NLO CORRECTIONS TO HARD PROCESS IN PARTON SHOWER MC — KrkNLO METHOD*

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A new method of combining an NLO-corrected hard process with an LO parton shower Monte Carlo, nicknamed KrkNLO, was proposed recently. It is simpler than well-established two other methods: MC@NLO and POWHEG. In this contribution, we present some results of extensive numerical tests of the new method for single Z-boson production at hadron colliders and numerical comparisons with two other methods as well as with NNLO calculations.

1. Introduction

The KrkNLO method of combining the NLO-corrected hard process with the LO parton shower Monte Carlo (PS MC) was proposed in Ref. [1] and tested numerically in Ref. [2] using a not very realistic parton shower Monte Carlo (PS MC). A recent paper [3] reports on implementation of the KrkNLO method within the Sherpa [4] and Herwig++ [5–7] PS MCs, presents a lot of numerical results from the new method, comparing them with fixed-order NLO results from the MCFM program (MC integrator) [8], NNLO results from DYNNLO [9] and matched results obtained using MC@NLO [10] and POWHEG [11].

Multitude of the results of Ref. [3] will be presented in the following only partly. On the other hand, let us describe briefly a wide range of other research performed by the Kraków group in this very active area of combining resummed NLO QCD calculations with PS MCs.

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Early activity (2004–06) on the parton MC and NLO QCD started with solving exactly the LO and NLO DGLAP evolution equations, using Markovian MC methods, MMC programs, see Refs. [12–14][1]. MMCs were used to test constrained MC (CMC) series of programs (2005–07), see Refs. [16, 17]. CMCs implement the same evolution with constrained/predefined final x, an alternative to backward evolution [18] in the PS MC, aiming at better control (NLO level) of the distributions generated by LO PS MC. CMCs were for single ladder/shower, without hard process, but with exclusive LO kernels and optionally with inclusive NLO kernels.

The path from DGLAP to parton shower MC was continued with the exercise in which two CMC modules and hard process matrix element were combined into complete PS MC for the Drell–Yan (DY) process, see *e.g.* Refs. [2, 19], albeit not upgraded with realistic parton distribution functions (PDFs) and kinematics. However, this kind of PS MC has been instrumental in testing various new ideas on implementing: *(i)* the NLO corrections in the exclusive evolution kernels in the initial-state ladders/showers many times, *(ii)* the NLO corrections to the hard process just once (finally resulting in the KrkNLO method) thanks to perfect numerical and algebraic control over the LO distributions.

Another branch of the research has covered the NLO corrections to PS MC, that is the problem of including the NLO corrections in an exclusive form into evolution (kernels) in the (initial-state) ladder/shower, which was never addressed before[2]. The first solution, albeit limited to non-singlet evolution kernels, was proposed and tested numerically in Refs. [20, 21], using the NLO kernels in the exclusive form calculated from scratch in the Curci–Furmanski–Petronzio [22] (CFP) framework. The non-singlet 2-real kernels were presented in Ref. [23]. A simplified and faster scheme was reported (with numerical tests) in Ref. [24]. An even simpler and faster scheme of the NLO-correcting PS MC (single initial-state ladder) was reported in Ustroń 2013, see Ref. [25]. Also the singlet evolution kernels are now almost complete (unpublished). It is a major problem to include consistently virtual corrections to exclusive kernels starting from the CFP scheme. The first solution was formulated (unpublished) exploiting recalculated virtual corrections in the CFP scheme to the non-singlet kernels [26]. The above breakthrough is important but points to: *(i)* the need of better understanding of the MC distributions in PS MC, *(ii)* especially their kinematics, the definition of the evolution variable, *etc.*

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[1] These MMCs were also capable to solve the CCFM [15] evolution plus DGLAP.
[2] Except of statements that it is for sure unfeasible.

2. The KrkNLO method

Methodology of the KrkNLO for the DY process was primarily defined in Ustroń 2011 [27], but without numerical tests. The first numerical validation of KrkNLO on top of Double-CMC PS was demonstrated in Ref. [2]. A more complete discussion of the KrkNLO scheme with the introduction of PDFs in the Monte Carlo (MC) factorisation scheme was provided in Ref. [1], but the MC implementation was still on top of the not-so-realistic Double-CMC PS. Finally, in recent Ref. [3], a new implementation on top of Sherpa and Her-wig++ (instead of two CMCs) was done. Comparisons of KrkNLO numerical results with NLO calculations of MCFM (fixed-order NLO), MC@NLO and POWHEG for the DY process were presented.

The central object in the KrkNLO method is a multiplicative NLO weight used for re-weighting LO parton shower events, which for the $q\bar{q}$ incoming partons takes the following compact form in terms of the standard Sudakov variables α and β and the LO PS differential distribution $\sigma^{\mathrm{LO}}_{n_{\mathrm{F}} n_{\mathrm{B}}}$, defined in Ref. [3]

$$d\sigma^{\mathrm{NLO}}_{n_{\mathrm{F}} n_{\mathrm{B}}} = \left(1 + \Delta_{\mathrm{VS}} + \sum_{i=1}^{n_{\mathrm{F}}} W^{[1]}_{q\bar{q}} \left(\tilde{\alpha}^{\mathrm{F}}_i, \tilde{\beta}^{\mathrm{F}}_i \right) + \sum_{j=1}^{n_{\mathrm{B}}} W^{[1]}_{q\bar{q}} \left(\tilde{\alpha}^{\mathrm{B}}_j, \tilde{\beta}^{\mathrm{B}}_j \right) \right) d\sigma^{\mathrm{LO}}_{n_{\mathrm{F}} n_{\mathrm{B}}} ,$$

$$W^{[1]}_{q\bar{q}} = \frac{d^5 \bar{\beta}_{q\bar{q}}}{d^5 \sigma^{\mathrm{LO}}_{q\bar{q}}} = \frac{d^5 \sigma^{\mathrm{NLO}}_{q\bar{q}} - d^5 \sigma^{\mathrm{LO}}_{q\bar{q}}}{d^5 \sigma^{\mathrm{LO}}_{q\bar{q}}} , \quad \Delta^{q\bar{q}}_{\mathrm{VS}} = \frac{\alpha_{\mathrm{s}}}{2\pi} C_{\mathrm{F}} \left[\frac{4}{3} \pi^2 - \frac{5}{2} \right] . \quad (2.1)$$

$$d^5 \sigma^{NLO}_{q\bar{q}} (\alpha, \beta, \Omega)$$
$$= \frac{C_{\mathrm{F}} \alpha_{\mathrm{s}}}{\pi} \frac{d\alpha d\beta}{\alpha\beta} \frac{d\varphi}{2\pi} d\Omega \left[\frac{d\sigma_0(\hat{s}, \theta_{\mathrm{F}})}{d\Omega} \frac{(1-\beta)^2}{2} + \frac{d\sigma_0(\hat{s}, \theta_{\mathrm{B}})}{d\Omega} \frac{(1-\alpha)^2}{2} \right] , \quad (2.2)$$
$$d^5 \sigma^{\mathrm{LO}}_{q\bar{q}} (\alpha, \beta, \Omega)$$
$$= d^5 \sigma^{\mathrm{F}}_{q\bar{q}} + d^5 \sigma^{\mathrm{B}}_{q\bar{q}} = \frac{C_{\mathrm{F}} \alpha_{\mathrm{s}}}{\pi} \frac{d\alpha d\beta}{\alpha\beta} \frac{d\varphi}{2\pi} d\Omega \frac{1 + (1-\alpha-\beta)^2}{2} \frac{d\sigma_0}{d\Omega} \left(\hat{s}, \hat{\theta} \right) . (2.3)$$

As pointed out in Ref. [2], for getting the complete NLO corrections to the hard process, it is enough to retain in the above sums over gluons $\sum_j$ only a single term, the one with the maximum k^2_{T} from one of the two showers[3]. In the case of the backward-evolution algorithm and k_{T}-ordering, the retained gluon is just the one which was generated first[4].

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[3] Independently of the ordering type, angular or k_{T}-ordering, in PS MC.

[4] This exploits the Sudakov suppression as in POWHEG, but there is no need of trun-cated showers for angular ordering.

Two essential ingredients in the KrkNLO method are: (1) completeness of the hard process phase space in PS MC and (2) the use of PDFs in the so-called MC factorisation scheme. In modern PS MCs, such as Sherpa and Herwig++, the phase-space completeness is luckily not a problem. PDFs in the MC factorisation scheme are obtained from PDFs in the $\overline{\text{MS}}$ scheme with the following transformation:

$$f_{q(\bar{q})}^{\text{MC}}\left(x, Q^2\right) = f_{q(\bar{q})}^{\overline{\text{MS}}}\left(x, Q^2\right)$$

$$+ \int_x^1 \frac{dz}{z} f_{q(\bar{q})}^{\overline{\text{MS}}}\left(\frac{x}{z}, Q^2\right) \Delta C_{2q}(z) \; + \int_x^1 \frac{dz}{z} f_g^{\overline{\text{MS}}}\left(\frac{x}{z}, Q^2\right) \Delta C_{2g}(z) \,,$$

$$\Delta C_{2g}(z) = \frac{\alpha_{\text{s}}}{2\pi} T_R \left\{ \left[z^2 + (1-z)^2\right] \ln \frac{(1-z)^2}{z} + 2z(1-z) \right\} \,,$$

$$\Delta C_{2q}(z) == \frac{\alpha_{\text{s}}}{2\pi} C_{\text{F}} \left[\frac{1+z^2}{1-z} \ln \frac{(1-z)^2}{z} + 1 - z \right]_+ \,. \tag{2.4}$$

Note that in the MC scheme, the quark PDF gets contribution from gluons. The gluon PDF can be untouched as long as we consider the DY process at NLO, $f_g^{\text{MC}}(x, Q^2) = f_g^{\overline{\text{MS}}}(x, Q^2)$. Plots of quark PDFs in the MC scheme and a detailed discussion why such PDFs are instrumental for assuring the completeness of the NLO corrections in the KrkNLO scheme can be found in Ref. [3].

3. Numerical results

Numerical results presented in Ref. [3] start with detailed comparisons of the KrkNLO results with the fixed-order results from the MCFM integrator [8]. We skip that and in Fig. 1 we show the comparisons of the KrkNLO results with these of MC@NLO and POWHEG. The overall pattern of the differences and their size of the order of 20% is typical for this kind of comparisons and is attributed to the missing NNLO corrections. In Fig. 11 of Ref. [3], the results corresponding to changing the factorisation and renormalisation scales by the factors of 2 and 1/2 confirm this statement. The actual size of the missing NNLO corrections can be seen in Fig. 2, where our results are compared to the fixed-order NNLO results of the DYNNLO program [9].
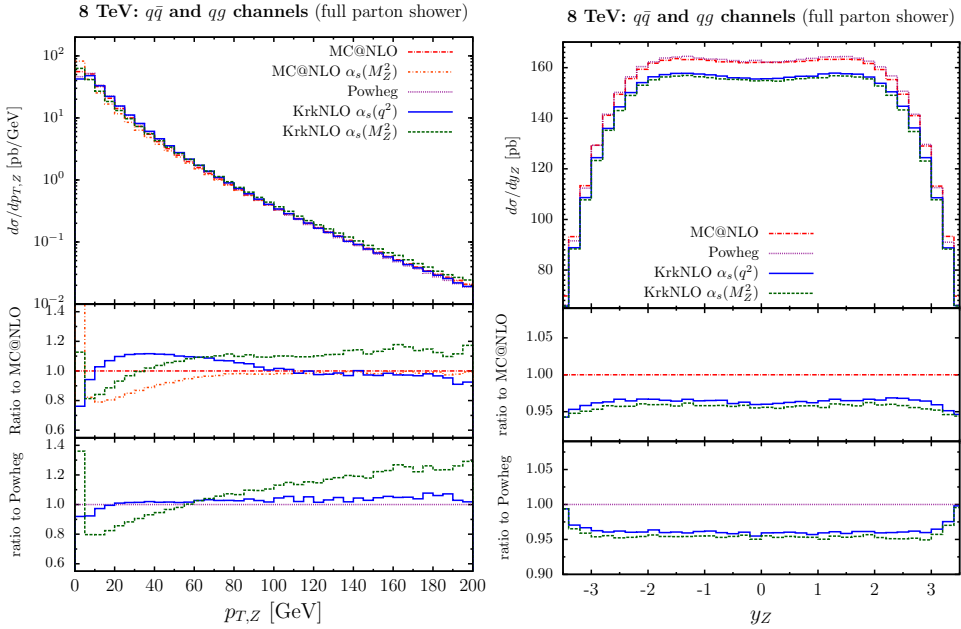
Fig. 1.   Distributions of the *Z*-boson transverse momentum (left) and rapidity (right) from the KrkNLO method compared with the MC@NLO and POWHEG results.
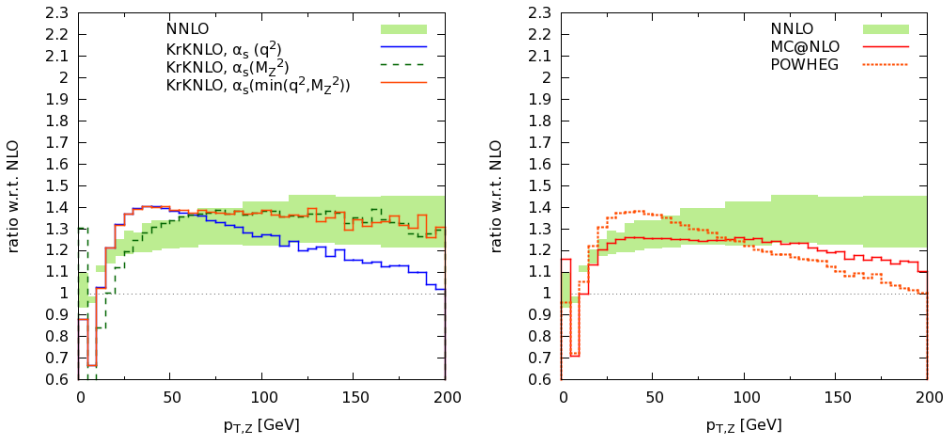

Fig. 2.   Distributions of the *Z*-boson transverse momentum from the KrkNLO method compared with the results from the DYNNLO program [9] (left). The results from MC@NLO and POWHEG are also shown (right).

For the sake of completeness, in Table I, we also present the corresponding results for the total cross section. All presented numerical results are taken from Ref. [3].

Values of the total cross section from the KrkNLO method compared with the MCFM, MC@NLO and POWHEG results.

	$\sigma_{\rm tot}^{q\bar{q}+qg}$ [pb]
MCFM	1086.5 ± 0.1
MC@NLO	1086.5 ± 0.1
POWHEG	1084.2 ± 0.6
KrkNLO $\alpha_{\rm s}\left(q^2\right)$	1045.4 ± 0.1
KrkNLO $\alpha_{\rm s}\left(M_Z^2\right)$	1039.0 ± 0.1

4. Summary and outlook

A new method of combining the NLO-corrected hard process with PS MC, called KrkNLO, was introduced and tested extensively for single Z-boson production at the LHC. It is much simpler than the MC@NLO and POWHEG methods at the expense of the introduction of PDFs in the new, so-called Monte Carlo, factorisation scheme. In the near future, this method will be applied to Higgs boson production and hopefully extended to the NNLO-corrected hard process, where its simplicity may be a very desirable feature.

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