CURRENT STATUS OF LHC PHYSICS: PRECISION THEORY VIEW*

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We discuss the current status of LHC physics from the perspective of precision theory predictions for the attendant QCD and EW higher order corrections. We focus on the interplay between the available data and the predictions for these data viewed in the context of the establishment of baselines for what is needed to exploit fully the discovery potential of the existing LHC data and the data expected by the end of the second LHC run (*i.e.*, 300 fb⁻¹). We conclude that significant improvement in the currently used theoretical predictions will be mandatory. Possible strategies to achieve such improvement are indicated.

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1. Introduction

As the LHC has operated successfully for a considerable period and has discovered [1] the long sought BEH [2], paving the way for the 2013 Nobel Prize in Physics for Profs. F. Englert and P. Higgs, and as the LHC is now in its Long Shutdown 1 as it prepares to move to higher energies, it is appropriate to assess the status of the physics purview of the LHC as it currently stands. This is our objective in what follows here.

With such an objective, it is appropriate as well to assume a particular vantage point, as one can well imagine a number of such. We will take the perspective of the theory of precision LHC physics, by which we mean predictions for LHC processes at the total precision tag of 1% or better. It is appropriate for any discussion from the perspective of precision theory for the status of physics purview of the LHC to set the attendant framework by recalling, at least in generic terms, why we *still* need the LHC in the first

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place. In the following discussion we shall begin with such recapitulation. In this way, the entirety of the effort required to realize and to extend the current purview of the physics for the LHC in a practical way can be more properly assessed.

Thus, in view of the discovery of the BEH boson we ask, "Why do we still need the LHC?" Many answers can be found in the original justifications for the colliding beam device and its detectors in Refs. [3–7]. We will call attention to a particular snapshot of the latter discussions with some eve toward the requirements of precision theory for LHC physics in view of the discovery of the BEH boson. More precisely, the LHC physics program still remains as a crucial step toward resolving fundamental outstanding issues in elementary particle physics: the big and little hierarchy problems, the number of families, the origin of Lagrangian fermion (and gauge boson) masses, baryon stability, the union of quantum mechanics and general theory of relativity, the origin of CP violation, the origin of the cosmological constant A. dark matter, ... The discovery of the BEH boson serves to help us refocus much of the considerable theory effort that has been invested in the New Physics (NP) that would still seem to be needed to solve all of these outstanding issues, that is to say, in the physics beyond the Standard Model 't Hooft-Veltman renormalized Glashow-Salam-Weinberg EW & Gross–Wilczek–Politzer QCD theory that seems to describe the quantum loop corrections in the measurements of electroweak and strong interactions at the shortest distances so far achieved in laboratory-based experiments.

We may still mention that superstring theory [8, 9] solves everything in principle but has trouble in practice: for example, it has more than 10^{500} candidate solutions for the vacuum state [10]. The ideas in superstring theory have helped to motivate many so-called string inspired models of NP such as [11] string-inspired GUTs, large extra dimensions, Kaluza–Klein excitations, ... We would list supersymmetric extensions of the SM, such as the MSSM, the CMSSM and the more recent pMSSM [11], as separate proposals from superstring motivated ideas, as historically this was the case. Modern approaches to the dynamical EW symmetry breaking (technicolor) such as little Brout–Englert–Higgs models [11] obtain as well. The list is quite long and LHC, especially now that it has found the BEH boson, will clearly help us shorten it.

One of the most provocative ideas continues to be the one which some superstring theorists [10] invoke to solve the problem of the large number of candidate superstring vacua: the anthropic principle, by which the solution is the one that allows us to be in the state in which we find ourselves. In the view of some [12], this would be the end of reductionist physics as we now know it. With the discovery of the BEH boson, has the chance that the LHC can even settle this discussion decreased? Perhaps not.

Recently, even newer paradigms are emerging which would foretell the need for new accelerated beam devices. In Ref. [13], it is shown that GUT's exist in which there are three or more heavy families such that SM guarks are paired with new heavy leptons and SM leptons are paired with new heavy guarks with a GUT scale $M_{\rm GUT} \sim 100$ TeV, so that it would be accessible to the VLHC device as discussed in Ref. [14] as well as at the FCC-ee device under discussion now at CERN [15]. The proton is stable because all the leptons to which it could decay are too heavy for the decay to occur. Indeed, many of the new states in such scenarios may already be accessible at the LHC. New ideas are also needed as a remedy for the superficially bad UV behavior of quantum gravity if one does not use string theory for the unification of the EW and QCD theories with quantum gravity and these new ideas also foretell the need for new accelerated beam devices as well as the possible existence of new signatures at the LHC. More specifically, more progress has been made recently on solving the problem of the UV behavior of quantum gravity in the context of local Lagrangian field theory methods [16–18]. Indeed, following the suggestion by Weinberg [19] that quantum gravity might have a non-trivial UV fixed point, with a finite dimensional critical surface in the UV limit, so that it would be asymptotically safe with an S-matrix that depends on only a finite number of observable parameters, in Refs. [16] strong evidence has been calculated using Wilsonian [20] field-space exact renormalization group methods to support Weinberg's asymptotic safety hypothesis for the Einstein-Hilbert theory^{1,2}. In a parallel but independent development [17], we have shown [23] that the extension of the amplitude-based, exact resummation theory of Ref. [24] to the Einstein-Hilbert theory leads to analogous UV-fixed-point behavior for the dimensionless gravitational and cosmological constants with the bonus that the resummed theory is actually UV finite when expanded in the resummed propagators and vertices to any finite order in the respective improved loop expansion. We designate the resummed theory as resummed quantum gravity. We emphasize that there is no known inconsistency between our analysis and those of Refs. [16, 21] or the leg renormalizability arguments in Ref. [18]. In Ref. [25] we use resummed quantum gravity in the context of the Planck scale cosmology of Refs. [26, 27]³, which is based on the approach of Refs. [16], to predict the value of the cosmological constant such that $\rho_A \cong (2.40 \times 10^{-3} \text{ eV})^4$. This latter result is close enough to the

¹ In addition, more evidence for Weinberg's asymptotic safety behavior has been calculated using causal dynamical triangulated lattice methods in Ref. [21].

 $^{^2}$ We also note that the model in Ref. [22] realizes many aspects of the effective field theory implied by the anomalous dimension of 2 at the UV-fixed point but it does so at the expense of violating Lorentz invariance.

 $^{^3}$ The attendant choice of the scale $k\sim 1/t$ used in Refs. [26, 27] was also proposed in Ref. [28].

observed value [29, 30], $((2.37 \pm 0.05) \times 10^{-3} \text{ eV})^4$, that it would require some [31] SUSY GUT scenarios to have flipped copies of their EW scale multiplets such that these copies would also be accessible at VLHC and the newly discussed FCC devices. We may also note that new approaches to dark matter such as Higgsogenesis [32] suggest new particles in the 100 TeV regime could obtain, again within reach of the VLHC and FCC devices. Finally, to end our illustration of the many new paradigms in the literature⁴, we note that in Ref. [33] the UV limit of theories such as quantum gravity is solved by the dynamical generation of non-perturbative large distance excitations called classicalons, which provide the necessary damping of the naively divergent UV behavior. When discussed in general terms, possible new signatures for the LHC, VLHC and FCC obtain [33].

Evidently, the new paradigms, admittedly only illustrated here in a limited way to set the stage of our discussion, must be taken seriously in analyzing the new LHC data. We continue to stress that we must be able to distinguish higher order SM processes from New Physics and that we must be able to probe New Physics precisely to distinguish among different New Physics scenarios. These requirements necessitate the era of precision theory for the LHC and justify it as a valid perspective from which to view the current status of LHC physics.

To give an overview of our discussion, we call attention to the summary of the observations of the BEH boson by ATLAS and CMS as reported in Ref. [34] on page 60 — see https://indico.cern.ch/event/242095/ session/2/contribution/5/material/slides/1.pdf. Given the greater than 9σ significance level in each collaboration's observation reported therein, we proceed in what follows by evaluating what will be needed from the standpoint of precision theory to make the corresponding improvement in our knowledge of the BEH boson and its properties and of the limits on its possible extensions/generalizations (NP physics) given the expected improvements in the integrated luminosity and energies during the future planned running of the LHC. We start in the next section with the BSM Higgs scenarios but we discuss as well the precision BEH boson studies from this latter perspective. We then address in Sec. 3 what issues obtain in developing the theory improvements so implied. We sum up in our Conclusions.

2. BSM and precision BEH scenarios

For a more detailed discussion of BSM scenarios relative to the current status of LHC physics, see Ref. [35]. Here, we use a few representative examples to illustrate the attendant view from the perspective of precision theory.

⁴ We apologize right now for all the new ideas we have not mentioned for reasons of space.

In Ref. [36], the constraints from LHC data on new neutral Higgs particles in the MSSM in their decays are used to set the limits on the respective MSSM parameter space as illustrated on page 10 (tau pairs) and on page 13 (b-quark pairs) — see https://indico.cern.ch/event/242095/session/ 2/contribution/6/material/slides/0.pdf. In Ref. [37] the general situation on the constraints of LHC data on BSM scenarios is reviewed and we call attention to its page 28 for a summary of the respective results as presented therein — see https://indico.cern.ch/event/242095/session/3/contribution/12/material/slides/0.pdf. When we look at the results just discussed, we ask, "How are these results obtained?" The important answer is that, in all cases, they are obtained from comparison of theoretical predictions with experimental data. This allows a future for LHC physics that entails precision studies of the properties of the BEH boson and precision studies of the possibilities for BSM physics, both of which open possibilities for fundamental discoveries at the LHC. We now develop this view of the future of LHC physics based on what has been achieved at this writing.

Starting first with the implications for precision BEH studies, we note respectively the results given on page 21 of Ref. [34] and on page 37 of Ref. [38] — see https://indico.cern.ch/event/242095/session/2/contribution/5/material/slides/1.pdf and https://indico.cern.ch/ event/242095/session/2/contribution/4/material/slides/0.pdf, respectively — on the current constraints from LHC data on $\sigma/\sigma_{\rm SM} \equiv \mu$ for the $H \to ZZ \to 4\ell H$ production as measured by the CMS and a similar set of constraints from ATLAS which entails the same channel as well as two others, the $H \to \gamma \gamma$ and $H \to WW \to \ell \nu \ell \nu$ channels. Here, $\sigma(\sigma_{\rm SM})$ is the cross section for the respective channels as observed (as predicted in the SM) respectively. What we see in these results, which are illustrative of the generic situation in many current BEH studies, is that we have a 10-12%value for the theoretical uncertainty $\Delta \mu_{\rm th}$ on μ versus the respective experimental uncertainty $\Delta \mu_{\text{expt}} \sim 20\%$. This is sufficient currently, but, as we see from Ref. [39], the expectation is that LHC will have delivered 300 fb^{-1} by 2021 and to achieve the analogous results we would need by that time $\Delta \mu_{\rm th} \lesssim 3.2\%$. This requires the precision SM theory with a provable precision tag, in analogy with what was done for LEP — see Refs. [40].

A similar situation obtains for BSM studies. We observe the results on page 7 and, again, on page 10 of Ref. [36] — see https://indico.cern.ch/ event/242095/session/2/contribution/6/material/slides/0.pdf — for the studies of a neutral Higgs decaying to $\tau \bar{\tau}$ from CMS and ATLAS as reviewed in Ref. [36]. These results feature the experimental precision of $\Delta \sigma_{\text{expt}} \cong 3.4\%$ to be compared with the theoretical precision $\Delta \sigma_{\text{th}} \cong 2.8\%$. With 300 fb⁻¹ of data, to get the analogous return on the data analysis we need $\Delta \sigma_{\text{th}} \lesssim 1\%$. This requires precision theory for both QCD and EW corrections for LHC physics. Continuing in this way, we note on page 20 of Ref. [37] — see https:// indico.cern.ch/event/242095/session/3/contribution/12/material/ slides/0.pdf — the data from the ATLAS search for new quark partners $T^{2/3}$, $B^{1/3}$ as reviewed therein. The current precisions, which yield the mass limits for 0.7 TeV for generic branching ratios [37], are $\Delta \sigma_{\text{expt}} \cong 10\%$ and $\Delta \sigma_{\text{th}} \cong 10\%$ and, for 300 fb⁻¹ one would need $\Delta \sigma_{\text{th}} \lesssim 2\%$. Again, this requires precision theory for the QCD and EW corrections to LHC physics.

The generic searches for SUSY have a similar message, as we see on page 22 of Ref. [41] — see https://indico.cern.ch/event/242095/session/3/ contribution/11/material/slides/0.pdf — which features a review of results for LHC SUSY searches using the MET spectrum for events with ≥ 1 jet. Currently, in the featured results, for $E_{\rm T}^{\rm miss} \gtrsim 75$ GeV we have the experimental error $\Delta \sigma_{\rm expt} \sim 25\%$ and the theoretical error $\Delta \sigma_{\rm th} \sim 40\%$, which imply the limits, using GGM (bino-like NLSP), $m(\tilde{q}) > 1.2$ TeV, $m(\tilde{g}) > 1.1$ TeV for the respective squark and gluino mass limits, for example. With 300 fb⁻¹ of data, to get a similar return on the data, we need $\Delta \sigma_{\rm th} \lesssim 10\%$, which, again, motivates precision theory for the QCD and EW corrections.

Indeed, the situation regarding the interplay between the current theoretical precision in LHC physics applications and the reach of the experimental probes of New Physics is illustrated and reviewed on pages 15, 17 and 18 of Ref. [42] — see https://indico.cern.ch/event/242095/session/3/contribution/10/material/slides/0.pdf. On page 15 of Ref. [42], we see that generic New Physics scenarios have the signature high $P_{\rm T}$ jets, high $P_{\rm T}$ leptons and $E_{\rm T}^{\rm miss}$, where in the SUSY case, if R-parity is violated there are additional jets and leptons instead of $E_{\rm T}^{\rm miss}$. This means that production of W, Z and top with additional jets provides significant background, so that the SM predictions for such production impact the effectiveness of the attendant experimental searches. Indeed, on page 17 of Ref. [42], we see the status of the comparison between some of the available theoretical predictions and the ATLAS data on the production of $Z/\gamma^*(\to \ell^+\ell^-) + n$ jets, $n \ge 0$ as reviewed therein. For a more inclusive observable such as the cross section for a given number of jets, the calculations featured do reasonably well relative to the respective values of $\Delta \sigma_{\text{expt}}$. For the more exclusive normalized differential distribution in $H_{\rm T}$, none of the calculations featured, Alpgen [43], Sherpa [44], BlackHat+Sherpa [45] are in good agreement with the data. Indeed, even though the BlackHat+Sherpa should have exact NLO results that Sherpa lacks, its agreement with the $H_{\rm T}$ spectrum is worse in the lower and higher regions of the spectrum! One has to stress that, if the experimentalists only use the NLO results shown on page 17 of Ref. [42], the discrepancy between the tree level and NLO results means that the error on the SM prediction therein approaches a factor of 2 at high values of $H_{\rm T}$ and any limits using this spectrum would need to reflect this uncertainty. The two sets of results on pages 17 and 18 of Ref. [42], taken together, both then show that one needs precise background predictions to realize exclusion to the respective kinematical limit: in the latter set, the current precision of the background predictions allows exclusion up to the region where $m_{\tilde{t}} = m_{\rm LSP} + m_t + 30$ GeV, where the case $m_{\tilde{t}} = 350$ GeV, $m_{\rm LSP} = 150$ GeV is illustrated.

Two main observations from BSM studies here are as follows. First, conclusive results go hand-in-hand with more precise theory. Second, the transverse degrees of freedom are essential to understand with precise predictions realized on an event-by-event basis. These observations support the need for exact resummation methods in the MC such as we have advocated in Refs. [46]. Indeed, the need for such an approach is even more acute if we consider the requirements for discovery of new heavy states: we see from the analyses above that such discovery requires strict control of the transverse degrees of freedom, which implies exact resummation methods in the MC, for both QCD and EW higher order effects. Where do we stand on this?

3. Where do we stand on the required precision theory?

We use the production of $\{Z/\gamma^*, W^{\pm}\}$ + jets, as reviewed by Ref. [41] with ATLAS differential measurements presented on its page 11 — see https://indico.cern.ch/event/242095/session/3/contribution/11/ material/slides/0.pdf, to illustrate a view of the comparison of theory and experiment for SM expectations relevant to background precision theory considerations. We do this because the large data sets for the single heavy gauge boson productions and decays to lepton pairs, which are data sets exceeding 10^7 events in ATLAS and CMS, are not available to us at this writing. Focusing on the transverse momentum spectra for the leading jet in the referenced data set [41], we see that, while the predictions shown for Blackhat/Sherpa [45], Sherpa [44] and Alpgen [43] are listed as satisfactory, there is some room for improvement: indeed, if we compare Blackhat/Sherpa NLO improvement of Sherpa with Sherpa, we see that between $p_{\rm T}^{\rm jet}$ of 200 GeV and 530 GeV the agreement with the data is degraded by the NLO correction? In the same region, Alpgen is generally above the data, Sherpa and Blackhat/Sherpa are generally below the data, where all three sets of results are never too far from the data when the errors are taken into account. We expect that the MC@NLO [47] results should agree with the low $p_{\rm T}^{\rm jet}$ regime (left-panel) and with the $N_{\text{jet}} \ge 0$ data point (right panel) and we see that this expectation holds. We note that all three multi-jet MC's are low at this latter right-panel point, with the NLO result missing it by the largest margin, $\sim 6\%$. These results show that significant improvement is needed if the analogous physics return from the analysis of the data would be desired at 300 fb⁻¹ of LHC data.

We note that, for a large fraction of the data presented on page 11 of Ref. [41], both $p_{\rm T}^{\rm jet} < 50$ GeV and $N_{\rm jet} = 0, 1$ hold. We conclude that, for normalized distributions, we must understand with good precision the respective $\Delta \sigma_{\rm th}$ in the regime $p_{\rm T}^{\rm jet} < 50$ GeV for $N_{\rm jet} \ge 0$ — a small error in the latter regime can result in a factor of a much larger error at large values of $p_{\rm T}^{\rm jet}$ and thereby change the reach of an analysis. Testing the theory predictions in this regime would be greatly aided by the release of the large (> 10⁷) samples of Z/γ^* decays to lepton pairs at ATLAS and CMS. We also stress that resummation [48] of the large higher order effects is essential here. Indeed, for the parton shower/ME matched exact NLO realizations in MC@NLO and Powheg [49], we have the physical theoretical precision estimate $\Delta \sigma_{\rm phys} \cong 10\%$. For the CSS [50] resummation as realized in RESBOS [51], we have [48] the estimate $\Delta \sigma_{\rm phys} \cong \mathcal{O}(Q_{\rm T}/Q) \cong 5\%$. For the SCT/SCET [52, 53] resummation approach, we have [48] the estimate $\Delta \sigma_{\rm phys} \cong \lambda = \sqrt{\Lambda/Q} \cong 5\%$. Here, we use the typical $\Lambda \cong 0.3$ GeV, $Q = M_Z$, and $Q_T \cong 5$ GeV. These estimates should be contrasted with what one expects from exact amplitude-based $QED \otimes QCD$ resummation, where [48, 54] $\Delta \sigma_{\rm phys} \lesssim 1\%$ is possible.

More precisely, standard resummations, such as that in Ref. [50] as realized in RESBOS, *etc.*, drop terms that are not resummed as we may illustrate as follows (see Ref. [48] for more details): writing the Drell–Yan [55] formula as given in Ref. [50] (we follow the notation of this latter reference here)

$$\frac{d\sigma}{dQ^{2}dydQ_{T}^{2}} \sim \frac{4\pi^{2}\alpha^{2}}{9Q^{2}s} \left\{ \int \frac{d^{2}b}{(2\pi)^{2}} e^{i\vec{Q}_{T}\cdot\vec{b}} \times \sum_{j} e_{j}^{2}\widetilde{W}_{j}(b^{*};Q,x_{A},x_{B})e^{\left\{-\ln\left(Q^{2}/Q_{0}^{2}\right)g_{1}(b)-g_{j/A}(x_{A},b)-g_{j/B}(x_{B},b)\right\}} + Y(Q_{T};Q,x_{A},x_{B}) \right\},$$
(1)

the important point is that exponentiated part of the right-hand side involves dropping terms $\mathcal{O}(Q_{\rm T}/Q)$ to all orders in $\alpha_{\rm s}$ so that, even if one knows the fixed-order based result for Y to $\mathcal{O}(\alpha_{\rm s}^N)$ to get exactness to the latter order, one is missing terms $\mathcal{O}(Q_{\rm T}/Q)$ in all orders in $\alpha_{\rm s}$ beyond $\mathcal{O}(\alpha_{\rm s}^N)^5$. Corresponding arguments hold for the other approaches to resummation in Refs. [52, 53].

What we want to call more attention to here is the following. In the usual starting point for the fully differential representation of a hard LHC scattering process,

⁵ Errors on the non-perturbative functions g_{ℓ} also yield $\Delta \sigma_{\rm phys}$.

$$d\sigma = \sum_{i,j} \int dx_1 dx_2 F_i(x_1) F_j(x_2) d\hat{\sigma}_{\rm res}(x_1 x_2 s) , \qquad (2)$$

where the $\{F_j\}$ and $d\hat{\sigma}_{res}$ are the respective parton densities and reduced hard differential cross section and where we use the subscript on the latter to indicate that it has been resummed for all large EW and QCD higher order corrections in a manner consistent with achieving a total precision tag of 1% or better for the total theoretical precision of (2), resummation may be carried via resummed collinear evolution on the $\{F_j\}$ and via soft resummation (non-collinear) on $d\hat{\sigma}_{res}$.

Considering first representations of resummation carried via $d\hat{\sigma}_{res}$, we note the QCD \otimes QED \equiv QED \otimes QCD exact amplitude-based resummation theory in Refs. [54], which gives

$$d\bar{\sigma}_{\rm res} = \sum_{n,m} d\bar{\sigma}_{nm}$$

$$\equiv e^{\rm SUM_{IR}(QCED)} \sum_{n,m=0}^{\infty} \frac{1}{n!m!} \int \prod_{j_1=1}^{n} \frac{d^3 k_{j_1}}{k_{j_1}}$$

$$\times \prod_{j_2=1}^{m} \frac{d^3 k'_{j_2}}{k'_{j_2}} \int \frac{d^4 y}{(2\pi)^4} e^{iy \cdot (p_1 + q_1 - p_2 - q_2 - \sum k_{j_1} - \sum k'_{j_2}) + D_{\rm QCED}}$$

$$\times \tilde{\beta}_{n,m} \left(k_1, \dots, k_n; k'_1, \dots, k'_m \right) \frac{d^3 p_2}{p_2^0} \frac{d^3 q_2}{q_2^0} , \qquad (3)$$

where $d\bar{\sigma}_{\rm res}$ is either the reduced cross section $d\hat{\sigma}_{\rm res}$ or the differential rate associated to a DGLAP-CS [56, 57] kernel involved in the evolution of the $\{F_j\}$ and where the *new* (YFS-style [24, 58]) *non-Abelian* residuals $\tilde{\beta}_{n,m}(k_1,\ldots,k_n;k'_1,\ldots,k'_m)$ have *n* hard gluons and *m* hard photons and we show the generic 2f final state with momenta p_2 , q_2 for definiteness. The infrared functions SUM_{IR}(QCED), $D_{\rm QCED}$ are given in Refs. [54, 59, 60]. The exactness of the result (3) means that its residuals $\tilde{\beta}_{n,m}$ allow a rigorous parton shower/ME matching via their shower-subtracted counterparts $\hat{\beta}_{n,m}$ [54].

Indeed, focusing on the DGLAP-CS theory itself and applying the formula in (3) to the calculation of the kernels, P_{AB} , we arrive at an improved IR limit of these kernels. In this IR-improved DGLAP-CS theory [59, 60], large IR effects are resummed for the kernels themselves. The resulting new resummed kernels, P_{AB}^{exp} [59, 60], yield a new resummed scheme for the PDF's and the reduced cross section:

$$F_j, \ \hat{\sigma} \ o \ F'_j, \ \hat{\sigma}'$$
 for
 $P_{gq}(z) \ o \ P_{gq}^{\exp}(z) = C_F F_{\rm YFS}(\gamma_q) e^{\frac{1}{2}\delta_q} \frac{1 + (1-z)^2}{z} z^{\gamma_q}, \quad etc.$

This new scheme gives the same value for σ in (2) with improved MC stability as discussed in Ref. [46]. Here, the YFS [24] infrared factor is given by $F_{\rm YFS}(a) = e^{-C_{\rm E}a}/\Gamma(1+a)$ where $C_{\rm E}$ is Euler's constant and we refer the reader to Ref. [59, 60] for the definition of the infrared exponents γ_q , δ_q as well as for the complete set of equations for the new $P_{AB}^{\rm exp}$. C_F is the quadratic Casimir invariant for the quark color representation.

For reference, to show how we make contact between the $\bar{\beta}_{n,m}$ and the differential distributions in MC@NLO we proceed as follows. We represent the MC@NLO differential cross section via [47]

$$d\sigma_{\rm MC@NLO} = \left[B + V + \int (R_{\rm MC} - C) d\Phi_R \right] d\Phi_B$$

$$\times \left[\Delta_{\rm MC}(0) + \int (R_{\rm MC}/B) \Delta_{\rm MC}(k_{\rm T}) d\Phi_R \right]$$

$$+ (R - R_{\rm MC}) \Delta_{\rm MC}(k_{\rm T}) d\Phi_B d\Phi_R, \qquad (4)$$

where B is Born distribution, V is the regularized virtual contribution, C is the corresponding counter-term required at exact NLO, R is the respective exact real emission distribution for exact NLO, $R_{\rm MC} = R_{\rm MC}(P_{AB})$ is the parton shower real emission distribution so that the Sudakov form factor is

$$\Delta_{\mathrm{MC}}(p_{\mathrm{T}}) = e^{\left[-\int d\Phi_{R} \frac{R_{\mathrm{MC}}(\Phi_{B}, \Phi_{R})}{B} \theta(k_{\mathrm{T}}(\Phi_{B}, \Phi_{R}) - p_{\mathrm{T}})\right]},$$

where as usual it describes the respective no-emission probability. The respective Born and real emission differential phase spaces are denoted by $d\Phi_A$, A = B, R, respectively. We may note further that the representation of the differential distribution for MC@NLO in (4) is an explicit realization of the compensation between real and virtual divergent soft effects discussed in Appendices of Refs. [59, 60] in establishing the validity of (3) for QCD — all of the terms on the RHS of (4) are infrared finite. Indeed, from comparison with (3) restricted to its QCD aspect we get the identifications, accurate to $\mathcal{O}(\alpha_s)$,

$$\frac{1}{2}\hat{\tilde{\beta}}_{0,0} = \bar{B} + \left(\bar{B}/\Delta_{\rm MC}(0)\right) \int \left(R_{\rm MC}/B\right) \Delta_{\rm MC}(k_{\rm T}) d\Phi_R,$$

$$\frac{1}{2}\hat{\tilde{\beta}}_{1,0} = R - R_{\rm MC} - B\tilde{S}_{\rm QCD}, \qquad (5)$$

where we defined [47]

$$\bar{B} = B \left(1 - 2\alpha_{\rm s} \Re B_{\rm QCD}\right) + V + \int (R_{\rm MC} - C) d\Phi_R,$$

and we understand here that the DGLAP-CS kernels in $R_{\rm MC}$ are to be taken as the IR-improved ones as we exhibit above [59, 60]. Here, we have written the QCD virtual and real infrared functions $B_{\rm QCD}$ and $\tilde{S}_{\rm QCD}$ respectively without the superscript *nls* for simplicity of notation and they are understood to be DGLAP-CS synthesized as explained in Refs. [54, 59, 60] so that we avoid doubling counting of effects. We also re-emphasize that we do not drop any effects here in (5). We observe further that, in view of (5), the way to the extension of frameworks such as MC@NLO to exact higher orders in $\{\alpha_{\rm s}, \alpha\}$ is, therefore, open via our $\hat{\beta}_{n,m}$ and will be taken up elsewhere [61].

Turning next to the representations of resummation carried by the $\{F_J\}$ we note the NLO exclusive improvement of the P_{AB} in the parton shower evolution as developed in Ref. [62]. As we see in Figs. 1 and 2, the proof of concept for the non-singlet NLO DGLAP evolution has been established with successful numerical tests of the ISR pure C_F^2 NLO MC. Similar results have been obtained for the FSR and results with more than two NLO insertions and with virtual corrections are also now available [62]. The way is, therefore, open for the complete NLO in the parton shower itself.

NLO-corrected middle-of-the-ladder kernel, C_F^2 Position of the NLO correction/insertion p can be anywhere in the ladder and we sum up over p:

$$\bar{D}_{B}^{[1]}(x,Q) = e^{-S_{ISR}} \sum_{n=0}^{\infty} \left\{ \begin{vmatrix} x & \cdots & x & z \\ \cdots & y & z & z \\ \cdots & z \\$$

Next step is to add more "NLO insertions" 2, 3 and so on ...

Fig. 1. Proof of concept for the non-singlet NLO DGLAP evolution from Ref. [62].



Fig. 2. Numerical tests with one and two NLO insertions in the NLO DGLAP evolution from Ref. [62].

Given that we have approaches that, in principle, can reach the sub-1% precision theoretical precision tag for LHC physics process, what is the current state of affairs in the comparison between the theoretical predictions in the refereed literature and the LHC data? We will illustrate the situation first with the results from ATLAS in Figs. 3 and 4 from Refs. [63]. In the former figure, some calculations available in the literature are compared to the combined ATLAS $e\bar{e}$ and $\mu\bar{\mu}$ data from 2010 for the differential $p_{\rm T}$ spectrum in single Z/γ^* production and decay to lepton pairs and in latter figure another set of calculations available in the literature are compared to the ATLAS data for the differential ϕ^*_{η} spectrum, where $\phi^*_{\eta} = \tan(\frac{1}{2}(\pi - \Delta\phi))\sin\theta^*$, with $\sin\theta^* = 1/\cosh(\Delta\eta/2)$. Here, $\Delta\phi$, $\Delta\eta$ are the respective differences in the azimuths and pseudo-rapidities of the two attendant leptons. We see in both cases that there is considerable need for improvement if we want to

have the same or a better reach in physics for 300 fb^{-1} of LHC data. Indeed, given that none of the calculations actually "looks" like the data for all values of the observables plotted, one could even question what reach in physics these data allow now at the LHC. Indeed, since there are other calculations available in the literature that are excluded from the comparisons in Figs. 3 and 4, this leaves the question even more unsettled.



Fig. 3. Comparisons of some theoretical predictions with the ATLAS $Z\gamma^* p_{\rm T}$ spectrum in single $Z\gamma^*$ production with decay to lepton pairs as given in the first reference in Refs. [63].



Fig. 4. Comparisons of some theoretical predictions with the ATLAS $Z\gamma^* \phi_{\eta}^*$ spectrum in single $Z\gamma^*$ production with decay to lepton pairs as given in the second reference in Refs. [63].

In Refs. [46], we have realized the new IR-improved DGLAP-CS theory in the Herwig6.5 environment with the new MC Herwiri1.031. We show in Figs. 5 comparison between the predictions for the $p_{\rm T}$ and rapidity spectra for the IR-improved MC Herwiri1.031 and the predictions for the well-known Herwig.5 MC without our IR-improvement in its parton showers, where in the case of Herwig6.5, we show spectra both for an intrinsic Gaussian $p_{\rm T}$ with r.m.s. value 2.2 GeV and without such intrinsic $p_{\rm T}$ for the proton constituents. For the unimproved case, the comparisons with the ATLAS $p_{\rm T}$ data [63] suggest that we need the Gaussian (intrinsic) $p_{\rm T}$ with r.m.s. value PTRMS \cong 2.2 GeV to get a good fit to both of the spectra, whereas for the IR-improved case (Herwiri1.031), we get good fits to both sets of spectra without the need of such a hard intrinsic $p_{\rm T}$. Both the IR-improved and the IR-unimproved MC's need the MC@NLO exact $\mathcal{O}(\alpha_{\rm s})$ correction to fit the CMS rapidity data [64] shown. The respective sets of χ^2 /d.o.f.'s are {1.37,0.70}, {0.72, 0.72}, and {2.23, 0.70} for the MC@NLO/Herwig6.510 (PTRMS = 2.2 GeV), MC@NLO/Herwiri1.031, and MC@NLO/Herwig6.510 (PTRMS = 0.0) predictions for the $p_{\rm T}$ and rapidity spectra.



Fig. 5. Comparison with LHC data: (left), CMS rapidity data on (Z/γ^*) production to e^+e^- , $\mu^+\mu^-$ pairs, the circular dots are the data, the light gray/green (dark gray/blue) lines are Herwig6.510 (Herwiri1.031); (right), ATLAS p_T spectrum data on (Z/γ^*) production to (bare) e^+e^- pairs, the circular dots are the data, the dark gray/blue (light gray/green) lines are Herwiri1.031 (Herwig6.510). In both (left) and (right) the dark gray/blue (light gray/green) squares are MC@NLO/Herwiri1.031 (Herwig6.510 (PTRMS = 2.2 GeV)). In (right), the light gray/green triangles are MC@NLO/Herwig6.510 (PTRMS = 0). These are otherwise untuned theoretical results.

Which of the predictions illustrated in Fig. 5 is better for precision QCD theory? We note that precocious Bjorken scaling in the SLAC-MIT experiments [65, 66], where scaling occurs already at $Q^2 \cong 1_+ \text{GeV}^2$, implies that PTRMS² should be small compared to 1_+GeV^2 . See the models of the proton wave function in Refs. [67], where in all cases PTRMS² $\ll 1_+ \text{GeV}^2$. This favors the IR-improved approach in Herwiri1.031. Moreover, the first

principles approach in Herwiri1.031 is not subject to arbitrary functional variation to determine its $\Delta \sigma_{\rm th}$. Experimentally, in principle, the events in the two cases MC@NLO/Herwig6.510 (PTRMS = 2.2 GeV) and MC@NLO/Herwiri1.031 should look different in terms of the properties of the rest of the particles in the events — this is under study [61]. Here, we show already the results in Figs. 6 and 7, which show that the two MC's predict



Fig. 6. Normalized vector boson mass spectrum at the LHC for $p_{\rm T}$ (lepton) > 20 GeV.



Fig. 7. Normalized vector boson $p_{\rm T}$ spectrum at the LHC for the ATLAS cuts as exhibited in Fig. 5 for the same conventions on the notation for the theoretical results with the vector boson $p_{\rm T} < 10$ GeV to illustrate the differences between the three predictions.

a 2.2% difference in the Z-peak and that, if we make a finer binning of the $p_{\rm T}$ spectra, 0.5 GeV/bin instead of 3 GeV/bin, we can distinguish all three cases, MC@NLO/Herwig6.510 (PTRMS = 2.2 GeV), MC@NLO/Herwiri1.031, and MC@NLO/Herwig6.510 (PTRMS = 0.0), with the type of precision data at the LHC ($\geq 10^7 Z/\gamma^*$ decays to lepton pairs). We await these exciting data. We stress that IR-improved DGLAP-CS theory increases the definiteness of precision determination of NLO parton shower MC's and improves such. More potential checks against experiment of the new IR-improved MC Herwiri1.031 are being pursued. We note that realizations of the new IR-improved showers are in progress for Herwig++ [68, 69], Pythia8 [70, 71] and Sherpa [72].

In the near future, in addition to more specific tests of observables such as ϕ_{η}^* , with and without $p_{\rm T}$ cuts on the respective Z/γ^* , as well as the analysis of the large and hopefully soon to be released > 10⁷ lepton pairs data samples form ATLAS and CMS, we are also pursuing the version Herwiri2.0 [73] in which the CEEX realization of higher order EW corrections in \mathcal{KK} MC [74] is realized in the Herwig6.5 environment and

TABLE I

$v_{\rm max}$	$\mathcal{K}\mathcal{K}$ sem Refer.	$\mathcal{O}(\alpha^3)_{\rm EEX3}$	$\mathcal{O}(\alpha^2)_{\text{CEEX}}$ intOFF	$\mathcal{O}(\alpha^2)_{\text{CEEX}}$									
$\sigma(v_{\rm max})$ [pb]													
0.01	0.9145 ± 0.0000	0.9150 ± 0.0004	0.9150 ± 0.0004	0.9323 ± 0.0004									
$\begin{array}{c} 0.10 \\ 0.30 \\ 0.50 \\ 0.70 \\ 0.90 \end{array}$	$\begin{array}{c} 1.0805\pm 0.0000\\ 1.1612\pm 0.0000\\ 1.1974\pm 0.0000\\ 1.2310\pm 0.0000\\ 1.6104\pm 0.0000 \end{array}$	$\begin{array}{c} 1.0807 \pm 0.0004 \\ 1.1615 \pm 0.0004 \\ 1.1977 \pm 0.0004 \\ 1.2312 \pm 0.0004 \\ 1.6128 \pm 0.0003 \end{array}$	$\begin{array}{c} 1.0808 \pm 0.0004 \\ 1.1616 \pm 0.0004 \\ 1.1981 \pm 0.0004 \\ 1.2317 \pm 0.0004 \\ 1.6114 \pm 0.0004 \end{array}$	$\begin{array}{c} 1.0920 \pm 0.0004 \\ 1.1691 \pm 0.0004 \\ 1.2036 \pm 0.0004 \\ 1.2357 \pm 0.0004 \\ 1.6148 \pm 0.0004 \end{array}$									
0.99	1.6218 ± 0.0000	1.6254 ± 0.0003	1.6244 ± 0.0004	1.6277 ± 0.0004									
$A_{\rm FB}(v_{ m max})$													
0.01	0.5883 ± 0.0000	0.5883 ± 0.0005	0.5883 ± 0.0005	0.6033 ± 0.0005									
$\begin{array}{c} 0.10 \\ 0.30 \\ 0.50 \\ 0.70 \\ 0.90 \end{array}$	$\begin{array}{c} 0.5882 \pm 0.0000 \\ 0.5879 \pm 0.0000 \\ 0.5875 \pm 0.0000 \\ 0.5848 \pm 0.0000 \\ 0.4736 \pm 0.0000 \end{array}$	$\begin{array}{c} 0.5881 \pm 0.0004 \\ 0.5879 \pm 0.0004 \\ 0.5874 \pm 0.0004 \\ 0.5845 \pm 0.0004 \\ 0.4722 \pm 0.0003 \end{array}$	$\begin{array}{c} 0.5881 \pm 0.0004 \\ 0.5879 \pm 0.0004 \\ 0.5875 \pm 0.0004 \\ 0.5846 \pm 0.0004 \\ 0.4728 \pm 0.0003 \end{array}$	$\begin{array}{c} 0.5966 \pm 0.0004 \\ 0.5932 \pm 0.0004 \\ 0.5912 \pm 0.0004 \\ 0.5868 \pm 0.0004 \\ 0.4748 \pm 0.0003 \end{array}$									
0.99	0.4710 ± 0.0000	0.4691 ± 0.0003	0.4697 ± 0.0003	0.4716 ± 0.0003									

Study of total cross section $\sigma(v_{\text{max}})$ and charge asymmetry $A_{\text{FB}}(v_{\text{max}})$, $d\bar{d} \rightarrow \mu^{-}\mu^{+}$, at $\sqrt{s} = 189$ GeV. See the text for definition of the energy cut v_{max} , see Ref. [75] for the scattering angle and M.E. type definitions.

the direct application of \mathcal{KK} MCto LHC processes with the release recently of \mathcal{KK} MC4.22 [75] in which the incoming beam choices are extended to $q\bar{q}, \mu\bar{\mu}, \tau\bar{\tau}, \nu_{\ell}\bar{\nu}_{\ell}, q = u, d, s, c, b, t, \ell = e, \mu, \tau$. Finally, we note that we have in mind as well the development of MC@NNLO or its equivalent, as that is the requirement for the sub-1% precision tag that will be needed to exploit fully the 300 fb⁻¹ LHC data sets.

We show in Table I and Fig. 8 the respective illustrative results for $d\bar{d} \rightarrow \mu\bar{\mu}$ at 189 GeV for its *v*-spectrum⁶ and for its physical precision test and in Fig. 9 the illustrative results for $pp \rightarrow u\bar{u} \rightarrow \ell\bar{\ell} + n\gamma$, where the proton PDF's for *u* and \bar{u} are replacing the beamsstrahlung distributions in \mathcal{KK} MC.



Fig. 8. Physical precision of CEEX ISR matrix element for $d\bar{d} \rightarrow \mu^{-}\mu^{+}$ at $\sqrt{s} =$ 189 GeV. See Table I for definition of cut-offs.

Take note that the results show the value $\Delta \sigma_{\rm th} \cong 0.1\%$ for the $d\bar{d}$ process at the typical energy cut near $v \cong 0.6$ and show effects of non-zero $p_{\rm T}$ in the multiple photon radiation at the LHC can be important. These matters are under study accordingly.

4. Outlook

We note that there are other efforts than those we have mentioned to improve resummation in progress. In the EW collinear regime, we call attention to Refs. [76], in QCD we note the new SCET approach of Ref. [78], and the new SCET based MC's in Refs. [79], etc. There are new NLO and new NNLO results: multi-leg NLO [80], $t\bar{t}$ at NNLO [81], etc. What we can say is that the full exploitation of the LHC discovery potential with 300 fb⁻¹ of data will need all such efforts in view of and in conjunction with what we have discussed above.

⁶ Here, v = 1 - s'/s in the usual notation.

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***********************	nergy average energy spread	photon mult.	witch	SR interferenc	niz. switch	niz. min. mass um weicht	photon mult.	num phot. ener.	mass, IR regul mult. enhanc.	(1) 11Beaml (1)	[Beam1 (3)	(1Beam2 (2) (1Beam2 (3)	(sumary)	00 P.V00 22.668	10 0.000 -245.458 16 18.370 -80.068 89 -6 132 -128 905	00 0.000 0.031 73 -12.238 -13.848	00 0.000 3477.332	00 0.000-3254.542 01 21.657 -20.217	16 -3.287 -59.851	(summary)	00 0.000 271.908	00 0.000 -6.542 47 1.133 244.401	47 -1.133 20.965	00 0.000-3493.458	01 14.537 242.396 54 -13.404 242.396 00 0.000 0.000
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Fig. 9. Sample output from \mathcal{KK} MC4.22 for $pp \to u\bar{u} \to l^-l^+ + n\gamma$.

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