

TOWARDS $\mathcal{B}(\bar{B} \rightarrow X_s \gamma)$ FOR AN ARBITRARY CHARM QUARK MASS*

ABDUR REHMAN

Faculty of Physics, University of Warsaw

Pasteura 5, 02-093 Warszawa, Poland

and

National Centre for Physics, Quaid-i-Azam University Campus

Shahdra Valley Road, Islamabad 45320, Pakistan

(Received October 19, 2015)

Constraints on New Physics from the $\bar{B} \rightarrow X_s \gamma$ branching ratio are very sensitive to uncertainties in the Standard Model prediction. However, some of the dominant m_c -dependent $\mathcal{O}(\alpha_s^2)$ corrections are currently estimated with the help of an interpolation in the charm quark mass m_c , which causes about $\pm 3\%$ uncertainty. They need to be calculated for the physical value of m_c . Here, we report on evaluation of all the necessary m_c -dependent ultraviolet counterterm contributions to the considered corrections.

DOI:10.5506/APhysPolB.46.2111

PACS numbers: 13.20.He, 12.38.Bx

1. Introduction

The decay $\bar{B} \rightarrow X_s \gamma$ is a loop-generated flavour-changing neutral current process. Some yet-undiscovered particles may appear in loop diagrams that contribute to its amplitude. Comparing the current experimental average [2] for its CP- and isospin-averaged branching ratio $\mathcal{B}_{s\gamma} = (3.43 \pm 0.22) \times 10^{-4}$ (for $E_\gamma > E_0 = 1.6$ GeV) with theoretical predictions in the Standard Model (SM) and beyond, one obtains bounds on masses and couplings of various exotic particles. For instance, the recently updated 95% C.L. bound on the charged Higgs boson mass in the Two-Higgs-Doublet Model II reads $M_{H^\pm} > 480$ GeV [3].

Calculations of $\mathcal{B}_{s\gamma}$ are conveniently carried out in the framework of a low-energy effective theory obtained after decoupling of the W -boson and all the heavier particles. The weak interactions are described then by the

* Presented at the XXXIX International Conference of Theoretical Physics “Matter to the Deepest”, Ustroń, Poland, September 13–18, 2015. Based on research performed together with M. Misiak and M. Steinhauser [1].

operators Q_i of either four-quark ($i = 1, \dots, 6$) or dipole type ($i = 7, 8$) — see, *e.g.*, Eqs. (1.5)–(1.6) of Ref. [4]. In a generic beyond-SM (BSM) model, one finds [3, 4]

$$\mathcal{B}_{s\gamma} \times 10^4 = (3.36 \pm 0.23) - 8.22 \Delta C_7 - 1.99 \Delta C_8, \quad (1)$$

where $\Delta C_{7,8}$ stand for the BSM contributions to the Wilson coefficients of the dipole operators at the renormalization scale $\mu_0 = 160$ GeV. In the above equation, only the linear terms in $\Delta C_{7,8}$ have been retained, and only $C_{7,8}$ have been allowed to contain non-negligible BSM contributions.

The uncertainty in Eq. (1) is the SM one. It has been obtained by combining in quadrature four types of uncertainties: non-perturbative (5%) [5], parametric (2%), higher-order ($\mathcal{O}(\alpha_s^3)$) perturbative (3%), and the one due to m_c -interpolation ambiguity in the perturbative $\mathcal{O}(\alpha_s^2)$ corrections (3%).

At present, the SM prediction for $\mathcal{B}_{s\gamma}$ agrees within uncertainties with the experimental average. However, more precise measurements are going to be performed at the Belle II experiment that is scheduled to begin operation in 2017. A more precise theoretical calculation is necessary to match the expected experimental accuracy. The two main issues are re-considering the estimates of non-perturbative effects, and eliminating the m_c -interpolation ambiguity in the perturbative $\mathcal{O}(\alpha_s^2)$ corrections. A calculation [1, 6] that contributes to removing the latter uncertainty is the topic of this report.

2. Interpolated parts of the $\mathcal{O}(\alpha_s^2)$ corrections

Perturbative corrections to $\mathcal{B}_{s\gamma}$ are studied by considering the partonic decay rate of the b quark into the photon and $X_s^p = s, sg, sgg, sq\bar{q}, \dots$,

$$\Gamma(b \rightarrow X_s^p \gamma) \sim \sum_{i,j=1}^8 C_i(\mu_b) C_j(\mu_b) \tilde{G}_{ij}(E_0, \mu_b), \quad (2)$$

where $C_i(\mu_b)$ denote the Wilson coefficients at the renormalization scale $\mu_b \sim m_b/2$. The quantities $\tilde{G}_{ij} = \tilde{G}_{ij}^{(0)} + \frac{\alpha_s}{4\pi} \tilde{G}_{ij}^{(1)} + \left(\frac{\alpha_s}{4\pi}\right)^2 \tilde{G}_{ij}^{(2)} + \mathcal{O}(\alpha_s^3)$ describe interferences of amplitudes generated by the operators Q_i and Q_j . The dependence of the decay rate on m_c starts to show up at $\mathcal{O}(\alpha_s)$ via matrix elements of the current–current operators Q_1 and Q_2 . At the Next-to-Next-to-Leading Order (NNLO), the most important m_c -dependent corrections are $\tilde{G}_{17}^{(2)}$ and $\tilde{G}_{27}^{(2)}$, which we shall commonly denote by $\tilde{G}_{(1,2)7}^{(2)}$.

The part of $\tilde{G}_{(1,2)7}^{(2)}$ that is proportional to the QCD β_0 -function has been known for arbitrary m_c since a long time [7, 8]. The same is true for contributions from massive quark loops on the gluon lines [9]. However, the

remaining part of $\tilde{G}_{(1,2)7}^{(2)}$ is only known in two limiting cases: for $m_c = 0$ [4] and for $m_c \gg m_b/2$ [10]. Its size at the physical value of $z = m_c^2/m_b^2$ is found via an interpolation in z [4]. The interpolation generates an uncertainty that is estimated at the $\pm 3\%$ level.

Evaluation of the missing corrections for an arbitrary value of z is underway. Here, we shall report on a calculation of the ultraviolet counterterm contributions to them. An extension of this work via inclusion of certain BSM operators is in progress [11].

Using the Cutkosky rules, we express the considered contributions in terms of two-scale (m_b and m_c) three-loop propagator integrals with unitarity cuts. They are reducible to a limited set of Master Integrals (MIs) with the help of standard integration-by-parts algorithms. The identified set of MIs turns out to be closed under differentiation with respect to z , which gives us a system of differential equations (DEs) for the MIs. This system is numerically solved along an ellipse in the complex z -plane, starting from initial conditions at large z . The initial conditions are found using asymptotic expansions, which effectively reduces our three-loop two-scale problem to a two-loop single-scale one. In the latter case, the MIs become very simple. Apart from the numerical solution, we have also evaluated all the MIs analytically as power-logarithmic expansions around $z = 0$, using a mixture of the DE and Mellin–Barnes methods.

3. Counterterm contributions to $\tilde{G}_{(1,2)7}^{(2)}$

Following Ref. [4], we consider $\tilde{G}_{(1,2)7}^{(2)}$ with no cut on the photon energy ($E_0 = 0$), and with skipped contributions from charm–quark loops on the gluon lines, together with the corresponding counterterms¹. An explicit formula for the renormalization of $\tilde{G}_{27}^{(2)}$ in such a case can be found in Eq. (2.61) of Ref. [6]. It generalizes Eq. (2.10) of Ref. [4] to arbitrary $z \neq 0$. Below, we split all the z -dependent counterterms into parts originating from the two- and three-particle cut diagrams, as indicated by the superscripts 2P and 3P, respectively:

$$\begin{aligned} \tilde{G}_{27}^{(1)\text{bare}} &= \tilde{G}_{27}^{(1)2\text{P}} + \tilde{G}_{27}^{(1)3\text{P}}, & \tilde{G}_{7(12)}^{(1)\text{bare}} &= \tilde{G}_{7(12)}^{(1)2\text{P}} + \tilde{G}_{7(12)}^{(1)3\text{P}}, \\ \tilde{G}_{27}^{(1)m} &= \tilde{G}_{27}^{(1)m,2\text{P}} + \tilde{G}_{27}^{(1)m,3\text{P}}. \end{aligned} \quad (3)$$

In the latter case, the superscript m denotes squaring one of the b -quark propagators in the diagrams which account for renormalization of m_b . The quantity $\tilde{G}_{7(12)}^{(1)\text{bare}}$ originates from an interference of the photonic dipole operator Q_7 with the evanescent operator Q_{12} . The latter operator vanishes in

¹ The charm–quark loop contributions are already known from Ref. [9].

four spacetime dimensions. All the z -dependent counterterms for Q_1 are related to those in Eq. (3) by a colour factor of $-\frac{1}{6}$, *i.e.*, $\tilde{G}_{17}^{(1)\text{bare}} = -\frac{1}{6}\tilde{G}_{27}^{(1)\text{bare}}$, $\tilde{G}_{7(11)}^{(1)\text{bare}} = -\frac{1}{6}\tilde{G}_{7(12)}^{(1)\text{bare}}$, and $\tilde{G}_{17}^{(1)m} = -\frac{1}{6}\tilde{G}_{27}^{(1)m}$.

Our results for all the quantities defined in Eq. (3) can be found in Refs. [1, 6]. As an example, let us discuss here

$$\tilde{G}_{27}^{(1)m,3P} = j_0(z) + \varepsilon j_1(z) + \mathcal{O}(\varepsilon^2) \quad (4)$$

evaluated in $D = 4 - 2\varepsilon$ dimensions. The functions $j_0(z)$ and $j_1(z)$ are displayed in Fig. 1. They exhibit logarithmic divergences when $z \rightarrow 0$. This fact manifests itself as an extra $1/\varepsilon$ pole when the corresponding interference term $\tilde{G}_{27}^{(1)m,3P}$ is calculated at $z = 0$ from the outset.

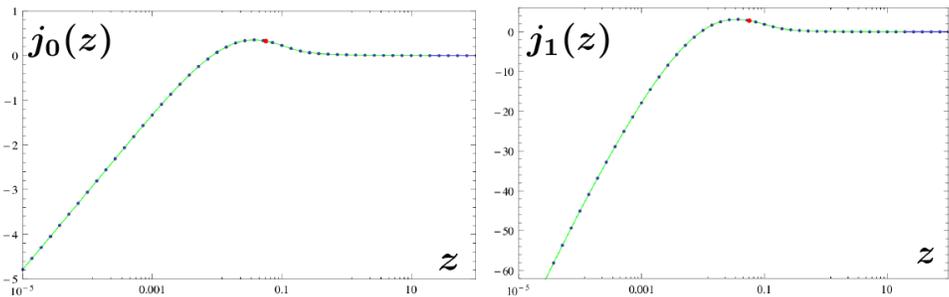


Fig. 1. The functions $j_i(z)$ defined in Eq. (4). See the text for interpretation of these curves.

In both plots, our results obtained with the help of a numerical solution to the DEs are shown by small (blue) dots. A bigger (red) dot indicates the physical point used as a central value in the phenomenological analysis of Refs. [3, 4], namely $z \simeq 0.0567$. The numerical solutions of the DEs were obtained with an initial condition at $z = 20$ evaluated using our large- z expansions. The curves describing these expansions for $z > 20$ are displayed by the solid (blue) lines. The remaining solid (green) lines show either the large- z expansions for $\frac{1}{4} < z < 20$, or the small- z expansions for $0 < z < \frac{1}{4}$. The physical $c\bar{c}$ production threshold at $z = \frac{1}{4}$ defines the convergence radii of both expansions.

As far as the three-particle-cut contribution from the evanescent operator is concerned, we have found $\tilde{G}_{7(12)}^{(1)3P} = -4\varepsilon(5 + \varepsilon)\tilde{G}_{27}^{(1)3P}$.

4. Outlook

A computation of the bare contributions to $\tilde{G}_{(1,2)7}^{(2)}$ for arbitrary z is, in principle, achievable using the same techniques as described in the previous

sections. However, the enterprise is considerably more involved. In the two-particle-cut case alone one encounters around 20000 scalar integrals, and around 500 MIs. Many yet-unknown single-scale MIs are expected to show up in the boundary conditions for the DEs. Given the complexity of the project, it is hard to predict the time scale of the bare NNLO calculation. However, one can realistically hope for its completion before Belle II starts collecting data in 2017.

5. Conclusions

At present, the experimental determination of $\mathcal{B}_{s\gamma}$ agrees very well with the SM prediction. A factor-of-two reduction of both the theoretical and experimental uncertainties is feasible in the future. On the theory side, the two main issues are re-considering estimates of the non-perturbative effects, and eliminating the m_c -interpolation in the perturbative NNLO contributions. Our current calculation has been a step towards a resolution of the latter issue.

This work has been supported in part by the National Science Centre (Poland) research project, Decision No. DEC-2014/13/B/ST2/03969, as well as by the National Centre for Physics (Pakistan).

REFERENCES

- [1] M. Misiak, A. Rehman, M. Steinhauser, in preparation.
- [2] Y. Amhis *et al.* [Heavy Flavor Averaging Group], [arXiv:1412.7515](https://arxiv.org/abs/1412.7515) [hep-ex].
- [3] M. Misiak *et al.*, *Phys. Rev. Lett.* **114**, 221801 (2015) [[arXiv:1503.01789](https://arxiv.org/abs/1503.01789)] [hep-ph].
- [4] M. Czakon *et al.*, *J. High Energy Phys.* **1504**, 168 (2015) [[arXiv:1503.01791](https://arxiv.org/abs/1503.01791)] [hep-ph].
- [5] M. Benzke, S.J. Lee, M. Neubert, G. Paz, *J. High Energy Phys.* **1008**, 099 (2010) [[arXiv:1003.5012](https://arxiv.org/abs/1003.5012)] [hep-ph].
- [6] A. Rehman, Ph.D. Thesis, University of Warsaw, 2015, <http://depotuw.ceon.pl/handle/item/1197>
- [7] Z. Ligeti, M.E. Luke, A.V. Manohar, M.B. Wise, *Phys. Rev. D* **60**, 034019 (1999) [[arXiv:hep-ph/9903305](https://arxiv.org/abs/hep-ph/9903305)].
- [8] K. Bieri, C. Greub, M. Steinhauser, *Phys. Rev. D* **67**, 114019 (2003) [[arXiv:hep-ph/0302051](https://arxiv.org/abs/hep-ph/0302051)].
- [9] R. Boughezal, M. Czakon, T. Schutzmeier, *J. High Energy Phys.* **0709**, 072 (2007) [[arXiv:0707.3090](https://arxiv.org/abs/0707.3090)] [hep-ph].
- [10] M. Misiak, M. Steinhauser, *Nucl. Phys. B* **764**, 62 (2007) [[arXiv:hep-ph/0609241](https://arxiv.org/abs/hep-ph/0609241)].
- [11] A. Rehman, in preparation.