\mu_0 \sim 1.4 \times 10^{16}$  GeV ( $M_H \sim 126$  GeV,  $M_t \sim 173.5$  GeV). Right: finite temperature delayed transition for  $\mu_0 \sim 6 \times 10^{17}$  GeV ( $M_H \sim 124$  GeV,  $M_t \sim 175$  GeV), the  $m_0^2$  term alone is flipping at about  $\mu_0 \sim 3.5 \times 10^{18}$  GeV.

Let us finally consider the behavior of  $v(\mu)$  in some more detail. The crucial point is that the running of  $v(\mu)$  is determined by the anomalous dimension of the Higgs potential parameter  $m^2$  and by the  $\beta$ -function related to the renormalization of  $\lambda$ . Its behavior has been investigated recently in Ref. [38]. For high energies, the second term of (25) is dominating, such that

$$\mu^2 \frac{d}{d\mu^2} \ln v^2(\mu^2) \sim -\frac{\beta_\lambda(\mu)}{\lambda(\mu)}.$$

The behavior of  $\lambda(\mu)$  and  $\beta_\lambda(\mu)$  has been studied recently in the context of vacuum stability of the SM Higgs sector in Refs. [8–13, 16, 17] and reveals that the beta function  $\beta_\lambda$  is negative up to a scale of about  $1.3 \times 10^{17}$  GeV, where it changes the sign. As already mentioned, above the zero  $\mu_\lambda$  of  $\beta_\lambda$ , the effective coupling starts to increase again and the key question is whether at the zero of  $\beta_\lambda$  the effective coupling is still positive. In the latter case, it will remain positive although small up to the Planck scale. In any case, at moderately high scales where  $\beta_\lambda < 0$ , and provided  $\lambda$  is still positive the following behavior is valid for the Higgs VEV

$$v^2(\mu^2)|_{\mu^2 \rightarrow \infty} \sim (\mu^2)^{-\frac{\beta_\lambda(\mu)}{\lambda(\mu)}} \rightarrow \infty, \quad (36)$$

which means that  $v^2(\mu^2)$  is increasing at these scales (where  $\beta_\lambda < 0$  and  $\lambda > 0$ ). The analyses Refs. [8–13, 16, 18] find that  $\lambda$  turns negative (unstable or meta-stable Higgs potential) before the beta function reaches its zero.

This may happen at rather low scales around  $10^9$  GeV. Some consequences we have mentioned above. In our approach, a zero of  $\lambda$  below the Planck scale would represent an essential singularity. According to our analysis,  $\lambda$  remains positive up to the zero of the beta function and as a consequence up to the Planck scale in agreement with Refs. [19, 38].

If  $\lambda = 0$  before  $\beta_\lambda = 0$ , the SM cannot be valid beyond that point  $\mu^*$  where  $\lambda(\mu^*) = 0$ . Note that in a renormalizable theory renormalization does not induce non-renormalizable higher-order terms, so one really has to give up the SM in its literal form. Here, one has to remember that the SM is an effective theory only and at higher scales non-renormalizable operators must come into play. The next relevant term would be  $\frac{\xi}{\Lambda_{\text{Pl}}^2} \frac{1}{6!} H^6$  with positive dimensionless coupling  $\xi$ , which keeps the system stable. Then, one has three parameters in the relevant part of the potential  $m^2$ ,  $\lambda$  and  $\xi$  and one may have more complicated vacuum structure, with metastable states *etc.* In other words, the case  $\lambda < 0$  cannot be discussed without extending the SM.

## 6. The impact on inflation

As the Higgs system persists to make sense back to times of the early universe, it is attractive to think that the SM Higgs field itself is responsible for the inflation era of the early universe, as originally thought by Guth [87] (see also Refs. [88–93]). Major phenomenological input on inflation comes from Cosmic Microwave Background observation, most recently from the Planck mission (see Ref. [82] and references therein). The “inflation term”, which comes in via the SM energy-momentum tensor, adds to the r.h.s. of the Friedmann equation

$$\ell^2 \left( V(\phi) + \frac{1}{2} \dot{\phi}^2 \right), \quad (37)$$

where  $\ell^2 = 8\pi G/3$ .  $M_{\text{Pl}} = (G)^{-1/2}$  is the Planck mass,  $G$  is Newton’s gravitational constant and for any quantity  $X$  we denote time derivatives by  $\dot{X}$ . In this section dealing with physics near the Planck scale  $\phi, V(\phi), \lambda$  and  $m$  denote the bare quantities (fields and parameters). Inflation requires an exponential growth  $a(t) \propto e^{Ht}$  of the Friedman–Robertson–Walker radius  $a(t)$  of the universe, where  $H(t) = \dot{a}/a(t)$  is the Hubble constant at cosmic time  $t$ .

The contribution of the Higgs to the energy momentum tensor amounts to a contribution to energy density and pressure given by

$$\rho_\phi = \frac{1}{2} \dot{\phi}^2 + V(\phi), \quad p_\phi = \frac{1}{2} \dot{\phi}^2 - V(\phi). \quad (38)$$

The second Friedman equation has the form  $\ddot{a}/a = -\frac{\ell^2}{2} (\rho + 3p)$  and the condition for growth  $\ddot{a} > 0$ , requires  $p < -\rho/3$  and hence  $\frac{1}{2} \dot{\phi}^2 < V(\phi)$ .

CMB observations strongly favor the slow-roll inflation  $\frac{1}{2}\dot{\phi}^2 \ll V(\phi)$  condition and favors the dark energy equation of state  $w = p/\rho = -1$ . Indeed, the Planck mission measured  $w = -1.13^{+0.13}_{-0.10}$ . The first Friedman equation reads  $\dot{a}^2/a^2 + k/a^2 = \ell^2 \rho$  and may be written as  $H^2 = \ell^2 \left[ V(\phi) + \frac{1}{2}\dot{\phi}^2 \right] = \ell^2 \rho$ . The kinetic term  $\dot{\phi}^2$  is controlled by  $\dot{H} = -\frac{3}{2}\ell^2 \dot{\phi}^2$  related to the observationally controlled deceleration parameter  $q(t) = -\ddot{a}/\dot{a}^2$ . In addition, we have the field equation

$$\ddot{\phi} + 3H\dot{\phi} = -V'(\phi) \equiv -dV(\phi)/d\phi. \quad (39)$$

By inflation  $k/a^2(t) \rightarrow 0$  ( $k = 0, \pm 1$  the normalized curvature), such that the universe looks effectively flat ( $k = 0$ ) for any initial  $k$ . Inflation looks to be universal for quasi-static fields  $\dot{\phi} \sim 0$  and  $V(\phi)$  large positive. Then  $a(t) \propto \exp(Ht)$  with  $H \simeq \ell\sqrt{V(\phi)}$ . This is precisely what the SM Higgs system in the symmetric phase suggests, if in the Higgs potential  $\lambda$  remains positive and the bare mass square  $m^2$  is positive too. As both  $\lambda$  and  $m^2$  for the first time are numerically fairly well known, quantitative conclusions on the inflation patterns should be possible solely on the basis of SM properties. The leading behavior is characterized by a free massive scalar field with potential  $V = \frac{m^2}{2}\phi^2$ , such that  $H^2 = (\dot{a}/a)^2 = \ell^2 \frac{m^2}{2}\phi^2$  and  $\ddot{\phi} + 3H\dot{\phi} = -m^2\phi$ , which is nothing but a harmonic oscillator with friction. A constant background field  $\phi \rightarrow \phi_0 + \phi$  would imply a dark energy term (cosmological constant) of the right sign. In contrast, after the phase transition triggered by the change of sign in the bare  $m^2$ , the scalar VEV implies a cosmological constant contribution  $-\frac{\lambda}{24}v^4$  of negative sign.

Note that, as required by the CMB horizon problem, the exponent  $Ht$  is much larger than unity if  $\phi$  exceeds the Planck mass at these times. Needed is  $N_e \simeq Ht > 60$  to solve the horizon problem. The inflation blow-up exponent is given by

$$N_e = \ln \frac{a(t_{\text{end}})}{a(t_{\text{initial}})} = \int_{t_i}^{t_e} H(t) dt = \int_{\phi_i}^{\phi_e} \frac{H}{\dot{\phi}} d\phi = -\frac{8\pi}{M_{\text{Pl}}^2} \int_{\phi_i}^{\phi_e} \frac{V}{V'} d\phi = H(t_e - t_i), \quad (40)$$

and  $N_e = H(t_e - t_i)$  is exact if  $H = \text{constant}$  *i.e.* when  $\rho = \rho_\Lambda$  is dominated by the cosmological constant, as it is expected for the SM. In the symmetric phase,  $V/V' > 0$  and hence  $\phi_i > \phi_e$ . Note that a rescaling of the potential does not affect inflation, but the relative weight of the terms is crucial. In fact, SM Higgs inflation is far from being self-evident. A detailed analysis is devoted to a forthcoming paper [94].

For the SM Higgs potential in the symmetric phase, denoting  $z \equiv \frac{\lambda}{6m^2}$ , and a potential  $V = V(0) + \Delta V(\phi)$  we have a term  $\frac{V(0)}{2m^2} \frac{1}{\phi} \frac{1}{1+z\phi^2}$  plus  $\frac{\Delta V}{V'} =$



$\frac{\phi}{4} \left(1 + \frac{1}{1+z\phi^2}\right)$  and thus with

$$\mathcal{I} = \int_{\phi_e}^{\phi_i} \frac{V}{V'} d\phi = \frac{V(0)}{2m^2} \left[ \ln \frac{\phi_i^2}{\phi_e^2} - \ln \frac{\phi_i^2 z + 1}{\phi_e^2 z + 1} \right] + \frac{1}{8} \left[ \phi_i^2 - \phi_e^2 + \frac{1}{z} \ln \frac{\phi_i^2 z + 1}{\phi_e^2 z + 1} \right]$$

we obtain

$$N_e = \frac{8\pi}{M_{\text{Pl}}^2} \mathcal{I}. \quad (41)$$

Note that  $V(0) = \frac{m^2}{2} \Xi + \frac{\lambda}{8} \Xi^2$  with  $\Xi = \frac{M_{\text{Pl}}^2}{16\pi^2}$ ,  $m^2$  and  $z = \frac{\lambda}{6m^2}$  all are known SM quantities!  $N_e$  large requires  $\phi_i \gg \phi_e$ . In our calculation, adopting **fixed** parameters as given at the Planck scale, and  $\phi_i \simeq 4.51 M_{\text{Pl}}$  as an initial field one obtains  $\phi_e = -1.32 \times 10^{-6} M_{\text{Pl}}$  which yields  $N_e \approx 65.83$ . The Higgs field in this constant coupling approximation starts oscillating for times  $t \gtrsim 200 M_{\text{Pl}}^{-1}$ . If we take into account the **running** of parameters as given by the standard  $\overline{\text{MS}}$  RG, we find  $\phi_e \simeq 3.73 \times 10^{-5} M_{\text{Pl}}$  and  $N_e \approx 65.05$  a value not too far above the phenomenologically required minimum bound. The Higgs field in this more adequate calculation is found to oscillate at much later times but still before the Higgs transition (see Fig. 7 below). A detailed analysis shows that the dynamical part of the Higgs potential  $\Delta V(\phi)$  decays exponentially, while  $V(0)$  the quasi cosmological constant is weakly scale dependent through  $m_0^2(\mu)$  and  $\lambda(\mu)$ , but has a zero not far above the Higgs transition point, as can be seen in Fig. 7 (see Ref. [94] for details).

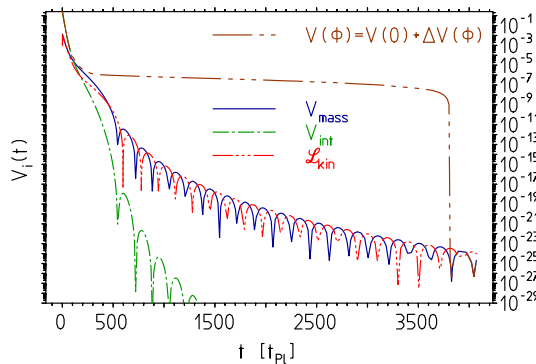


Fig. 7. The mass-, interaction- and kinetic-term of the bare Lagrangian in units of  $M_{\text{Pl}}^4$  as a function of time. The vacuum term  $V(0)$  gets nullified across the vacuum rearrangement somewhat above the Higgs transition point.

The inflation scenario suggested by the present analysis is a Gaussian potential with small anharmonic perturbations, since  $m^2$  is predicted to be large, while  $\lambda$  remains small. This picture should be valid in the renormalizable effective field theory regime below about  $10^{17}$  GeV. Going to higher energies, details of the cutoff system are expected to come into play, effectively in form of dimension 6 operators as leading corrections. These corrections are expected to get relevant only closer to the Planck scale. I expect that the observed value of dark energy has to be considered as a phenomenological constraint. The reason is that  $\rho_\Lambda$  is dependent on the Higgs field magnitude, which is not fixed by other observations, except maybe by CMB inflation data. In addition, we have to keep in mind that our scenario is very sensitive to the basic parameters  $C(\mu)$  and  $\lambda(\mu)$ , which were obtained by evolving coupling parameters over 16 orders of magnitude in scale. This cries for high precision physics to really settle the issue. High precision physics could become the tool in probing and investigating early cosmology. Note that given the SM couplings everything is essentially (besides the Higgs field strength) an SM prediction without any extra assumption. What we also learn is that the quartic “divergences” contributing to the vacuum energy, like the quadratic “divergences” affecting the Higgs mass, play an important role in promoting the SM Higgs to the inflaton, and inflation to be an unavoidable consequence of the SM.

## 7. Summary and outlook

We adopt that the new particle found by ATLAS and CMS at the LHC is the SM Higgs and we argue about the specific value found for the Higgs mass  $M_H = 125.9 \pm 0.4$  GeV and its impact for the SM itself. As noted quite some time ago in Ref. [6], stability of the SM vacuum up to the Planck scale is just what it now turns out to be and this looks to be more than just an accident. It signals a higher self-consistency of the SM than anticipated before. Provided the Higgs potential remains stable, there is no non-perturbative Higgs issue, no Landau pole nor any other problem. Surprisingly, except from the moderately increasing Abelian  $U(1)_Y$  gauge coupling, all other effective couplings behave asymptotically free, which renders Planck scale physics accessible by perturbative methods. The amazing thing is that this is the result of an intricate conspiracy of the several interactions “unified” in the SM. Besides the Higgs self-coupling  $\lambda$ , the top-quark Yukawa coupling  $y_t$ , the strong interaction coupling  $\alpha_s$ , as well as the gauge couplings  $g$  and  $g'$  turn out to be important in the conspiracy responsible for the stability of the SM ground state. Note that the full knowledge of the RG coefficients is needed to obtain a stable solution, while approximations like

the gaugeless one may suggest a false metastable situation. I also would say that complete four-loop calculations of the  $\beta$ -function coefficients would be highly desirable.

Concerning the vacuum stability, we should keep in mind: the Higgs mass miraculously turns out to have a value as it was expected from vacuum stability considerations, given the unexpectedly heavy top quark. As we have seen, it is a tricky conspiracy among the SM couplings which allows for a regular and stable extrapolation up to the Planck scale. If the SM misses to have a stable vacuum, why does it just miss it almost not as several related analyses Refs. [7–18] find? As also discussed in Refs. [95–97], very different scenarios would follow if the main condition of vacuum stability and the existence of a sign change of the Higgs potential mass term below the Planck mass scale would not be satisfied. At present, the Higgs potential stability issue looks to be almost entirely a matter of the precise value of the  $\overline{\text{MS}}$  top-quark Yukawa coupling at an appropriate matching scale. This issue is not settled in my opinion. First of all, we are left with the question what the top quark mass measured by experiments precisely means. For our analysis, we have identified the PDG top-quark mass entry with the on-shell mass. The second point concerns the missing higher order corrections in the matching conditions, which could help to clarify the situation. Problems in this direction have been discussed in Ref. [38] (see also Ref. [18] and references therein).

If our LEESM scenario is realistic, meaning that there is no essential non-SM physics up the Planck scale, the Higgs not only provides masses to the SM particles, but also supplies the necessary dark energy triggering inflaton. To settle the issues of inflation, the EW phase transition and baryogenesis, a very precise knowledge of the SM parameters becomes more crucial than ever. To achieve much better control on SM parameter-evolution, over 16 orders of magnitude in scale, becomes a key issue of particle physics and early cosmology. If we look at the leading coefficient of the quadratic divergence (29), we see that the top Yukawa term is enhanced by a factor 6 relative to the Higgs coupling, which means that a precise top-quark Yukawa coupling measurement is most crucial and should have highest priority at an ILC, where a threshold scan could provide much more reliable information (see *e.g.* [98–101]). This is, of course, an issue only if there is not a lot of yet unknown stuff which could obscure the situation. Almost equally important is a precise knowledge of the Higgs self-coupling. Especially, the inflation data are constraining the possible values in  $(\lambda, y_t)$ -plane at  $M_{\text{Pl}}$  dramatically. It is very surprising that such a possible window actually seems to exist. It also implies that higher order perturbative corrections are more

important than ever, as a tool to deepen our understanding of fundamental phenomena. Precision physics maybe a key tool to monitor the unknown bare physics at very high scales.

We understand the SM as a low energy effective emergence of some unknown physical system, we may call it “ether”, which is residing at the Planck scale with the Planck length as a “microscopic” length scale. Note that the cutoff, though very large, in any case is finite. Correspondingly, counterterms are finite. In such a kind of low energy effective scenario, quadratic and quartic “divergences” play an important role when we approach the bare system at the Planck scale. One key quantity here is the Higgs mass counterterm, which is given by  $\delta M_H^2 = \frac{M_{\text{Pl}}^2}{16\pi^2} C(\mu)$ , with  $C(\mu = M_{\text{Pl}}) \simeq 0.282$ , where  $C$  is the coefficient of the quadratic divergence of the bare Higgs mass given in Eqs. (2), (30). Note also that the bare mass  $m(\mu = M_{\text{Pl}})/M_{\text{Pl}} = 0.0295$  is almost two orders of magnitude below  $M_{\text{Pl}}$ . Note that in the broken phase at the EW scale  $C(\mu = v) \sim -6.7$ . Our main observation is that for appropriate input parameters the quadratically enhanced Higgs mass counterterm as a function of the renormalization scale exhibits a zero somewhat below the Planck scale. The zero implies a change of sign of the bare Higgs mass, which is responsible for the Higgs mechanism as a first order phase transition at about  $\mu_0 \sim 1.4 \times 10^{16}$  GeV in the  $T = 0$  SM. Above this scale, the system is in the unbroken phase *i.e.* the Higgs VEV is vanishing and all modes besides the remaining complex Higgs doublet fields are ideally massless. The second quantity which is dramatically enhanced by cutoff effects is the cosmological constant  $V(0) = \langle V(\phi) \rangle$  (Higgs vacuum loops), which yields a contribution to the dark energy density  $\delta\rho_\Lambda = \frac{M_{\text{Pl}}^4}{(16\pi^2)^2} X(\mu)$ , where  $X(\mu) = \frac{1}{8} (2C(\mu) + \lambda(\mu))$  with  $X(\mu = M_{\text{Pl}}) \simeq 0.088$ . As  $\lambda(\mu_0)$  is small  $X(\mu)$  has a zero not far below the zero of  $C(\mu)$ . Thus, as for the Higgs mass, there is a matching point between the renormalized low energy cosmological constant and the bare one seen at short distances. Again, that the bare cosmological constant is huge in the symmetric phase is not in conflict with the observed tiny dark energy density of today.

In the symmetric phase, we naturally have very heavy Higgses, while the light Higgs in the broken phase is a consequence of the phase transition itself, because all SM masses, including the Higgs itself, are proportional to the Higgs VEV  $v$ , which is an order parameter and hence naturally is a low energy quantity. The key point is that before the Higgs mechanism has taken place, the large positive bare Higgs mass-square term in the Higgs potential provides a huge dark energy term which triggers inflation and the four heavy Higgses represent the inflaton. Slow-roll inflation ends because of

the exponential decay of the Higgs fluctuation fields after short time, while  $V(0)$  persists to be large until it is nullified somewhat before the Higgs transition point and the subsequent EW phase transition.

The EW phase transition, due to finite temperature effects, takes place always a little below the Higgs transition scale. In our case, the EW transition essentially coincides with the Higgs transition, *i.e.* it takes place at  $\mu_0 \sim 1.4 \times 10^{16}$  GeV not near  $\mu \sim v \simeq 246.22$  GeV or elsewhere far below a typical GUT scale. This must have a definite impact on baryogenesis, and commonly accepted assumptions (see *e.g.* Refs. [86, 102–105] and references therein) have to be reconsidered.

In the symmetric phase, during inflation, the heavy Higgses decay into massless fermions, which provides the reheating of the universe which dramatically cools down by the inflation. The Higgs decay width  $\Gamma(H \rightarrow \bar{f}f) = \frac{M_H y_f^2}{16\pi}$  can be large for massless fermions, depending on the Yukawa couplings. Produced are preferably the modes with largest Yukawa coupling like the yet massless  $t\bar{t}$ -,  $t\bar{b}$ -,  $b\bar{t}$ -pairs, the latter two modes via the “charged” Higgses and rates proportional to  $y_t y_b$ . While  $b\bar{b}$ -production is suppressed by  $y_b^2/y_t^2(M_{\text{Pl}}) \sim 4.4 \times 10^{-4}$ ,  $\tau$ -production follows with a branching fraction  $y_\tau^2/(3y_t^2)(M_{\text{Pl}}) \sim 2.4 \times 10^{-4}$  *etc.* During and/or shortly after the EW phase transition, the heavier quarks decay into the lighter ones (the strongly coupled into the weakly coupled) channeled by the CKM-matrix [106]. Therefore, most of the normal matter is a decay product of top and bottom quarks and their anti-quarks. In this scenario, most normal matter must have undergone CP-violating decay processes. This is certainly an important ingredient for baryogenesis.

Here, we also should remind that QED, the electric charge assignments and massless photon radiation *etc.* are only defined after symmetry breaking  $\text{SU}(3)_c \otimes \text{SU}(2)_L \otimes \text{U}(1)_Y \rightarrow \text{SU}(3)_c \otimes \text{U}(1)_{\text{em}}$ . This certainly requires more detailed studies including the question which scalars couple to the vacuum. Another interesting aspect: in the symmetric phase  $\text{SU}(2)_L$  is unbroken in the very early phase of the universe. In the symmetric phase, there could exist heavy  $\text{SU}(2)$  bound states<sup>14</sup> which would bind energy and could be dark matter candidates. Thus cold dark matter could be frozen energy, very similar to ordinary matter, which is mainly hadronic binding energy (nucleon masses), while the masses of the elementary fields induced by the Higgs mechanism constitute an almost negligible contribution to normal matter in the universe. The way matter clusters and populates the universe is, of course, determined by the details of the Yukawa- and Higgs-sector and the particular form of the EW phase transition.

<sup>14</sup> Note that the basic parameters of this  $\text{SU}(2)$  is known and below close to the Planck scale actually has a coupling slightly stronger than the one of the  $\text{SU}(3)$  sector (see Table I).

A final remark about the reliability of numerical estimates given: precise numbers are expected to change not only when input parameters change, but may be affected by including higher order corrections. The role of what matching conditions precisely are used and justified to convert physical into  $\overline{\text{MS}}$  parameters remains a key issue. One also should be aware that what we used as a Planck scale in terms of the  $\overline{\text{MS}}$  scale  $\mu$  may differ by non-trivial factors. If the Planck medium would be a lattice of spacing  $a_{\text{Pl}}$  the effective Planck energy scale could be  $\tilde{\Lambda}_{\text{Pl}} = \pi/a_{\text{Pl}} = \pi M_{\text{Pl}}$  a factor of  $\pi$  higher than what we have assumed throughout the analysis. A similar ambiguity may be due to the **convention** adopted when defining the  $\overline{\text{MS}}$  scheme. We may ask whether the minimal MS parametrization would not be more closely related to the bare parameters, relevant at the Planck scale, than the  $\overline{\text{MS}}$  ones. We have  $\ln \mu_{\text{MS}}^2 = \ln \mu_{\overline{\text{MS}}}^2 + \gamma - \ln 4\pi$  thus  $\mu_{\overline{\text{MS}}}/\mu_{\text{MS}} = \exp \frac{\ln 4\pi - \gamma}{2} \simeq 2.66$ , which tells us that we should be aware of the fact that the identification of the renormalization scale with a physical Planck scale may be fairly arbitrary within some  $O(1)$  factors. What it means is that the matching scale looks to be ambiguous within a factor  $\sim 3$ , while the beta-functions do not change. While the  $\overline{\text{MS}}$  parametrization is fixed at the EW scale phenomenologically (up to possible matching condition uncertainties) it is conceivable that the identification  $\mu = M_{\text{Pl}}$  is requiring phenomenological adjustment as well, via indirect constraints from properties of the EW phase transition and the observed inflation profile, for example.

We have not discussed the possibility that the sterile right-handed neutrinos, which must exist in order to allow for non-vanishing neutrino masses, are Majorana particles. In this case, it would be natural that the singlet Majorana neutrinos have huge masses in the symmetric as well as in the broken phase, not protected by any of the SM symmetries. As is well known, this would provide the most natural explanation for the smallness of the neutrino masses by the resulting seesaw mechanism. When the singlet neutrino mass term is very high near  $M_{\text{Pl}}$  it would not affect the running of the other couplings also because a singlet has not any direct couplings to other fields. In addition, as the singlet Majorana mass is not subject to mass-coupling relations (it is intrinsic and not generated by a Higgs type mechanism) it is actually decoupling, although it leaves its trace in scaling the neutrino masses to very small values. Nevertheless, it would be interesting to investigate the scale dependence of the effective heavy Majorana masses and to study their influence on inflation, the EW phase transition and the dark matter problem, within the LEESM framework.

We also remark that gravity as we see it at long distances in our scenario emerges from the “ether” system exhibiting an intrinsic fundamental cutoff. Therefore, there is no reason why we should expect gravity to be quantized in the sense of a local relativistic renormalizable QFT at the Planck scale. Also for gravity the low energy manifestation is expected to be what is obtained from a low energy expansion (see Ref. [32] for a corresponding consideration).

Our findings do not exclude the existence of new physics as far as it does not spoil the gross features of the SM which are important: the stability of the Higgs potential and the existence of a zero in the coefficient of the quadratic divergences. Also important for the understanding of the today very small dark energy density is the zero in the coefficient of the corresponding quartically enhanced contribution.

In any case, after all relevant ingredients of the SM are confirmed and parameters have been determined within narrow error bands, many issues in early cosmology are likely direct predictable consequences of properties of the Higgs system and its embedding into the SM. This insight opens completely new possibilities for the solution of open problems. So far, the LEESM scenario has more phenomenological support than any of the other known beyond the SM scenarios. However, the inflation scenario is very sensitive to the precise SM parameter input values. This is not surprising as we try to extrapolate over 16 orders of magnitude in scale. In addition, there are two unknown inputs in the game which affect inflation. One is the magnitude of the Higgs field near the Planck scale, the other the renormalized mass  $m$  in the symmetric phase. These two inputs are constrained by what we think to know about inflation, slow-roll, equation of state, Gaussianity, spectral index, in particular. After all, the SM hides more secrets than answers and we are far from having worked out all its consequences nor have we understood many of its *why so’s*. This is an attempt to understand the SM as a conspiracy theory. More and more the SM looks to me to work like a fine Swiss clockwork.

As a final remark about the role of the SM Higgs let me point out the following: for some time at and after the Big Bang, the Higgs is the particle which is directly attached to gravity. It is the only SM particle which directly talks to the vacuum in the early universe (much later after the QCD phase transition also quark and gluon condensates develop VEVs). The Higgs is the one producing negative pressure and hence blowing continuously energy into the expanding universe. Amazingly, understanding the physics of the early universe now depends vitally on the precise determination and understanding of parameters like the top-quark mass and the Higgs mass, and the precise values of their couplings. Seeing more of the “ether” residing at the Planck scale is now a matter of high precision physics.

The key questions are “Where in SM or SM+ parameter space is the hot spot, which makes the Higgs be the inflaton?”, “Does the Higgs play the master role in the early evolution of our universe?”. Higher order effects, moderate additions to the SM like a Peccei–Quinn [107] axion sector and its impact on the strong CP problem could still also play a role in the fine-tuning conspiracy. On the other hand, what looks to be a straightforward possible renormalizable extension of the SM like a fourth fermion family seems to be definitely ruled out in our scenario.

In any case, our LEESM Higgs scenario offers a number of new aspects not considered so far and are worth being investigated in much more detail. Key point of the present analysis are the  $\overline{\text{MS}}$  input parameters at the  $Z$  mass scale, evaluated in terms of the matching conditions as studied in Ref. [38] for central values  $M_H = 126$  GeV and  $M_t = 173.5$  GeV. Any kind of possible and necessary fine tuning, by adjusting  $\lambda(M_H)$  or  $y_t(M_t)$ , has not been analyzed in detail so far. Moderate tuning or more accurate predictions of the SM input certainly will provide a more reliable prediction of the SM Higgs inflation pattern. The sensitivity to details is pronounced. The more it is remarkable that we found the spot where the SM provides dark energy and inflation “automatically”, and reasonably close to what we know from observation. For the first time we have a chance to get information about inflation from the SM alone and we can make predictions which are not just more or less direct consequences of some more or less plausible assumptions. The main new point is that we find the Higgs potential to be stable up to the Planck scale and that the coefficient of the  $M_{\text{Pl}}^2$  enhanced Higgs potential mass term changes sign sufficiently below the Planck scale.

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## Appendix

### *The ingredients for one-loop matching*

The one-loop on-shell counterterms may be expressed in terms of the known scalar integrals

$$A_0(m) = -m^2 (\text{Reg} + 1 - \ln m^2)$$

$$B_0(m_1, m_2; s) = \text{Reg} - \int_0^1 dz \ln(-sz(1-z) + m_1^2(1-z) + m_2^2z - i\varepsilon) ,$$

with

$$\text{Reg} = \frac{2}{\varepsilon} - \gamma + \ln 4\pi + \ln \mu_0^2 \equiv \ln \mu^2 .$$

In addition, we define  $\mathcal{C} = \frac{\sqrt{2}G_\mu}{16\pi^2}$  and  $\mathcal{C}_\mu = \frac{\sqrt{2}G_\mu}{16\pi^2} \ln \mu^2$ . Furthermore,  $c_W^2 = \frac{M_W^2}{M_Z^2}$  and  $s_W^2 = 1 - c_W^2$ . Sums over fermion contributions we write  $\sum_{f_s}$  for sums over individual fermions, and  $\sum_{f_d}$  for sums over fermion doublets.  $Q_f$  denotes the fermion charge,  $a_f = Q_f s_W^2 \mp \frac{1}{4}$  the  $Zf\bar{f}$  vector coupling and by  $b_f = \pm \frac{1}{4}$  the axial-vector couplings. A color factor  $N_c = 3$  applies for quarks.

$\overline{\text{MS}}$  counterterms:

$$\begin{aligned} \left. \frac{\delta v^{-1}}{v^{-1}} \right|_{\overline{\text{MS}}} &= +\mathcal{C}_\mu \left( 3 \frac{M_Z^4}{M_H^2} + 6 \frac{M_W^4}{M_H^2} - \frac{3}{2} M_Z^2 - 3 M_W^2 + \frac{3}{2} M_H^2 \right. \\ &\quad \left. + \sum_{f_d} \left[ -4 \frac{m_1^2 + m_2^2}{M_H^2} + m_1^2 + m_2^2 \right] \right) , \\ \left. \frac{\delta M_Z^2}{M_Z^2} \right|_{\overline{\text{MS}}} &= +\mathcal{C}_\mu \left( -6 \frac{M_Z^4}{M_H^2} - 12 \frac{M_W^4}{M_H^2} + \frac{11}{3} M_Z^2 + \frac{14}{3} M_W^2 - 28 c_W^2 M_W^2 - 3 M_H^2 \right. \\ &\quad \left. + \sum_{f_s} \left[ 8 \frac{m_f^4}{M_H^2} - 2 m_f^2 + \frac{22}{27} (M_Z^2 - 2 M_W^2) + \frac{40}{27} c_W^2 M_W^2 \right] \right) , \\ \left. \frac{\delta M_W^2}{M_W^2} \right|_{\overline{\text{MS}}} &= +\mathcal{C}_\mu \left( -6 \frac{M_Z^4}{M_H^2} - 12 \frac{M_W^4}{M_H^2} + 3 M_Z^2 - \frac{68}{3} M_W^2 - 3 M_H^2 \right. \\ &\quad \left. + \sum_{f_d} \left[ 8 \frac{m_1^4 + m_2^4}{M_H^2} - 2 (m_1^2 + m_2^2) + \frac{4}{3} M_W^2 \right] \right) , \\ \left. \frac{\delta M_H^2}{M_H^2} \right|_{\overline{\text{MS}}} &= +\mathcal{C}_\mu \left( -3 M_Z^2 - 6 M_W^2 + 3 M_H^2 + \sum_{f_s} [2 m_f^2] \right) , \end{aligned}$$

$$\begin{aligned}
\left. \frac{\delta m_t}{m_t} \right|_{\overline{\text{MS}}} &= +C_\mu \left( 12 \frac{M_t^4}{M_H^2} + 12 \frac{M_b^4}{M_H^2} - 3 \frac{M_Z^4}{M_H^2} - 6 \frac{M_W^4}{M_H^2} - \frac{3}{2} M_b^2 + \frac{3}{2} M_t^2 - \frac{4}{3} M_Z^2 \right. \\
&\quad \left. + \frac{20}{3} M_W^2 - \frac{16}{3} c_W^2 M_W^2 - \frac{3}{2} M_H^2 - \frac{16}{3} c_W^2 s_W^2 M_Z^2 \right), \\
\left. \frac{\delta m_b}{m_b} \right|_{\overline{\text{MS}}} &= +C_\mu \left( 12 \frac{M_t^4}{M_H^2} + 12 \frac{M_b^4}{M_H^2} - 3 \frac{M_Z^4}{M_H^2} - 6 \frac{M_W^4}{M_H^2} + \frac{3}{2} M_b^2 - \frac{3}{2} M_t^2 + \frac{2}{3} M_Z^2 \right. \\
&\quad \left. + \frac{2}{3} M_W^2 - \frac{4}{3} c_W^2 M_W^2 - \frac{3}{2} M_H^2 - \frac{4}{3} c_W^2 s_W^2 M_Z^2 \right),
\end{aligned}$$

On-shell counterterms:

$$\begin{aligned}
\frac{\delta e}{e} &= C s_W^2 M_W^2 \left( \frac{38}{3} + 14 \frac{A_0(M_W)}{M_W^2} - \frac{8}{3} \sum_{f_s} Q_f^2 \left( 1 + \frac{A_0(m_f)}{m_f^2} \right) \right), \\
\frac{\delta v^{-1}}{v^{-1}} &= \frac{\delta e}{e} - \frac{1}{2 s_W^2} \left( s_W^2 \frac{\delta M_W^2}{M_W^2} + c_W^2 \frac{\delta M_Z^2}{M_Z^2} \right), \\
\delta M_H^2 &= C \left( A_0(M_H) (3 M_H^2) + A_0(M_Z) (M_H^2 + 6 M_Z^2) + B_0(M_H, M_H, M_H^2) \left( \frac{9}{2} M_H^4 \right) \right. \\
&\quad + B_0(M_Z, M_Z, M_H^2) \left( \frac{1}{2} M_H^4 - 2 M_H^2 M_Z^2 + 6 M_Z^4 \right) + A_0(M_W) (2 M_H^2 + 12 M_W^2) \\
&\quad + B_0(M_W, M_W, M_H^2) (M_H^4 - 4 M_H^2 M_W^2 + 12 M_W^4) \\
&\quad \left. + \sum_{f_s} [A_0(m_f) (-8 m_f^2) + B_0(m_f, m_f, M_H^2) (2 M_H^2 m_f^2 - 8 m_f^4)] \right), \\
\delta M_Z^2 &= C \left( -\frac{2}{3} M_H^2 M_Z^2 + 4 \frac{M_Z^4}{M_H^2} M_Z^2 - \frac{2}{9} M_Z^4 + 8 \frac{M_W^4}{M_H^2} M_Z^2 - \frac{20}{9} M_W^2 M_Z^2 + \frac{16}{3} M_W^4 \right. \\
&\quad - 16 c_W^2 M_W^4 + A_0(M_H) \left( \frac{1}{3} M_H^2 + 2 M_Z^2 \right) + A_0(M_Z) \left( -\frac{1}{3} M_H^2 + 6 \frac{M_Z^4}{M_H^2} + \frac{2}{3} M_Z^2 \right) \\
&\quad + A_0(M_W) \left( 12 \frac{M_W^2}{M_H^2} M_Z^2 + \frac{16}{3} M_W^2 - 16 c_W^2 M_W^2 + \frac{2}{3} M_Z^2 \right) \\
&\quad + B_0(M_Z, M_H, M_Z^2) \left( -\frac{4}{3} M_H^2 M_Z^2 + \frac{1}{3} M_H^4 + 4 M_Z^4 \right) \\
&\quad + B_0(M_W, M_W, M_Z^2) \left( \frac{16}{3} M_W^2 M_Z^2 - \frac{68}{3} M_W^4 - 16 c_W^2 M_W^4 + \frac{1}{3} M_Z^4 \right) \\
&\quad + \sum_{f_s} \left[ \left( \frac{32}{3} M_Z^2 m_f^2 - \frac{16}{9} M_Z^4 \right) (a_f^2 + b_f^2) \right. \\
&\quad + A_0(m_f) \left( -8 \frac{M_Z^2}{M_H^2} m_f^2 + \frac{32}{3} M_Z^2 (a_f^2 + b_f^2) \right) \\
&\quad \left. \left. + B_0(m_f, m_f, M_Z^2) \left( \frac{32}{3} M_Z^2 m_f^2 (a_f^2 - 2 b_f^2) + \frac{16}{3} M_Z^4 (a_f^2 + b_f^2) \right) \right] \right),
\end{aligned}$$

$$\begin{aligned}
 \delta M_W^2 = & \mathcal{C} \left( -\frac{2}{3} M_H^2 M_W^2 + 8 \frac{M_W^6}{M_H^2} + 4 \frac{M_Z^4}{M_H^2} M_W^2 - \frac{112}{9} M_W^4 - \frac{2}{3} M_Z^2 M_W^2 \right. \\
 & + A_0(M_H) \left( 2 M_W^2 + \frac{1}{3} M_H^2 \right) \\
 & + A_0(M_Z) \left( 6 \frac{M_Z^2}{M_H^2} M_W^2 + \frac{8}{3} M_W^2 - 8 c_W^2 M_W^2 + \frac{1}{3} M_Z^2 \right) \\
 & + A_0(M_W) \left( 12 \frac{M_W^4}{M_H^2} - 4 M_W^2 - \frac{1}{3} M_H^2 - \frac{1}{3} M_Z^2 \right) \\
 & + B_0(M_H, M_W, M_W^2) \left( -\frac{4}{3} M_H^2 M_W^2 + \frac{1}{3} M_H^4 + 4 M_W^4 \right) \\
 & + B_0(M_Z, M_W, M_W^2) \left( -\frac{68}{3} M_W^4 - 16 c_W^2 M_W^4 + \frac{1}{3} M_Z^4 + \frac{16}{3} M_Z^2 M_W^2 \right) \\
 & + B_0(0, M_W, M_W^2) (-16 s_W^2 M_W^4) + \sum_{f_d} \left[ \frac{4}{3} M_W^2 (m_1^2 + m_2^2) - \frac{4}{9} M_W^4 \right. \\
 & + A_0(m_1) \left( -8 \frac{M_W^2}{M_H^2} m_1^2 + \frac{4}{3} M_W^2 - \frac{2}{3} (m_1^2 - m_2^2) \right) \\
 & + A_0(m_2) \left( -8 \frac{M_W^2}{M_H^2} m_2^2 + \frac{4}{3} M_W^2 + \frac{2}{3} (m_1^2 - m_2^2) \right) \\
 & \left. \left. + B_0(m_1, m_2, M_W^2) \left( \frac{4}{3} M_W^4 - \frac{2}{3} M_W^2 (m_1^2 + m_2^2) - \frac{2}{3} (m_1^2 - m_2^2)^2 \right) \right] \right), \\
 \delta m_\tau = & m_\tau \mathcal{C} \left( 4 \frac{M_W^4}{M_H^2} + 2 \frac{M_Z^4}{M_H^2} - 3 M_W^2 + \frac{3}{2} M_Z^2 \right. \\
 & + A_0(M_H) + A_0(M_Z) \left( 3 \frac{M_Z^2}{M_H^2} + 6 \frac{M_W^2}{m_\tau^2} - 4 c_W^2 \frac{M_W^2}{m_\tau^2} - \frac{5}{2} \frac{M_Z^2}{m_\tau^2} \right) \\
 & + A_0(M_W) \left( 6 \frac{M_W^2}{M_H^2} - \frac{M_W^2}{m_\tau^2} + \frac{1}{2} \right) + A_0(m_\tau) \left( -2 \frac{M_W^2}{m_\tau^2} + \frac{5}{2} \frac{M_Z^2}{m_\tau^2} + 1 \right) \\
 & + B_0(m_\tau, M_H, m_\tau^2) \left( -\frac{1}{2} M_H^2 + 2 m_\tau^2 \right) + B_0(m_\tau, M_Z, m_\tau^2) \\
 & \times \left( 6 M_W^2 M_Z^2 / m_\tau^2 + 12 M_W^2 - 8 c_W^2 M_W^2 - 4 \frac{M_W^4}{m_\tau^2} - \frac{7}{2} M_Z^2 - \frac{5}{2} \frac{M_Z^4}{m_\tau^2} \right) \\
 & + B_0(m_\tau, 0, m_\tau^2) (-8 M_W^2 + 8 c_W^2 M_W^2) \\
 & \left. + B_0(m_{\nu_\tau}, M_W, m_\tau^2) \left( \frac{1}{2} M_W^2 - \frac{M_W^4}{m_\tau^2} + \frac{1}{2} m_\tau^2 \right) + \sum_{f_s} A_0(m_f) \left( -4 \frac{m_f^2}{M_H^2} \right) \right),
 \end{aligned}$$

$$\begin{aligned}
\delta M_b = & M_b \mathcal{C} \left( -\frac{4}{9} s_W^2 M_W^2 + 2 \frac{M_Z^4}{M_H^2} + 4 \frac{M_W^4}{M_H^2} - \frac{13}{18} M_Z^2 - \frac{11}{9} M_W^2 + \frac{4}{9} \frac{M_W^4}{M_Z^2} \right. \\
& + A_0(M_H) + A_0(M_W) \left( \frac{1}{2} + 6 \frac{M_W^2}{M_H^2} - \frac{1}{2} \frac{M_t^2}{M_b^2} - \frac{M_W^2}{M_b^2} \right) \\
& + A_0(M_Z) \left( 3 \frac{M_Z^2}{M_H^2} - \frac{5}{18} \frac{M_Z^2}{M_b^2} + \frac{2}{9} \frac{M_W^2}{M_b^2} - \frac{4}{9} \frac{M_W^2}{M_Z^2} \frac{M_W^2}{M_b^2} \right) \\
& + A_0(M_t) \left( \frac{1}{2} + \frac{1}{2} \frac{M_t^2}{M_b^2} + \frac{M_W^2}{M_b^2} \right) + A_0(M_b) 4 s_W^2 \left( \frac{1}{3} \frac{M_W^2}{M_b^2} \right) \\
& + A_0(M_b) \left( 1 + \frac{5}{18} \frac{M_Z^2}{M_b^2} - \frac{2}{9} \frac{M_W^2}{M_b^2} + \frac{4}{9} \frac{M_W^2}{M_Z^2} \frac{M_W^2}{M_b^2} \right) + B_0(M_t, M_W, M_b^2) \\
& \times \left( \frac{1}{2} M_b^2 - M_t^2 + \frac{1}{2} \frac{M_t^4}{M_b^2} + \frac{1}{2} M_W^2 + \frac{1}{2} M_W^2 \frac{M_t^2}{M_b^2} - \frac{M_W^4}{M_b^2} \right) \\
& + B_0(M_b, M_H, M_b^2) \left( 2 M_b^2 - \frac{1}{2} M_H^2 \right) + B_0(M_b, M_Z, M_b^2) \\
& \times \left( \frac{17}{18} M_Z^2 - \frac{5}{18} \frac{M_Z^4}{M_b^2} + \frac{4}{9} M_W^2 + \frac{2}{9} M_W^2 \frac{M_Z^2}{M_b^2} - \frac{8}{9} \frac{M_W^4}{M_Z^2} - \frac{4}{9} \frac{M_W^4}{M_b^2} \right) \\
& \times \sum_{f_s} A_0(m_f) \left( -4 \frac{m_f^2}{M_H^2} \right) \Bigg) \\
\delta M_t = & M_t \mathcal{C} \left( -\frac{16}{9} s_W^2 M_W^2 + 2 \frac{M_Z^4}{M_H^2} + 4 \frac{M_W^4}{M_H^2} - \frac{1}{18} M_Z^2 - \frac{29}{9} M_W^2 + \frac{16}{9} \frac{M_W^4}{M_Z^2} \right. \\
& + A_0(M_H) + A_0(M_W) \left( \frac{1}{2} + 6 \frac{M_W^2}{M_H^2} - \frac{1}{2} \frac{M_b^2}{M_t^2} - \frac{M_W^2}{M_t^2} \right) \\
& + A_0(M_Z) \left( 3 \frac{M_Z^2}{M_H^2} - \frac{17}{18} \frac{M_Z^2}{M_t^2} + \frac{20}{9} \frac{M_W^2}{M_t^2} - \frac{16}{9} \frac{M_W^2}{M_Z^2} \frac{M_W^2}{M_t^2} \right) \\
& + A_0(M_t) 4 s_W^2 \left( \frac{4}{3} \frac{M_W^2}{M_t^2} \right) + A_0(M_t) \left( 1 + \frac{17}{18} \frac{M_Z^2}{M_t^2} - \frac{20}{9} \frac{M_W^2}{M_t^2} + \frac{16}{9} \frac{M_W^2}{M_Z^2} \frac{M_W^2}{M_t^2} \right) \\
& + A_0(M_b) \left( \frac{1}{2} + \frac{1}{2} \frac{M_b^2}{M_t^2} + \frac{M_W^2}{M_t^2} \right) + B_0(M_t, M_H, M_t^2) \left( 2 M_t^2 - \frac{1}{2} M_H^2 \right) \\
& + B_0(M_t, M_Z, M_t^2) \\
& \times \left( -\frac{7}{18} M_Z^2 - \frac{17}{18} \frac{M_Z^4}{M_t^2} + \frac{40}{9} M_W^2 + \frac{20}{9} M_W^2 \frac{M_Z^2}{M_t^2} - \frac{32}{9} \frac{M_W^4}{M_Z^2} - \frac{16}{9} \frac{M_W^4}{M_t^2} \right) \\
& + B_0(M_b, M_W, M_t^2) \left( \frac{1}{2} \frac{M_b^4}{M_t^2} - M_b^2 + \frac{1}{2} M_t^2 + \frac{1}{2} \frac{M_W^2}{M_t^2} M_b^2 + \frac{1}{2} M_W^2 - \frac{M_W^4}{M_t^2} \right) \\
& \times \sum_{f_s} A_0(m_f) \left( -4 \frac{m_f^2}{M_H^2} \right) \Bigg) .
\end{aligned}$$

The on-loop corrections give the dominant contribution in the matching relations. Two-loop results may be found in Ref. [54, 55, 81] and in Refs. quoted in Section 3.

## REFERENCES

- [1] G. Aad *et al.* [ATLAS Collaboration], *Phys. Lett.* **B716**, 1 (2012); *Science* **338**, 1576 (2012).
- [2] S. Chatrchyan *et al.* [CMS Collaboration], *Phys. Lett.* **B716**, 30 (2012); *Science* **338**, 1569 (2012).
- [3] S.L. Glashow, *Nucl. Phys.* **B22**, 579 (1961); S. Weinberg, *Phys. Rev. Lett.* **19**, 1264 (1967); A. Salam, in: *Elementary Particle Theory*, ed. N. Svartholm, Amquist and Wiksells, Stockholm 1969, p. 376.
- [4] H. Fritzsch, M. Gell-Mann, H. Leutwyler, *Phys. Lett.* **47**, 365 (1973); H.D. Politzer, *Phys. Rev. Lett.* **30**, 1346 (1973); D. Gross, F. Wilczek, *Phys. Rev. Lett.* **30**, 1343 (1973); S. Weinberg, *Phys. Rev. Lett.* **31**, 494 (1973).
- [5] P.W. Higgs, *Phys. Lett.* **12**, 132 (1964); *Phys. Rev. Lett.* **13**, 508 (1964); *Phys. Rev.* **145**, 1156 (1966); F. Englert, R. Brout, *Phys. Rev. Lett.* **13**, 321 (1964); G.S. Guralnik, C.R. Hagen, T.W.B. Kibble, *Phys. Rev. Lett.* **13**, 585 (1964); T.W.B. Kibble, *Phys. Rev.* **155**, 1554 (1967).
- [6] T. Hambye, K. Riesselmann, *Phys. Rev.* **D55**, 7255 (1997).
- [7] M. Holthausen, K.S. Lim, M. Lindner, *J. High Energy Phys.* **1202**, 037 (2012).
- [8] F. Bezrukov, M.Yu. Kalmykov, B.A. Kniehl, M. Shaposhnikov, *J. High Energy Phys.* **1210**, 140 (2012).
- [9] G. Degrandi *et al.*, *J. High Energy Phys.* **1208**, 098 (2012).
- [10] S. Alekhin, A. Djouadi, S. Moch, *Phys. Lett.* **B716**, 214 (2012).
- [11] L.N. Mihaila, J. Salomon, M. Steinhauser, *Phys. Rev. Lett.* **108**, 151602 (2012).
- [12] K.G. Chetyrkin, M.F. Zoller, *J. High Energy Phys.* **1206**, 033 (2012).
- [13] I. Masina, *Phys. Rev.* **D87**, 053001 (2013).
- [14] A.V. Bednyakov, A.F. Pikelner, V.N. Velizhanin, *J. High Energy Phys.* **1301**, 017 (2013).
- [15] A.V. Bednyakov, A.F. Pikelner, V.N. Velizhanin, *Phys. Lett.* **B722**, 336 (2013).
- [16] K.G. Chetyrkin, M.F. Zoller, *J. High Energy Phys.* **1304**, 091 (2013).
- [17] A.V. Bednyakov, A.F. Pikelner, V.N. Velizhanin, *Nucl. Phys.* **B875**, 552 (2013).
- [18] D. Buttazzo *et al.*, *J. High Energy Phys.* **1312**, 089 (2013).
- [19] A.V. Bednyakov, A.F. Pikelner, V.N. Velizhanin, *Nucl. Phys.* **B879**, 256 (2014).

- [20] G. 't Hooft, *Nucl. Phys.* **B33**, 173 (1971); **B35**, 167 (1971); G. 't Hooft, M. Veltman, *Nucl. Phys.* **B50**, 318 (1972).
- [21] F. Jegerlehner, *Quantum Field Theory and Statistical Mechanics*, in: *Trends in Elementary Particle Theory*, H. Rollnik, K. Dietz (eds.), Lecture Notes in Physics, Vol. 37, Springer, Berlin 1975, pp. 114–170.
- [22] M.J.G. Veltman, *Acta Phys. Pol. B* **12**, 437 (1981).
- [23] R. Decker, J. Pestieau, [arXiv:hep-ph/0512126](#).
- [24] G. Degrassi, A. Sirlin, *Nucl. Phys.* **B383**, 73 (1992).
- [25] Z.Y. Fang, G. Lopez Castro, J.L. Lucio, J. Pestieau, *Mod. Phys. Lett.* **A12**, 1531 (1997).
- [26] F. Jegerlehner, *Nucl. Phys. Proc. Suppl.* **B37**, 129 (1994).
- [27] M.J.G. Veltman, *Nucl. Phys.* **B7**, 637 (1968).
- [28] C.H. Llewellyn Smith, *Phys. Lett.* **B46**, 233 (1973).
- [29] J.S. Bell, *Nucl. Phys.* **B60**, 427 (1973).
- [30] J.M. Cornwall, D.N. Levin, G. Tiktopoulos, *Phys. Rev. Lett.* **30**, 1268 (1973) [*Erratum-ibid.* **31**, 572 (1973)].
- [31] J.M. Cornwall, D.N. Levin, G. Tiktopoulos, *Phys. Rev.* **D10**, 1145 (1974) [*Erratum-ibid.* **D11**, 972 (1975)].
- [32] F. Jegerlehner, *Helv. Phys. Acta* **51**, 783 (1978).
- [33] F. Jegerlehner, *The ‘Ether-World’ and Elementary Particles*, in: *Theory of Elementary Particles*, H. Dorn, D. Lüst, G. Weight (eds.), WILEY-VCH, Berlin 1998, pp. 386–392 [[arXiv:hep-th/9803021](#)].
- [34] K.G. Wilson, *Phys. Rev.* **B4**, 3174 (1971).
- [35] K.G. Wilson, *Phys. Rev.* **B4**, 3184 (1971).
- [36] K.G. Wilson, M.E. Fisher, *Phys. Rev. Lett.* **28**, 240 (1972).
- [37] K.G. Wilson, J.B. Kogut, *Phys. Rep.* **12**, 75 (1974).
- [38] F. Jegerlehner, M.Yu. Kalmykov, B.A. Kniehl, *Phys. Lett.* **B722**, 123 (2013).
- [39] H. Aoki, S. Iso, *Phys. Rev.* **D86**, 013001 (2012).
- [40] M.S. Al-sarhi, I. Jack, D.R.T. Jones, *Z. Phys.* **C55**, 283 (1992).
- [41] Y. Hamada, H. Kawai, K.-y. Oda, *Phys. Rev.* **D87**, 053009 (2013).
- [42] D.R.T. Jones, *Phys. Rev.* **D88**, 098301 (2013).
- [43] A. Pais, G.E. Uhlenbeck, *Phys. Rev.* **79**, 145 (1950).
- [44] W. Pauli, F. Villars, *Rev. Mod. Phys.* **21**, 434 (1949).
- [45] F. Jegerlehner, *Phys. Rev.* **D16**, 397 (1977).
- [46] K. Symanzik, *Commun. Math. Phys.* **45**, 79 (1975).
- [47] J. Zinn-Justin, in: *Cargèse Lectures in Physics*, 1973, Ed. E. Brèzin (unpublished); E. Brèzin, J.C. Le Guillou, J. Zinn-Justin, *Field Theoretical Approach to Critical Phenomena*, in: *Phase Transitions and Critical Phenomena*, C. Domb, M.S. Green (Eds.), Academic, New York 1976, Vol. 6, pp. 125–247.

- [48] C.G. Callan, Jr., *Phys. Rev.* **D2**, 1541 (1970).
- [49] K. Symanzik, *Commun. Math. Phys.* **18**, 227 (1970).
- [50] P. Hegde, K. Jansen, C.J.D. Lin, A. Nagy, arXiv:1310.6260 [hep-lat].
- [51] G. 't Hooft, M.J.G. Veltman, *Nucl. Phys.* **B44**, 189 (1972).
- [52] G. 't Hooft, *Nucl. Phys.* **B61**, 455 (1973).
- [53] F. Jegerlehner, *The Anomalous Magnetic Moment of the Muon*, Springer Tracts in Modern Physics, Vol. 226, (2008), p. 102.
- [54] F. Jegerlehner, M.Y. Kalmykov, O. Veretin, *Nucl. Phys. Proc. Suppl.* **116**, 382 (2003).
- [55] F. Jegerlehner, M.Y. Kalmykov, O. Veretin, *Nucl. Phys.* **B658**, 49 (2003).
- [56] J. Fleischer, F. Jegerlehner, *Phys. Rev.* **D23**, 2001 (1981).
- [57] A. Sirlin, R. Zucchini, *Nucl. Phys.* **B266**, 389 (1986).
- [58] N. Gray, D.J. Broadhurst, W. Grafe, K. Schilcher, *Z. Phys.* **C48**, 673 (1990).
- [59] J. Fleischer, F. Jegerlehner, O.V. Tarasov, O.L. Veretin, *Nucl. Phys.* **B539**, 671 (1999) [Erratum-*ibid.* **B571**, 511 (2000)].
- [60] K.G. Chetyrkin, M. Steinhauser, *Phys. Rev. Lett.* **83**, 4001 (1999).
- [61] K.G. Chetyrkin, M. Steinhauser, *Nucl. Phys.* **B573**, 617 (2000).
- [62] K. Melnikov, T. von Ritbergen, *Phys. Lett.* **B482**, 99 (2000).
- [63] A.L. Kataev, V.T. Kim, *Phys. Part. Nucl.* **41**, 946 (2010).
- [64] R. Hempfling, B.A. Kniehl, *Phys. Rev.* **D51**, 1386 (1995).
- [65] F. Jegerlehner, M.Y. Kalmykov, *Nucl. Phys.* **B676**, 365 (2004).
- [66] F. Jegerlehner, M.Y. Kalmykov, *Acta Phys. Pol. B* **34**, 5335 (2003).
- [67] F. Jegerlehner, M.Y. Kalmykov, *Nucl. Instrum. Methods* **A534**, 299 (2004).
- [68] S.P. Martin, *Phys. Rev.* **D72**, 096008 (2005).
- [69] D. Eiras, M. Steinhauser, *J. High Energy Phys.* **0602**, 010 (2006).
- [70] M. Faisst, J.H. Kühn, O. Veretin, *Phys. Lett.* **B589**, 35 (2004).
- [71] T. Appelquist, J. Carazzone, *Phys. Rev.* **D11**, 2856 (1975).
- [72] J.C. Taylor, *Gauge Theories of Weak Interactions*, Cambridge Monographs on Mathematical Physics, 1976, pp. 124–127.
- [73] E. Kraus, *Ann. Phys.* **262**, 155 (1998).
- [74] J. Beringer *et al.* [Particle Data Group], *Phys. Rev.* **D86**, 010001 (2012).
- [75] A. Juste, arXiv:1310.0799 [hep-ph].
- [76] D.R.T. Jones, *Nucl. Phys.* **B87**, 127 (1975).
- [77] D.R.T. Jones, *Phys. Rev.* **D25**, 581 (1982).
- [78] M.E. Machacek, M.T. Vaughn, *Nucl. Phys.* **B222**, 83 (1983).
- [79] M.E. Machacek, M.T. Vaughn, *Nucl. Phys.* **B236**, 221 (1984).
- [80] M.E. Machacek, M.T. Vaughn, *Nucl. Phys.* **B249**, 70 (1985).
- [81] F. Jegerlehner, M.Y. Kalmykov, O. Veretin, *Nucl. Phys.* **B641**, 285 (2002).

- [82] P.A.R. Ade *et al.* [Planck Collab.], [arXiv:1303.5062 \[astro-ph.CO\]](#); [arXiv:1303.5082 \[astro-ph.CO\]](#); [arXiv:1303.5084 \[astro-ph.CO\]](#).
- [83] D.A. Kirzhnits, *JETP Lett.* **15**, 529 (1972); D.A. Kirzhnits, A.D. Linde, *Phys. Lett.* **B72**, 471 (1972).
- [84] S. Weinberg, *Phys. Rev.* **D9**, 3357 (1974); L. Dolan, R. Jackiw, *Phys. Rev.* **D9**, 3320 (1974); D.A. Kirzhnits, A.D. Linde, *JETP* **40**, 628 (1974).
- [85] D.A. Kirzhnits, A.D. Linde, *Ann. Phys.* **101**, 195 (1976).
- [86] M. Dine *et al.*, *Phys. Rev.* **D46**, 550 (1992).
- [87] A.H. Guth, *Phys. Rev.* **D23**, 347 (1981).
- [88] A.A. Starobinsky, *Phys. Lett.* **B91**, 99 (1980).
- [89] A.D. Linde, *Phys. Lett.* **B108**, 389 (1982); **B129**, 177 (1983).
- [90] A. Albrecht, P.J. Steinhardt, *Phys. Rev. Lett.* **48**, 1220 (1982).
- [91] V.F. Mukhanov, G.V. Chibisov, *JETP Lett.* **33**, 532 (1981) [*Pisma Zh. Eksp. Teor. Fiz.* **33**, 549 (1981)].
- [92] V.F. Mukhanov, *JETP Lett.* **41**, 493 (1985) [*Pisma Zh. Eksp. Teor. Fiz.* **41**, 402 (1985)].
- [93] V.F. Mukhanov, H.A. Feldman, R.H. Brandenberger, *Phys. Rep.* **215**, 203 (1992).
- [94] F. Jegerlehner, *Acta Phys. Pol. B* **45**, 1215 (2014), this issue.
- [95] L.G. Bian, [arXiv:1303.2402 \[hep-ph\]](#).
- [96] I. Masina, M. Quiros, *Phys. Rev.* **D88**, 093003 (2013).
- [97] Y. Hamada, H. Kawai, K.-y. Oda, *Prog. Theor. Exp. Phys.* **023B02**, (2014).
- [98] A.H. Hoang, T. Teubner, *Phys. Rev.* **D58**, 114023 (1998).
- [99] A.H. Hoang *et al.*, *Eur. Phys. J.* **C2**, 1 (2000).
- [100] H. Baer *et al.* [arXiv:1306.6352 \[hep-ph\]](#).
- [101] K. Agashe *et al.* [Top Quark Working Group], [arXiv:1311.2028 \[hep-ph\]](#).
- [102] W. Buchmüller, Z. Fodor, A. Hebecker, *Nucl. Phys.* **B447**, 317 (1995).
- [103] K. Kajantie, M. Laine, K. Rummukainen, M.E. Shaposhnikov, *Phys. Rev. Lett.* **77**, 2887 (1996).
- [104] V.A. Rubakov, M.E. Shaposhnikov, *Usp. Fiz. Nauk* **166**, 493 (1996) [*Phys. Usp.* **39**, 461 (1996)].
- [105] D.J.H. Chung, A.J. Long, L.-T. Wang, *Phys. Rev.* **D87**, 023509 (2013).
- [106] N. Cabibbo, *Phys. Rev. Lett.* **10**, 531 (1963); M. Kobayashi, K. Maskawa, *Prog. Theor. Phys.* **49**, 652 (1973).
- [107] R.D. Peccei, H.R. Quinn, *Phys. Rev. Lett.* **38**, 1440 (1977).