STUDY OF COLLECTIVE ROTATIONAL ENHANCEMENT FACTORS FOR NUCLEAR LEVEL DENSITIES USING THE MICROSCOPIC THEORY OF INTERACTING FERMIONS

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Determination of accurate nuclear level densities is of crucial importance for a variety of nuclear physics aspects and its related technologies. Therefore, there have been many theoretical and experimental efforts to determine nuclear level densities for variety of nuclei. In this research, the effects of changing structure and collective excitations on some deformed nuclei with axial symmetry were studied using microscopic generalized superfluid model (MGSFM) and experimental data. It was shown that the experimental data can be reproduced by a level density formalism developed for nuclei with static deformation.

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1. Introduction

Nuclear level density (NLD) is one of the most interesting concepts in nuclear physics with numerous applications such as basic research in nuclear structure, nuclear medicine (radioisotopes production), statistical calculation of reactor physics, activation methods, transport theory, shielding, and astrophysics. NLD is an important parameter in statistical analysis of nuclear reactions, in calculating cross section of reactions such as those involved in the formation of compound nuclei and pre-equilibrium reactions, and calculating the rate of gamma decay of highly excited nuclei. Besides, NLD has a significant role in the statistical study of nuclei as many body systems from which all thermodynamic quantities can be extracted. Estimating thermodynamic quantities such as entropy, temperature, and heat capacity as

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functions of excitation energy is one of the fundamental challenges in nuclear physics. These thermodynamic quantities depend on statistical properties of nuclear many-body systems. Determining NLD is the key point in estimating these thermodynamic quantities [1, 2]. Numerous experimental and theoretical models have been proposed for studying NLD and each has pros and cons. Many of these models relate the excited levels only to intrinsic excitation in which excitation energy is distributed between limited numbers of nucleons. However, there is another kind of excitation, especially at low energies, that cannot be justified by intrinsic excitation. These levels are produced from total or partial distribution of energy on the whole system of fermions. Collective excitations are generated by vibrational and rotational motions. In the microscopic theory of interacting fermions, the formation of rotational bonds influences the intrinsic level densities. Experimental data on NLD are often obtained from counting discrete low levels. This method of spectroscopy is limited to very low energies in which experimental resolution is good enough for splitting the spectrum peaks. In slightly higher energies, these peaks overlap and then go to the continuous spectrum. Therefore, the experimental data based on low-lying levels are only complete up to a specific energy (completeness) and after that, number of levels is underestimated. In such a situation, nuclear levels are only described using the statistical approach and NLD function.

A new experimental method has been developed to enable extracting NLD on a wide range of excitation energies [3, 4]. In this method, particle–γ coincidences have been measured to obtain γ-ray spectra as a function of excitation energy for a few nuclei and then NLDs for ^{162}Dy, ^{166}Er, ^{172}Yb, ^{232}Th and ^{238}U nuclei are extracted [5, 6]. The ground states of these nuclei are expected to be deformed. This is of interest to study the level density of deformed nuclei and compare the results with the relevant experimental values. The effects of deformation and collective rotational excitations for these nuclei are considered in calculations using the mechanism of rotational collective enhancements in microscopic theory of interacting fermions. Accuracy of calculated results is then evaluated by comparing with the new experimental data.

2. Microscopic theory of interacting fermions

NLD in microscopic models is calculated directly from a realistic average potential of single particle levels that include shell effects, deformation and pairing. Pairing interaction is one of the most important residual interactions which is considered in microscopic calculations through the BCS theory. This model is the microscopic version of Generalized Super Fluid Model (GSFM) in which a phase transition is occurred from superfluidity

behavior at low energy toward a normal state at excited energy. For this case, a mechanism is defined similar to the effective Hamiltonian for Cooper pairs in the BCS theory. Therefore, the problem is reduced to the BCS quasi-particle as non-interacting fermions. For this, the second quantization formalism is applied [7–10]. There are many parameters in the Hamiltonian that describe a system of interacting fermions [11]. Such a Hamiltonian is transformed to a Hamiltonian of a system composed of non-interacting quasi-particles with quasi-particle energy (E) which can be approximately diagonal. The intrinsic state and level density are calculated with statistical partition function as below

$$\omega_{\rm intr}(E_x) = \frac{e^{S(E_x)}}{(2\pi)^{3/2}\sqrt{\det D}}\,, \tag{1}$$

$$\rho_{\rm intr}(E_x) = \frac{\omega_{\rm intr}(E_x)}{\sqrt{2\pi\sigma^2}}\,, \tag{2}$$

where $S(E_x) = \ln Z(\beta_0, \alpha_0) + \beta_0 E_x - \alpha_{p0} Z - \alpha_{n0} N$ is the entropy of the system and $\det D$ is a 3×3 determinant of second order partial differentials of partition function $\ln Z(\beta_0, \alpha_0)$ *versus* β and α_i in the saddle point. In microscopic formalism, grand partition function is determined by realistic single particle levels [7–10, 12]

$$\ln Z(\beta, \alpha_i) = \sum_{q=n,p} \left\{ -\beta \sum_k \left(\epsilon_q^k - \lambda_q - E_q^k \right) + \sum_k \ln\left(1 + e^{-\beta E_q^k}\right) - \beta \frac{\Delta_q^2}{G_q} \right\}, \tag{3}$$

where E_q^k, λ_q, ϵ_q^k, Δ_q, G_q are quasi-particle energy, chemical potential, single particle energy levels, pairing parameter and gap parameter for each kind of particles ($q = p, n$), respectively. Other necessary thermodynamic quantities, including entropy and $\det D$ can be obtained by this equation. For nuclei with axial symmetry, intrinsic states are determined by quantum number Ω that shows the projection of total intrinsic angular momentum on the axial symmetry of nuclei. Projection of Ω for a determined intrinsic state is obtained from combination of unpaired single particle excitations. An intrinsic density state of a Ω with a normal distribution (Gaussian shape) is estimated by

$$\omega_{\rm intr}(E_x, \Omega) = \frac{\omega_{\rm intr}^{\rm tot}(E_x)}{\sqrt{2\pi\sigma_\parallel^2}} \exp\left(-\frac{\Omega^2}{2\sigma_\parallel^2}\right), \tag{4}$$

where $\omega_{\rm intr}^{\rm tot}(E_x)$ is intrinsic density state or total density of particle and $\sigma_\parallel^2$ is spin cut-off factor that explains width of spin distribution. In foregoing

calculations, we will assume that each intrinsic state which is determined by $|\Omega|$ has a rotational band that is constructed based on it. These bands are a collection of levels with the same parity. If there are not many states for which $K \neq \Omega$, it can be assumed that $K = \Omega$ with K as the projection of total angular momentum of I on the symmetry axis. Level density for a special I is obtained by summing up all intrinsic states with $|K| \leq I$

$$\rho(E_x, I) = \sum_{K \leq I} \omega_{\text{intr}} \left[E_x - E_{\text{rot}}(K, I), \Omega = K \right] . \tag{5}$$

In $E_{\text{rot}}(K, I)$ which is rotational energy [12–14]

$$E_{\text{rot}}(K, I) = \begin{cases} \frac{I(I+1)-K^2}{2\sigma_\perp^2} + \frac{K^2}{2\sigma_\parallel^2} & \text{for } e\text{–}e \text{ nuclei (axially symetric)}, \\ \frac{I(I+1)-K(K+1)}{2\sigma_\parallel^2} & \text{for large deformed odd–}A \text{ nuclei}. \end{cases} \tag{6}$$

If $E_{\text{rot}}(K, I)$ is negligible compared to total excitation energy, the above equation can be shown as [7, 9]

$$\rho(E_x, I) = \frac{1}{2} \sum_{K=-I}^{I=K} \frac{\omega_{\text{intr}}^{\text{tot}}(E_x)}{\sqrt{2\pi\sigma_\parallel^2(E_x)}} e^{-\frac{K^2}{2\sigma_\parallel^2(E_x)}} e^{\frac{I(I+1)-K^2}{2\sigma_\perp^2(E_x)}} , \tag{7}$$

$$\rho_{\text{def}}(E_x) = \sum_I \rho(E_x, I) \tag{8}$$

in which $\rho_{\text{def}}(E_x)$ is total level density (by introducing the effect of collective rotational excitations). The spin cut-off parameter $\sigma^2 = \langle M^2 \rangle$ represents the width of the angular momentum distribution of the level density. $\sigma_\parallel^2$ designates the parallel spin cut-off parameter, obtained from the projection of the angular momentum of the single-particle states on the symmetry axis. It can be calculated by using microscopic theory and a realistic scheme of single particle levels as [7, 9]

$$\sigma_\parallel^2(E_x) = \frac{1}{2} \sum_{q=n,p} \sum_k \Omega_q^{k^2} \operatorname{sech}^2 \left(\frac{\beta E_q^k}{2} \right) \tag{9}$$

in which $q = p, n$ are neutrons and protons and β is inverse of temperature, T. $E_q^k = \sqrt{(\epsilon_q^k - \lambda_q)^2 + \Delta_q^2}$ is quasi-particle energy. Also, the spin cut-off factor σ^2 is related to $\Im$ inertial moment of nuclei by $\sigma^2 = \frac{\Im T}{\hbar^2}$. Therefore, one can use $\sigma_\parallel^2(E_x) = \frac{\Im_\parallel T}{\hbar^2}$, in which $\Im_\parallel$ is inertial moment around a parallel axis with nuclei symmetry axis. Similarly, using the above equation $\sigma_\perp^2(E_x)$

is related to an axis perpendicular to a nuclear symmetry axis. Since the relation of inertial moment perpendicular to nuclear symmetry axis of a rigid spherical shape is stated as $\Im_\perp = \Im_0(1+\beta_2/3)$, we can consider $\sigma_\perp^2(E_x)$ as $\sigma_\perp^2(E_x) = \sigma_\parallel^2(E_x)[(1+\beta_2/3)/(1-2\beta_2/3)]$ [15]. Here, this approximation is only used to obtain $\sigma_\perp^2(E_x)$ from $\sigma_\parallel^2(E_x)$ and β_2 deformation parameter. Pairing correlations in the theory of BCS is considered in constant G estimation. The BCS equation gap parameter Δ_q and chemical potential λ_q are determined as a function of pairing power, G_q by equations

$$N_q = \sum_k \left[1 - \frac{\epsilon_q^k - \lambda_q}{E_q^k} \tanh\left(\frac{1}{2}\beta_0 E_q^k\right)\right], \qquad q = n, p, \tag{10}$$

$$\frac{2}{G_q} = \sum_k \frac{1}{E_q^k} \tanh\left(\frac{1}{2}\beta E_q^k\right) \tag{11}$$

in which N_q refers to the number of neutrons and protons. It must be mentioned that the NLD for a special angular momentum is calculated by Eq. (7). In this equation, state density is calculated by a realistic collection of single particle level with grand partition function method for a system of interacting fermions.

3. Phenomenological vibrational and rotational collective enhancements

According to the hybrid Bohr and Mottelson model, each single particle level can be the start of another vibrational or rotational band. Hence, the level density is increased as compared to intrinsic models [12]. By using adiabatic hypothesis in which intrinsic and accumulated excitations are assumed to be independent of each other, it can be shown that accumulated effects can be explained by multiplying rotational and vibrational collective enhancement factors in intrinsic level density

$$\rho_{\text{coll}}(E_x, J, \pi) = K_{\text{rot}}(E_x) K_{\text{vib}}(E_x) \rho_{\text{int}}(E_x, J, \pi)\,. \tag{12}$$

Since these accumulated levels are effective only in low energies, their effects are vanished with increasing energy similar to shell effects. Therefore, these factors must be balanced in high energies. To this end, we multiplied them by damping factors.

4. Vibrational collective effects

Vibrational levels exist in all nuclei and follow Bose–Einstein statistics. Therefore, they can be explained by using a Bosonian partition function

without any limitation on the occupying numbers. In this method, by using changes in entropy and excitation energies that result from the vibrational modes $(\delta S, \delta U)$ which are calculated base on the Bose gas model, these factors can be estimated as [16, 17]

$$K_{\rm vib}(T) = \exp(\delta S - \delta U/T)\,, \tag{13}$$

$$\delta S = \sum_i (2\lambda_i + 1)[(1+n_i)\ln(1+n_i) - n_i \ln n_i]\,, \tag{14}$$

$$\delta U = \sum_i (2\lambda_i + 1)\omega_i n_i \tag{15}$$

in which $T = \sqrt{U/a}$ is a nuclear temperature. δU and δS are changes in excitation energy and entropy, respectively. In addition, ω_i, λ_i and n_i are energies, multipolarities and occupation numbers for vibrational excitations in a determined temperature, respectively. The damping of vibrational enhancements is estimated by defining occupying numbers with increasing energy as

$$n_i = \exp\left(-\frac{\gamma_i}{2\omega_i}\right) \Big/ \exp\left(\frac{\omega_i}{T}\right) - 1\,, \tag{16}$$

where γ_i is the width of vibrational excitations

$$\gamma_i = 0.0075 A^{1/3} \left(\omega_i^2 + (2\pi T)^2\right)\,. \tag{17}$$

Quadrupole and octupoles are the most important multipolarities in these calculations $(\lambda_i = 2, 3)$. The energies of phonons for quadrupole and octupole are estimated as

$$\omega_2 = 65 A^{-5/6}/(1 + 0.05\delta W)\,, \tag{18}$$

$$\omega_3 = 100 A^{-5/6}/(1 + 0.05\delta W)\,. \tag{19}$$

There are alternative approaches for estimation of the vibrational collective enhancement factor such as liquid drop model that gives $K_{\rm vib}(T) = \exp(0.0555 A^{2/3} T^{5/3})$ [18], but the Bose gas relationship is most commonly used expression in nuclear reaction codes.

5. Rotational collective effects

As compared to vibrational excitations, rotational excitations have more contribution to increasing accumulated level density of a nucleus ($K_{\rm vib}(T) \sim 3$, while $K_{\rm rot} \sim$ 10–100). Rotational excitations depend on the nucleus shape. Except for the magic nuclei which are completely spherical at their ground state, other nuclei are somehow distorted at their ground state. Hence, these

factors can be used for most nuclei. Basically, $K_{\rm rot}$ is considered equal to vertical spin cut-off factor [15]. However, to introduce damping in rotational enhancement with increasing energy, the following semi-empirical equation can be used [19]

$$K_{\rm rot} = \frac{\left(\sigma_{\perp}^2 - 1\right)}{1 + \exp\left(\frac{U - U_c}{d_c}\right)}, \tag{20}$$

$$U_c = 120\beta_2^2 A^{1/3}, \tag{21}$$

$$d_c = 1400\beta_2^2/A^{2/3}. \tag{22}$$

In order to estimate rotational enhancement factor from the microscopic relationships, first microscopic intrinsic level density $\rho_{\rm intr}^{\rm mic}(E_x) = \frac{\omega_{\rm intr}^{\rm tot}(E_x)}{\sqrt{2\pi\sigma_{\parallel}^2}}$ and total level density $\rho_{\rm def}^{\rm mic}(E_x) = \sum_I \rho(E_x, I)$ are calculated. Then, since $\rho_{\rm def} = K_{\rm rot}\rho_{\rm int}$, microscopic rotational enhancement factor is obtained as

$$K_{\rm rot}^{\rm mic} = \frac{\rho_{\rm def}^{\rm mic}}{\rho_{\rm intr}^{\rm mic}}. \tag{23}$$

For taking into account the damping of this rotational enhancement with increasing energy, Eq. (20) can be used again exept that $K_{\rm rot} = \sigma_{\perp}^2$ is replaced by $K_{\rm rot}^{\rm mic}$. Than,

$$K_{\rm rot,damp}^{\rm mic} = \frac{\left(K_{\rm rot}^{\rm mic} - 1\right)}{1 + \exp\left(\frac{U - U_c}{d_c}\right)}. \tag{24}$$

In addition, in this microscopic model it is possible to adjust the results with experimental low level data by using the following equation and its two empirically adjustable parameters, (c, p) [20]

$$\rho_{\rm renorm}(E_x, J, \pi) = \exp\left[c\sqrt{E_x}\right] \rho_{\rm mic}\left(E_x - p, J, \pi\right). \tag{25}$$

Using this approach, microscopic model will be flexible with accessible experimental data. Also, for pairing constants, functional equation is used [21–23]

$$G_z = \frac{2.0}{Z^{0.7}}, \qquad G_N = \frac{2.25}{N^{0.7}}. \tag{26}$$

6. Results and discussion

In this research, low level experimental data of nucleus were used for adjusting this model. It should be noted that these levels are complete

below certain excitation energy. Afterward, because of the missed levels, level densities are underestimated. Figure 1 compares the obtained results from these methods for vibrational and rotational enhancement factors of ^{166}Er nucleus.

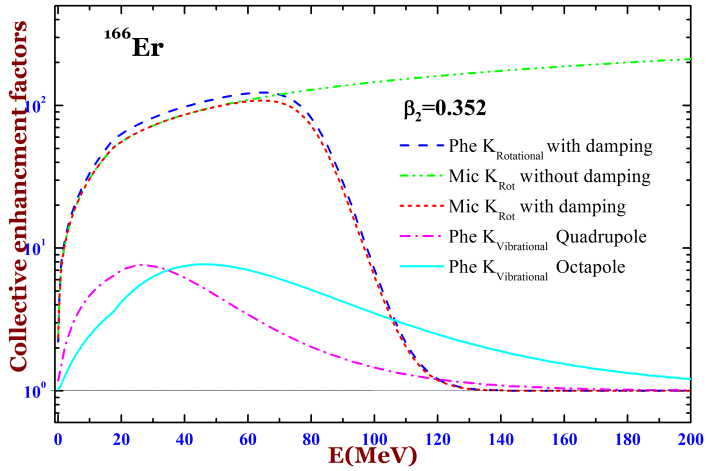

Fig. 1. Comparison of phenomenological and microscopic methods for vibrational and rotational enhancement factors of nucleus.

These results show a good agreement between two methods of calculating rotational enhancements. In this study, the NLDs were extracted using experimental data of the Oslo group for several nuclei which are expected to have large static deformation in their ground state [6, 24–27]. This was carried out by using the microscopic formalism and phenomenological study for calculating the effects of deformation and collective excitations and the estimated NLDs were then compared with the experimental results. The results are shown in Fig. 2.

The results are in good agreement with the new experimental data for various deformation parameters β_2 for each nucleus. In the microscopic theory, these deformations mainly depend on the shell configuration. The deformations shown in the figures are in good agreement with the reported results [29]. To estimate the effect of nucleus deformation on the NLD, the microscopic calculations were performed assuming spherical shape for nuclei. Therefore, both microscopic and phenomenological methods can be used for introducing the rotational collective effects in calculating NLDs for nuclei with static prolate deformation. The results of these calculations have a significant impact on the prediction of many reactions particularly the ones in the fission decay channel.

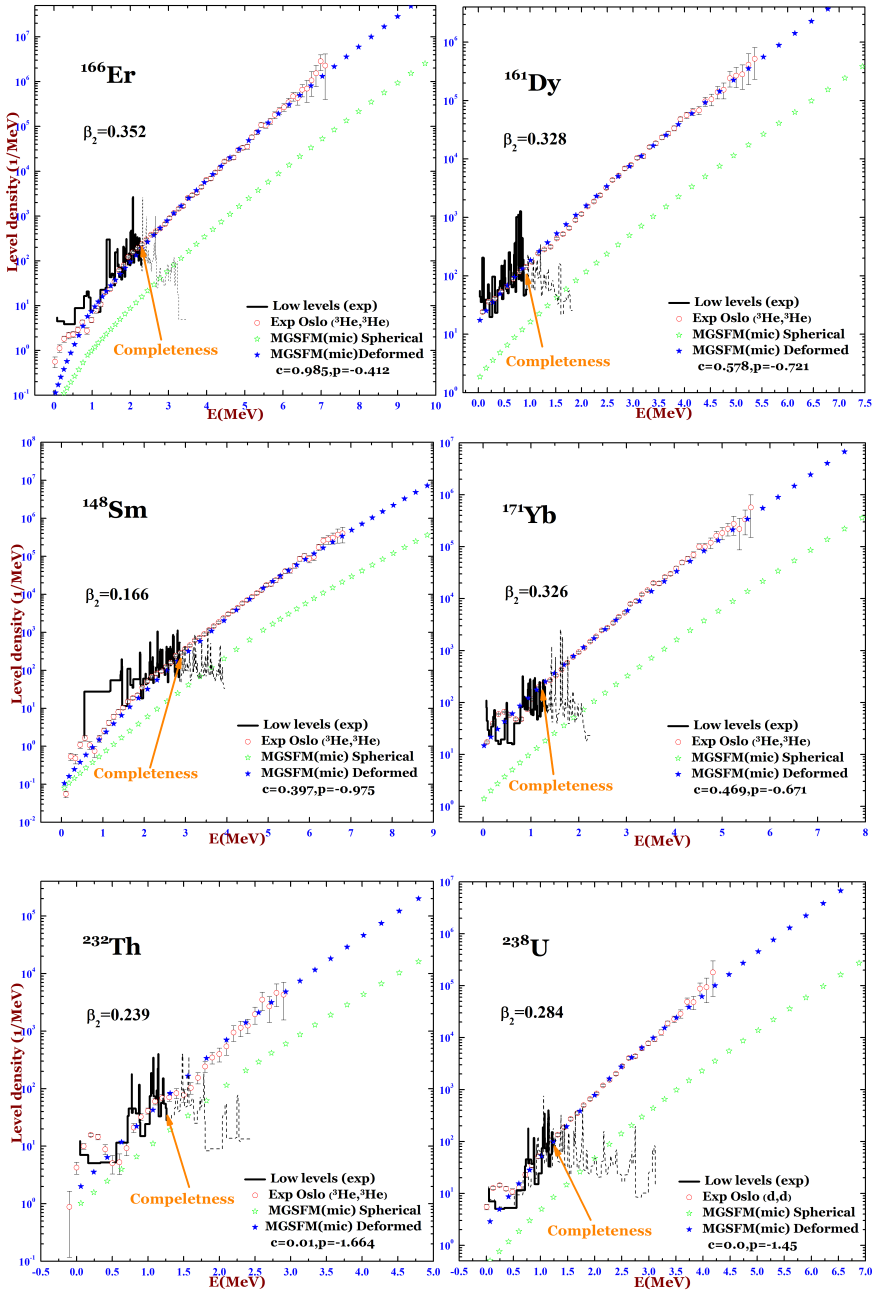

Fig. 2. Comparison of microscopic generalized superfluid model calculations considering deformation effects with experimental data of the Oslo group [6, 24–27] and experimental low-lying levels [28].

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