HYDRODYNAMIC MODELS OF ULTRARELATIVISTIC COLLISIONS*

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We discuss the use of the hydrodynamic model for the description of the evolution of dense matter formed in ultrarelativistic heavy-ion collisions. The collective flow observed in heavy-ion collisions at the BNL Relativistic Heavy Ion Collider and at the CERN Large Hadron Collider is consistent with the assumption that a fireball of strongly interacting matter is formed. Experimental results from p–Pb and d–Au collisions show similar phenomena, which suggests that collective expansion appears in small systems as well. We review the recent application of the hydrodynamic model to small systems and discuss limitations and possible further checks of this scenario.

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1. Introduction

Nuclear collisions at ultrarelativistic energies lead to the formation of a dense fireball of quark–gluon plasma [1–4]. The observation of the elliptic flow, the asymmetry in the particle spectra between the in and out of the reaction plane directions, is a strong evidence in favor of the existence of collective flow [5–10]. The interpretation of these experimental results assumes the expansion of a droplet of strongly interacting medium.

If the systems evolves close to local thermodynamic equilibrium, relativistic hydrodynamic equations

$$\partial_\mu T^{\mu\nu} = 0 \tag{1}$$

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can be used to describe the dynamics of the local energy density ϵ, pressure P and flow velocity u^μ, where the energy momentum tensor

$$T^{\mu\nu} = (\epsilon + P)u^\mu u^\nu - Pg^{\mu\nu} + \pi^{\mu\nu} + \Pi\left(g^{\mu\nu} - u^\mu u^\nu\right) . \tag{2}$$

The stress tensor $\pi^{\mu\nu}$ and the bulk viscosity correction Π are solutions of dynamical equation in the second order viscous hydrodynamic framework [11–22]. The shear and bulk viscosity coefficients are important characteristics of the quark–gluon plasma. The estimated small value of the shear viscosity to entropy ratio η/s is not far from the AdS/CFT estimate $\eta/s = 0.08$ [23], which shows that the medium formed in heavy-ion collisions is strongly interacting. The extraction of the shear viscosity coefficient is a difficult task, as it requires the comparison of model calculations to experimental data on the elliptic and triangular flow at different collision centralities [24–27]. The problem comes from the uncertainty on the initial values of the spatial ellipticity and triangularity. Another issue is related to the temperature dependence of η/s; in particular, η/s can be very different in the plasma and the hadronic phase [17, 19].

The shape of the overlap region in the collision fluctuates from event to event and the eccentricity increases due to these fluctuations [28]. For each initial state, the hydrodynamic evolution is performed independently [18, 29–34]. The appearance of the triangular deformation from fluctuations brings in a qualitatively new observation, a non-zero triangular flow [35]. The initial density of the fireball is generated from a Monte Carlo model, the Glauber model, f-KLN, IP-Glasma, URQMD, or AMTP. The density and flow velocity evolves according to hydrodynamic equations and is driven by pressure gradients in the fireball. The collective expansion ends at the freeze-out hypersurface. That surface is usually defined as a constant temperature surface, or equivalently as a cut-off in local energy density. For smaller densities the collective expansion does not occur; individual hadrons are emitted from the freeze-out hypersurface. After freeze-out only hadron rescattering, resonance decay or creation can occur. In the model calculations presented below, we use Glauber Monte Carlo initial conditions [36], event-by-event hydrodynamic evolution with bulk and shear viscosity [37], and the Therminator code to simulate statistical hadron emission at freeze-out and subsequent resonance decays [38, 39].

The fireball is elongated in the longitudinal direction (space-time rapidity) and in the transverse direction it is deformed. The azimuthal deformation can be effectively parametrized using n-order eccentricity coefficients

$$\epsilon_n e^{in\phi_n} = \frac{\int \rho(x,y) e^{in\phi} r^n dxdy}{\int \rho(x,y) r^n dxdy} , \tag{3}$$

where ϕ_n defines the n-order event plane direction. The transverse momentum spectra of emitted particles can be written as

$$\frac{dN}{d_\perp^2 dy} = \frac{dN}{2\pi p_\perp dp_\perp d\phi dy}\left(1 + 2v_1\cos(\phi-\psi_1) + 2v_2\cos(2(\phi-\psi_2)) + 2v_3\cos(3(\phi-\psi_3)) + \ldots\right). \quad (4)$$

For $n = 2$ and 3, the hydrodynamic response is approximately linear [30]

$$v_n \simeq A\epsilon_n\,. \quad (5)$$

The hydrodynamic response A depends on the details of the hydrodynamic evolution, in particular, it is sensitive to the value of the shear viscosity. The flow component in the two-particle correlation function in relative azimuthal angle $\Delta\phi$ and relative pseudorapidity $\Delta\eta$ is approximately independent of $\Delta\eta$, while its harmonic expansion in azimuthal angle is given by the flow coefficients

$$C(\Delta\phi,\Delta\eta) \propto 1 + 2v_1^2\cos(\Delta\phi) + 2v_2^2\cos(2\Delta\phi) + 2v_3^2\cos(3\Delta\phi) + \ldots \quad (6)$$

The two-dimensional plot of the correlation function has a same- ($\Delta\phi \simeq 0$) and away-side side ($\Delta\phi \simeq \pi$) ridge (Fig. 1). Non-flow correlations contribute to the two-particle correlation $C(\Delta\phi,\Delta\eta)$: the jet-like correlations, resonance decays, and the local charge conservation at small $\Delta\eta$ and $\Delta\phi$ [40],

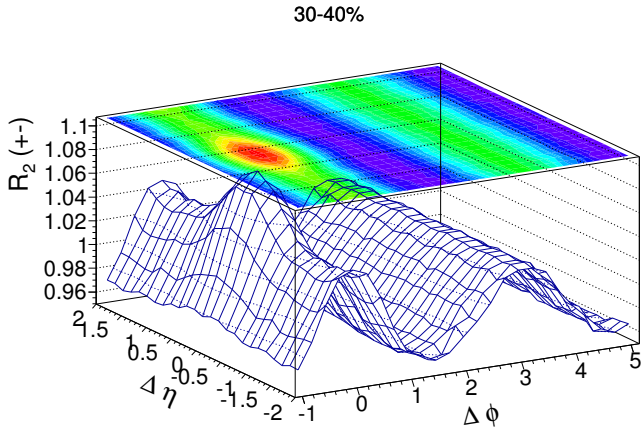


Fig. 1. Two particle correlations function ($R(\Delta\phi,\Delta\eta) = C(\Delta\phi,\Delta\eta)$) in relative pseudorapidity and relative azimuthal angle for unlike charged hadrons emitted with $p_\perp > 0.8$ GeV in 30–40% centrality Au–Au collisions at 200 GeV. The calculation is based on event-by-event viscous hydrodynamics with local charge correlations included.

and transverse momentum conservation in the away-side ridge region [41]. Experimental estimates of the flow coefficients from the second order cumulants are equivalent to an event average of the two-particle correlation function $\langle v_n^2 \rangle = \langle C(\Delta\phi) \cos(n\Delta\phi) \rangle$, which sums up the flow fluctuations as well as the average flow.

The space-time pattern of particle emission from the expanding fluid can be extracted from the same particle interferometry correlations [42, 43]. The interferometry radii measure the size of the emission region for pairs of particles of a given momentum [44]. The value of the femtoscopy radii serves as an estimate of the size of the fireball at freeze-out, and it is consistent with the size of the initial fireball assumed in hydrodynamic models, supplemented with the increase during the collective expansion phase. The reduction of the interferometry radii with the average momentum of the pair indicates a strong correlation between the flow and position. This correlation can be reproduced in hydrodynamic calculations when a realistic, hard equation of state is used [45–48].

2. Collective flow in small systems

Experimental results from relativistic heavy-ion collisions present strong evidence for the formation of a dense fireball that expands collectively. Ultrarelativistic d–Au and p–Pb collisions have been performed in order to study phenomena unrelated to plasma formation and to obtain reference data for heavy-ion experiments [49]. On the other hand, extrapolations of the initial energy density from peripheral Pb–Pb to p–Pb collisions indicate that collective expansion could take place in p–Pb collisions at the LHC. The hydrodynamic model predicts a significant transverse expansion of the fireball formed in high multiplicity p–Pb collisions [50].

The observed two-particle correlation functions in p–Pb collisions [51–53] are qualitatively similar to the A–A case, as two ridge-like structures elongated in the pseudorapidity direction are clearly visible. These structures can be explained as due to the collective flow and the transverse momentum conservation [54]. The elliptic and triangular collective flow components, together with the $\cos(\Delta\phi)$ contribution from momentum conservation, qualitatively reproduce the observed projected correlation function $C(\Delta\phi)$ (Fig. 2). In the one-dimensional correlation function presented in Fig. 2, the short range non-flow correlations are reduced using a cut $|\Delta\eta| > 2$ in the projection. We note that a similar mechanism could explain the ridge structures observed in the high multiplicity p–p collisions [55], but definite conclusions are more difficult here due to stronger non-flow contributions [56]. The observed two ridge structure of the correlation function can arise due to initial state effects [57–60], leading an enhancement of the gluon emission at small angles. It is important to be able to disentangle the two scenarios.

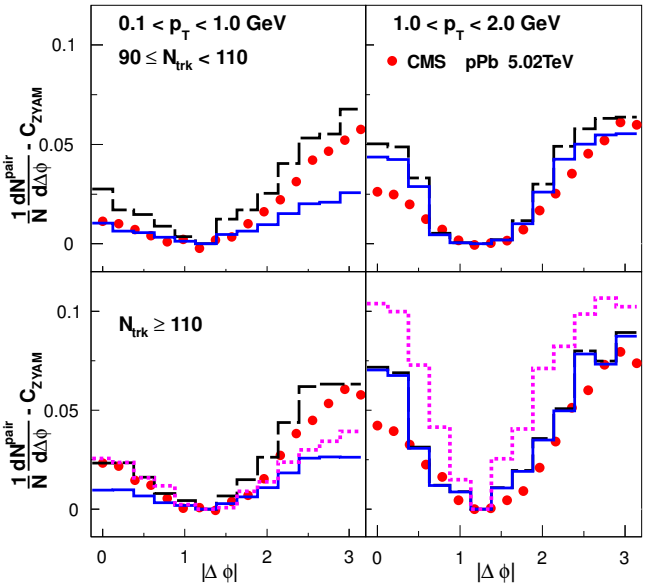


Fig. 2. The ZYAM subtracted correlation function in p–Pb collisions. The CMS measurement [51] is shown as filled circles. The results of the hydrodynamic model with the normalized correlation functions are shown with the solid lines. The dashed lines represent the ZYAM subtracted results of the hydrodynamic model and no rescaling. The dotted lines show the results obtained with an initial time of 0.2 instead of 0.6 fm/c (from [54]).

The extraction of the flow coefficients v_2 and v_3 in p–Pb collisions is difficult due to significant non-flow correlations. Methods involving subtraction of peripheral from central correlation functions, employing the rapidity gap, or higher order cumulants can be used for that purpose [61–63]. The elliptic and triangular flow of charged particles in high multiplicity p–Pb events is well described by the hydrodynamic model [64] (Fig. 3). Qualitatively similar results are obtained in hydrodynamic calculations using various assumptions about the initial density [65–68]. In p–Pb collisions the initial density is formed from a small number of independent sources. This leads to the approximate equality of eccentricities from higher order cumulants $v_2\{4\} \simeq v_2\{6\} \simeq v_2\{8\}$ [69–71].

The fireball is smaller and lives shorter than in A–A collisions. It makes the quantitative prediction of the hydrodynamic model more sensible to the assumed initial state scenario or to changes in phenomenological parameters. The shape of the fireball depends on the modeling of the energy deposition on small scales and should be described using subnuclear degrees of freedom [64, 66]. The amount of the transverse flow generated changes noticeably

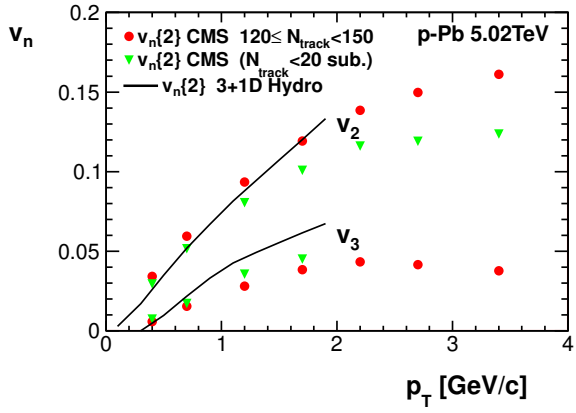


Fig. 3. v_2 and v_3 for charged particles from the hydrodynamic calculation compared to CMS Collaboration data [62] (from [72]).

when the initial thermalization time or the freeze-out density are lowered. More importantly, we should be aware that the applicability of second order viscous hydrodynamics is less justified in small systems, when velocity gradients are large.

The elliptic flow coefficient as function of transverse momentum, $v_2(p_\perp)$, splits for different particles. In particular, the elliptic flow of pions is larger than for protons, for $p_\perp < 1.5$ GeV. This appears in hydrodynamic models as the mass splitting of the elliptic flow. The results for the elliptic flow of identified particles reproduce qualitatively the experimental pion–proton splitting [72] (Fig. 4).

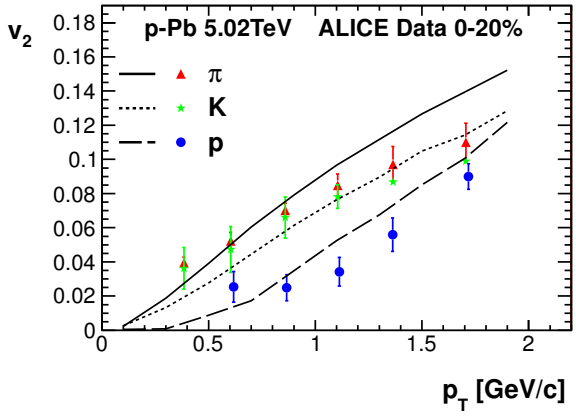


Fig. 4. $v_2(p_\perp)$ for pions, kaons, and protons from the hydrodynamic model, compared to ALICE Collaboration data [63] (from [72]).

The momentum of particles emitted from a moving fluid element gets a contribution from the collective velocity, it is bigger for massive particles. This yields a mass hierarchy in the average transverse momentum of particles [74]. Scenarios without collective expansion can predict the increase of the average transverse flow with particle mass in p–p collisions [75], in accordance with experimental results, but models based on the convolution of independent nucleon–nucleon collisions cannot reproduce the experimental results in p–Pb interactions [76]. An example of the average transverse momentum from a superposition model (HIJING) is given in the right panel of Fig. 5, where the value of $\langle p_\perp \rangle$ and its mass splitting are smaller than in the experiment. A hydrodynamic calculation [72] can reproduce the mass hierarchy of the average transverse momenta (Fig. 5, left panel). The rapidity dependence of the average transverse momentum could serve as a way to disentangle between the collective expansion and color glass condensate scenarios [77]. In the hydrodynamic model, the transverse push is smaller when going to the proton side, while the reverse is true in the color glass condensate approach.

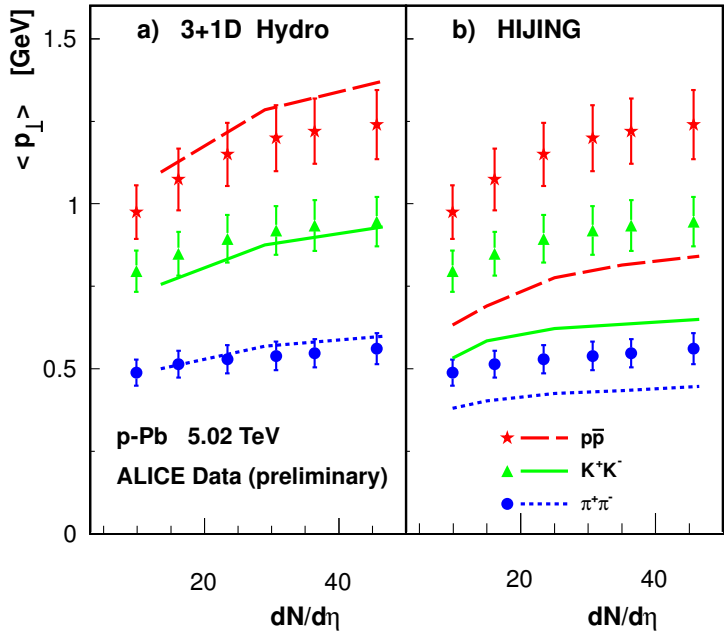


Fig. 5. Average transverse momentum for pions, kaons, and protons in p–Pb collisions from the hydrodynamic model (left panel) and from the HIJING model (right panel), compared to ALICE Collaboration data [73] (from [72]).

The scenario based on the formation and expansion of a dense fireball brings the question about the possibility of measuring its size. The interferometry radii could measure the size of the fireball at freeze-out and its momentum dependence could provide information on the transverse flow. If the initial size of the fireball increases during the expansion, the femtoscopy radii measured in p–Pb collisions should be larger than in p–p interaction [78]. The initial size of the interaction region is small in the IP-glasma scenario [66], hence measuring such small radii in the experiment would indicate that the expansion does not happen.

The collective flow in d–A collisions is another interesting possibility. It has been noted that the ellipticity in high multiplicity d–A collisions is big, with the central (highest-multiplicity) events corresponding to the deuteron hitting the bigger nucleus side-wise [50]. An intrinsic deformation of the fireball appears, unlike for p–Pb collisions where the deformation is entirely due to fluctuations. The two-particle correlation functions in the d–Au collisions at 200 GeV have been analyzed using similar methods as for the p–Pb collisions at the LHC energies [79]. The extracted elliptic flow coefficient is large as expected from hydrodynamic calculations, while the triangular flow is negligible.

3. Discussion

3.1. Summary of results

The relativistic heavy-ion research program has provided a strong evidence for the formation of strongly interacting quark–gluon plasma in A–A collisions. Recent experimental results indicate that final state interaction followed with collective evolution could also be important in ultrarelativistic collisions of small on large systems, p–Pb at 2.76 TeV and d–Au collisions at 200 GeV. Observations favoring this scenario are:

- The observation of elliptic and triangular flow in p–Pb collisions [61–63], consistent with model calculations [64–68].
- An even larger elliptic flow in d–Au collisions [79], in line with hydrodynamic predictions [50].
- The mass hierarchy of the elliptic flow and of the average transverse flow [63, 73, 80].
- Similarity between p–Pb collisions and peripheral Pb–Pb collisions [62, 81].

The experimental results listed in the previous section motivate further studies:

- The analysis of the proton–proton and peripheral A–A collision carried out in a similar way as in p–A and d–A collisions. Such a program could be used to find the possible onset of collectivity in small systems. The study of the onset of jet quenching in very peripheral A–A and p–A collisions would give additional information on the nature of the matter formed in small systems.

- The energy scan of p–A and d–A collisions to find the onset of collectivity as a function of the energy density.

- Experimental studies of collisions of small deformed projectiles on big nuclei. This subject is discussed in the next subsection.

3.2. Why small on big collisions?

We now briefly discuss motivations for performing experiments with a very asymmetric projectile and target, other than to study various aspects of the initial state dynamics [49]. In view of the results indicating that collective flow appears in such collisions, it is important to study such systems in more detail. The extraction of flow coefficients is difficult because of significant non-flow contributions. It should be kept in mind that alternative scenarios based on the color glass condensate approach are used to interpret the observations. Therefore, further experiments are needed to validate or disprove the collectivity in small systems.

A mechanism to control the eccentricity has been discussed for d–A collisions [50]. The collision of a deformed projectile with a large nucleus can be viewed as a small deformed (in this case a dumbbell shaped) nucleus hitting a wall. The orientation of the projectile determines the ellipticity of the fireball. By triggering on high multiplicity events we choose collisions where the projectile makes the largest damage when colliding, *i.e.*, events with a large number of participants. The configurations relevant in this case are those where the deuteron hits the larger nucleus side-wise. Thus we expect the largest ellipticity for the most central collisions (Fig. 6). The PHENIX Collaboration indeed observes a large v_2 in d–Au collisions [79], consistent with such estimates. A larger nucleus with a quadrupole deformation, such as ^{9}Be, could be used instead of the deuteron.

When using a small deformed projectile of triangular shape [82], such as triton or ^{3}He, a significant triangular flow should appear. Hydrodynamic simulations show that v_3 in ^{3}He–Au is larger than in d–Au or p–Au collisions [68].

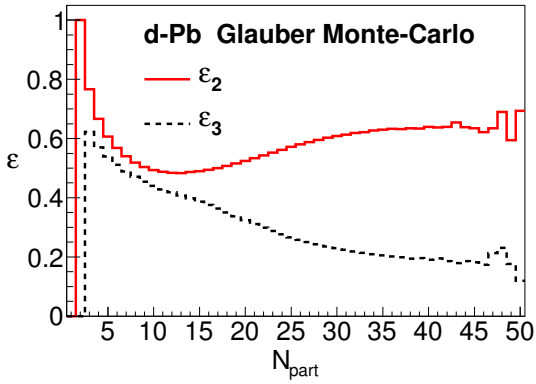


Fig. 6. The ellipticity and triangularity in d–Pb collisions as function of the number of participants (from [50]).

A very promising and interesting systems to study the triangularity of the projectile is to use ^{12}C [83]. Nucleons in the ground state of the ^{12}C nucleus are strongly clustered into three α particles (see, *e.g.*, [84] for an early review, [85] for some history, or [86–89] for references). For ^{12}C–^{208}Pb events with a large number of participants, the triangularity can be as high as 0.3 (Fig. 7). Moreover, due to the intrinsic triangular shape of the carbon, the triangularity of the fireball increases with the number of participants (in analogy to the ellipticity in the d–A case shown in Fig. 6), providing a vivid qualitative signal of the clusterization. We note that the size of the interaction region and the multiplicity are much larger than in ^{3}He–Au collisions, ranging up to 85 wounded nucleons for the highest RHIC energy. Thus the collective scenario is anticipated for the ^{12}C–Au collisions. Finally, we stress that from the quantum-mechanical point view, the flow analysis would present a unique way to get snapshots of the intrinsic wave function of the carbon nucleus at the instant of the collision.

Performing a series of experiments using small projectiles hitting a large nucleus would clarify the role of the final state interactions. If the collective expansion is valid, one expects a moderate v_2 with a smaller v_3 in p–A collisions, a large v_2 and negligible v_3 in d–A or ^{9}Be–A collisions, and a large v_3 and v_2 in ^{3}He–A or ^{12}C–A collisions, with specific correlations with multiplicity [83].

3.3. Limits of collectivity

Experimental indications of collective expansion in small systems rise the question about the limits of applicability of the hydrodynamic model. At the very early stage of the collision the system evolves far from equilibrium.

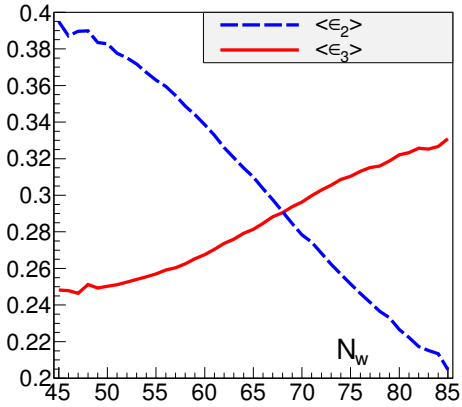


Fig. 7. The Glauber-model prediction for the event-averaged ellipticity and triangularity of the initial fireball created in the ^{12}C–^{208}Pb collisions at the highest RHIC energies of $\sqrt{s_{NN}} = 200$ GeV. We use the mixed Glauber model with $\sigma_{NN} = 42$ mb and the fraction of the binary collisions $a = 0.145$.

A strong pressure asymmetry is expected between the longitudinal and transverse direction. Since the total evolution time is shorter for p–A collisions, one hopes to be able to investigate this early transient stage. Quantitative predictions require going beyond the framework of the near-equilibrium relativistic hydrodynamics [90, 91].

If the gradients of the transverse velocity get big in a small system, the hydrodynamic approach would cease to be valid even for the transverse expansion. This means that the second order viscous hydrodynamics breaks down. Phenomenologically, as long as the scale at which hadrons are formed is smaller than the size of the system, local correlations between flow and position (collective flow) can appear. It remains a challenge to provide a theoretical framework able to give quantitative predictions in that case.

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REFERENCES

[1] W. Florkowski, *Phenomenology of Ultra-Relativistic Heavy-Ion Collisions*, World Scientific Publishing Company, Singapore 2010.

[2] U.W. Heinz, R. Snellings, *Annu. Rev. Nucl. Part. Sci.* **63**, 123 (2013).

[3] C. Gale, S. Jeon, B. Schenke, *Int. J. Mod. Phys.* **A28**, 1340011 (2013).

[4] M. Luzum, H. Petersen, *J. Phys. G: Nucl. Part. Phys.* **41**, 063102 (2014) [arXiv:1312.5503 [nucl-th]].
[5] B.B. Back *et al.* [PHOBOS Collaboration], *Nucl. Phys.* **A757**, 28 (2005).
[6] J. Adams *et al.* [STAR Collaboration], *Nucl. Phys.* **A757**, 102 (2005).
[7] K. Adcox *et al.* [PHENIX Collaboration], *Nucl. Phys.* **A757**, 184 (2005).
[8] K. Aamodt, *et al.* [ALICE Collaboration], *Phys. Rev. Lett.* **106**, 032301 (2011).
[9] G. Aad *et al.* [ATLAS Collaboration], *Phys. Rev. Lett.* **105**, 252303 (2010).
[10] S. Chatrchyan *et al.* [CMS Collaboration], *J.High Energy Phys.* **1108**, 141 (2011).
[11] W. Israel, J. Stewart, *Ann. Phys.* **118**, 341 (1979).
[12] H. Song, U.W. Heinz, *Phys. Rev.* **C77**, 064901 (2008).
[13] A.K. Chaudhuri, *Phys. Rev.* **C74**, 044904 (2006).
[14] K. Dusling, D. Teaney, *Phys. Rev.* **C77**, 034905 (2008).
[15] P. Romatschke, *Int. J. Mod. Phys.* **E19**, 1 (2010).
[16] M. Luzum, P. Romatschke, *Phys. Rev.* **C78**, 034915 (2008) [*Erratum ibid.* **79**, 039903 (2009)].
[17] P. Bożek, *Phys. Rev.* **C81**, 034909 (2010).
[18] B. Schenke, S. Jeon, C. Gale, *Phys. Rev. Lett.* **106**, 042301 (2011).
[19] H. Niemi *et al.*, *Phys. Rev. Lett.* **106**, 212302 (2011).
[20] Y. Akamatsu, S.-I. Inutsuka, C. Nonaka, M. Takamoto, *J. Comput. Phys.* **256**, 34 (2014).
[21] L. Del Zanna *et al.*, *Eur. Phys. J.* **C73**, 2524 (2013).
[22] I. Karpenko, P. Huovinen, M. Bleicher, arXiv:1312.4160 [nucl-th].
[23] P.K. Kovtun, D.T. Son, A.O. Starinets, *Phys. Rev. Lett.* **94**, 111601 (2005).
[24] P. Romatschke, U. Romatschke, *Phys. Rev. Lett.* **99**, 172301 (2007).
[25] H. Song, U.W. Heinz, *J. Phys. G* **36**, 064033 (2009).
[26] C. Gale *et al.*, *Phys. Rev. Lett.* **110**, 012302 (2013).
[27] M. Luzum, J.-Y. Ollitrault, *Nucl. Phys.* **A904–905**, 377c (2013).
[28] B. Alver *et al.* [PHOBOS Collaboration], *J. Phys. G* **34**, S907 (2007).
[29] H. Petersen, G.-Y. Qin, S.A. Bass, B. Müller, *Phys. Rev.* **C82**, 041901 (2010).
[30] F.G. Gardim, F. Grassi, M. Luzum, J.-Y. Ollitrault, *Phys. Rev.* **C85**, 024908 (2012).
[31] P. Bożek, W. Broniowski, *Phys. Rev.* **C85**, 044910 (2012).
[32] Z. Qiu, C. Shen, U. Heinz, *Phys. Lett.* **B707**, 151 (2012).
[33] H. Niemi, G. Denicol, H. Holopainen, P. Huovinen, *Phys. Rev.* **C87**, 054901 (2013).
[34] L. Pang, Q. Wang, X.-N. Wang, *Phys. Rev.* **C86**, 024911 (2012).

[35] B. Alver, G. Roland, *Phys. Rev.* **C81**, 054905 (2010).
[36] M. Rybczyński, G. Stefanek, W. Broniowski, P. Bożek, *Comput. Phys. Commun.* **185**, 1759 (2014) [arXiv:1310.5475 [nucl-th]].
[37] P. Bożek, *Phys. Rev.* **C85**, 034901 (2012).
[38] A. Kisiel, T. Tałuć, W. Broniowski, W. Florkowski, *Comput. Phys. Commun.* **174**, 669 (2006).
[39] M. Chojnacki, A. Kisiel, W. Florkowski, W. Broniowski, *Comput. Phys. Commun.* **183**, 746 (2012).
[40] P. Bożek, W. Broniowski, *Phys. Rev. Lett.* **109**, 062301 (2012).
[41] N. Borghini, P.M. Dinh, J.-Y. Ollitrault, *Phys. Rev.* **C62**, 034902 (2000).
[42] U.A. Wiedemann, U.W. Heinz, *Phys. Rep.* **319**, 145 (1999).
[43] M.A. Lisa, S. Pratt, R. Soltz, U. Wiedemann, *Annu. Rev. Nucl. Part. Sci.* **55**, 357 (2005).
[44] S. Akkelin, Y. Sinyukov, *Phys. Lett.* **B356**, 525 (1995).
[45] W. Broniowski, M. Chojnacki, W. Florkowski, A. Kisiel, *Phys. Rev. Lett.* **101**, 022301 (2008).
[46] S. Pratt, *Phys. Rev. Lett.* **102**, 232301 (2009).
[47] P. Bożek, *Phys. Rev.* **C83**, 044910 (2011).
[48] I. Karpenko, Y. Sinyukov, K. Werner, *Phys. Rev.* **C87**, 024914 (2013).
[49] C.A. Salgado, *J. Phys. G* **38**, 124036 (2011).
[50] P. Bożek, *Phys. Rev.* **C85**, 014911 (2012).
[51] S. Chatrchyan *et al.* [CMS Collaboration], *Phys. Lett.* **B718**, 795 (2013).
[52] B. Abelev *et al.*, [ALICE Collaboration], *Phys. Lett.* **B719**, 29 (2013).
[53] G. Aad *et al.* [ATLAS Collaboration], *Phys. Rev. Lett.* **110**, 182302 (2013).
[54] P. Bożek, W. Broniowski, *Phys. Lett.* **B718**, 1557 (2013).
[55] V. Khachatryan *et al.* [CMS Collaboration], *J. High Energy Phys.* **09**, 091 (2010).
[56] P. Bożek, *Eur. Phys. J.* **C71**, 1530 (2011).
[57] K. Dusling, R. Venugopalan, *Phys. Rev. Lett.* **108**, 262001 (2012).
[58] K. Dusling, R. Venugopalan, *Phys. Rev.* **D87**, 051502 (2013).
[59] K. Dusling, R. Venugopalan, *Phys. Rev.* **D87**, 094034 (2013).
[60] K. Dusling, R. Venugopalan, *Phys. Rev.* **D87**, 054014 (2013).
[61] G. Aad *et al.* [ATLAS Collaboration], *Phys. Lett.* **B725**, 60 (2013).
[62] S. Chatrchyan *et al.* [CMS Collaboration], *Phys. Lett.* **B724**, 213 (2013).
[63] B.B. Abelev *et al.* [ALICE Collaboration], *Phys. Lett.* **B726**, 164 (2013).
[64] P. Bożek, W. Broniowski, *Phys. Rev.* **C88**, 014903 (2013).
[65] G.-Y. Qin, B. Müller, *Phys. Rev.* **C89**, 044902 (2014).
[66] A. Bzdak, B. Schenke, P. Tribedy, R. Venugopalan, *Phys. Rev.* **C87**, 064906 (2013).

[67] K. Werner *et al.*, arXiv:1307.4379 [nucl-th].

[68] J. Nagle *et al.*, arXiv:1312.4565 [nucl-th].

[69] A. Bzdak, P. Bożek, L. McLerran, *Nucl. Phys.* **A927**, 15 (2014) [arXiv:1311.7325 [hep-ph]].

[70] L. Yan, J.-Y. Ollitrault, *Phys. Rev. Lett.* **112**, 082301 (2014).

[71] A. Bzdak, V. Skokov, arXiv:1312.7349 [hep-ph].

[72] P. Bożek, W. Broniowski, G. Torrieri, *Phys. Rev. Lett.* **111**, 172303 (2013).

[73] B.B. Abelev *et al.* [ALICE Collaboration], *Phys. Lett.* **B728**, 25 (2014) [arXiv:1307.6796 [nucl-ex]].

[74] P.F. Kolb, J. Sollfrank, U.W. Heinz, *Phys. Rev.* **C62**, 054909 (2000).

[75] A. Ortiz *et al.*, *Phys. Rev. Lett.* **111**, 042001 (2013).

[76] A. Bzdak, V. Skokov, *Phys. Lett.* **B726**, 408 (2013).

[77] P. Bożek, A. Bzdak, V. Skokov, *Phys.Lett.* **B728**, 662 (2014).

[78] P. Bożek, W. Broniowski, *Phys. Lett.* **B720**, 250 (2013).

[79] A. Adare *et al.* [PHENIX Collaboration], *Phys. Rev. Lett.* **111**, 212301 (2013).

[80] S. Chatrchyan *et al.* [CMS Collaboration], arXiv:1307.3442 [hep-ex].

[81] G. Basar, D. Teaney, arXiv:1312.6770 [nucl-th].

[82] A.M. Sickles [PHENIX Collaboration], arXiv:1310.4388 [nucl-ex].

[83] W. Broniowski, E.R. Arriola, *Phys. Rev. Lett.* **112**, 112501 (2014).

[84] J. Blatt, V. Weisskopf, *Theoretical Nuclear Physics*, John Wiley and Sons, New York 1952.

[85] D. Brink, *History of Cluster Structure in Nuclei*, in: *J. Phys. Conf. Ser.*, Vol. **111**, IOP Publishing, 2008, p. 012001.

[86] D. Brink, Proc. Int. School Enrico Fermi, Course, **36**.

[87] M. Freer, *Rep. Prog. Phys.* **70**, 122149 (2007).

[88] K. Ikeda, T. Myo, K. Kato, H. Toki, *Clusters in Nuclei*, *Lect. Notes Phys.* **818**, Vol. 1, Springer, 2010.

[89] C. Beck, *Clusters in Nuclei*, *Lect. Notes Phys.* **848**, Vol. 2, Springer, 2012.

[90] W. Florkowski, M. Martinez, R. Ryblewski, M. Strickland, *PoS* **ConfinementX**, 221 (2012) [arXiv:1301.7539 [nucl-th]].

[91] M.P. Heller, R.A. Janik, P. Witaszczyk, *Phys. Rev. Lett.* **108**, 201602 (2012).