CALCULATION OF MULTI-SCALE, MULTI-LOOP INTEGRALS USING SecDec 3*

S. Borowka

Institute for Physics, University of Zurich
Winterthurerstr. 190, 8057 Zurich, Switzerland

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In this paper, the publicly available program SecDec is briefly described. Its main virtues and new features are summarized, including suggestions for an optimal usage of the program.

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1. Introduction

In light of the increasing precision achieved by the experiments at the Large Hadron Collider, the uncertainties on theoretical predictions need to decrease at equal speed. Background processes need to be understood at the same level of accuracy as the predictions of signals, so that the Standard Model can be confirmed or deviations from it be discovered. This requires the computation of higher orders in perturbation theory. For a considerable number of processes, predictions at NNLO would be desirable for the LHC Run 2 and beyond. These involve the computation of complicated, often massive, two-loop diagrams. In particular, multi-loop multi-scale integrals are highly challenging for currently available analytical techniques. However, they are more easily accessible with numerical evaluation approaches. One public program to evaluate such diagrams is SecDec [1–4], which relies on the method of sector decomposition [5–7] to factorize UV and IR singularities. Using subtraction terms, the singularities can be extracted and their coefficients integrated numerically. SecDec is available from `http://secdec.hepforge.org/`. Other public implementations are the programs sector_decomposition [8, 9] and Fiesta [10–12].

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2. Applicability of the program SecDec

The program SecDec is designed for multifold applications. Many recent improvements were implemented for Feynman multi-loop integrals. Nonetheless, more general parametric integrals and integrals which deviate from the standard form of Feynman integrals can be computed as well.

A scalar Feynman integral G in D dimensions at L loops with N propagators, where the propagators can have arbitrary, not necessarily integer powers ν_j, has the following representation in the momentum space:

$$G = \int \prod_{l=1}^{L} d^D\kappa_l \, \frac{1}{\prod\limits_{j=1}^{N} P_j^{\nu_j}\left(\{k\},\{p\},m_j^2\right)}\,, \tag{1}$$

$$d^D\kappa_l = \left(i\pi^{\frac{D}{2}}\right)^{-1} d^D k_l\,,\; P_j\left(\{k\},\{p\},m_j^2\right) = q_j^2 - m_j^2 + i\delta\,, \tag{2}$$

where the q_j are linear combinations of external momenta p_i and loop momenta k_l. Multi-loop Feynman integrals with scalar, rank R or inverse propagator numerators can be handled by SecDec within the *loop* setup. Prefactors dependent on the dimensional regulator ε are allowed, any scale dependence must be explicitly given by the user. Internally, the user input of the aforementioned loop integral specifications is transformed into a representation in terms of Feynman parameters. After the loop momentum integration, the general expression for a scalar Feynman integral reads

$$G = \frac{(-1)^{N_\nu}}{\prod_{j=1}^{N}\Gamma(\nu_j)}\Gamma(N_\nu - LD/2)\int\limits_0^\infty \prod_{j=1}^{N} dx_j \; x_j^{\nu_j-1}\,\delta\left(1-\sum_{l=1}^{N} x_l\right)\frac{\mathcal{U}^{N_\nu-(L+1)D/2}(\vec{x})}{\mathcal{F}^{N_\nu-LD/2}(\vec{x},s_{ij})} \tag{3}$$

with $N_\nu = \sum_{j=1}^{N}\nu_j$, see *e.g.* Ref. [13] for the definition of general Feynman loop integrals of rank R. The graph polynomials $\mathcal{U}$ and $\mathcal{F}$ in Eq. (3) contain the sub-UV and IR divergences of an integral, respectively. The polynomials can be constructed from topological cuts by SecDec or from the explicit specification of propagators in momentum space in the math.m input file.

When numerical checks at intermediate stages of an analytical (multi-) loop calculation are of interest, or when the integral at hand is simply too complicated for a direct numerical evaluation, an analytical preparation can be beneficial. Then, the integral is no longer of the form of Eq. (3). Such integrals, as long as they match the following structure,

$$G_{\text{user}} = P(\varepsilon)\int\limits_0^1 \prod_{j=1}^{N} \mathrm{d}x_j \; x_j^{a_j(\varepsilon)}\,\mathcal{N}(\vec{x},s_{ij},\varepsilon)\,\mathcal{U}^{\mathrm{expoU}(\varepsilon)}(\vec{x},s_{ij})\,\mathcal{F}^{\mathrm{expoF}(\varepsilon)}(\vec{x},s_{ij}) \tag{4}$$

can be handled within the *userdefined* setup of SecDec. In Eq. (4), the function $\mathcal{N}$ may contain products of polynomials with either a direct or exponential dependence on ε. The functions $\mathcal{U}$ and $\mathcal{F}$ may have negative exponents, and powers $a_j < 0$ are allowed. The function $\mathcal{F}$ is chosen as reference for the analytical continuation of the integrand to the physical region. For further details, the reader is referred to Refs. [2, 3, 14].

More general parametric functions are handled in the *general* setup of SecDec. Examples are phase space integrals, where IR divergences are regulated dimensionally, or hypergeometric functions. This setup allows for an arbitrary number of products in the integrand, where each of them can have a negative exponent. While the poles of the integrand are factorized using sector decomposition, it is sometimes of interest to include additional ε-dependent functions, which may depend on the integration parameters but do not contain any non-factorized poles. These functions can be masked in so-called "dummy functions". The latter feature is implemented in version 3 of the program.

3. Brief summary of the operational sequence

The program SecDec processes the user input further by factorizing the poles of the integrand using either one of the two heuristic [7, 13] iterated sector decomposition strategies or one of the two deterministic [4, 15] ones. The user can choose among the four, see Ref. [4] for details. While the heuristic algorithms used to lead to more compact expressions to be integrated numerically, they are not guaranteed to terminate. The other two algorithms are based on the algebraic geometry and cannot lead to an infinite recursion by construction. While the original strategy by Kaneko and Ueda [15, 16] generated the lowest number of sectors compared to the other available strategies [5, 17–19], its resulting functions turned out to converge slower than the ones resulting from the simplest heuristic strategy. By contrast, our new composed strategy `G2` [4] outperforms all others in terms of the number of sectors created during the decomposition, see [4], and in terms of convergence during numerical integration.

After the factorization of poles, an analytical continuation of the integrand into the complex plane is performed if applicable, see Ref. [2] for details. The last algebraic step is the subtraction of the poles and expansion in ε, such that the coefficients of a Laurent series in ε are obtained in the form of parametric functions. Up to this point, kinematic invariants are left symbolic. Explicit values are only inserted at the numerical stage. This has the advantage that looping over ranges of kinematic points is facilitated as only the numerical integration step has to be performed repeatedly. On the other hand, sometimes only one or two kinematic points of highly complicated integrals are of interest for numerical checks. In this case, including values for the kinematic invariants from the start simplifies the functions which need to be integrated numerically at the end. Explicit numerical values can be specified in addition to the definition of the scalar products of the external momenta in `ScalarProductRules` in the math.m input file, *e.g.* `ScalarProductRules={..., s->5.1}`.

4. New features at one glance

The following list features improvements made to SecDec with the upgrade to version 3:

— Two additional deterministic sector decomposition algorithms based on computational geometry are implemented.

— Numerators can be given in terms of inverse propagators.

— Loop integrals with linear propagators can be handled.

— Additional ε-dependent symbolic functions can be included in parametric integrals.

— The restructured user input helps in building interfaces to reduction programs [20–23] or Loopedia [24].

— The two numerical integrators CQuad [25] and NIntegrate [26] are included in addition to the implementation of the updated version of Cuba [27–29]. CQuad is the fastest adaptive one-dimensional parameter integrator on the market and is chosen automatically by SecDec when it can be used.

— The usage of batch systems is facilitated and scans over parameter ranges are accelerated.

5. Summary

These proceedings highlight the applicability of the publicly available program SecDec. The operational sequence of the latter and its new features are summarized and suggestions for an optimal usage of the program are given.

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REFERENCES

[1] J. Carter, G. Heinrich, *Comput. Phys. Commun.* **182**, 1566 (2011) [arXiv:1011.5493 [hep-ph]].

[2] S. Borowka, J. Carter, G. Heinrich, *Comput. Phys. Commun.* **184**, 396 (2013) [arXiv:1204.4152 [hep-ph]].

[3] S. Borowka, G. Heinrich, *Comput. Phys. Commun.* **184**, 2552 (2013) [arXiv:1303.1157 [hep-ph]].

[4] S. Borowka *et al.*, *Comput. Phys. Commun.* **196**, 470 (2015) [arXiv:1502.06595 [hep-ph]].

[5] K. Hepp, *Commun. Math. Phys.* **2**, 301 (1966).

[6] M. Roth, A. Denner, *Nucl. Phys.* **B479**, 495 (1996) [arXiv:hep-ph/9605420].

[7] T. Binoth, G. Heinrich, *Nucl. Phys.* **B585**, 741 (2000) [arXiv:hep-ph/0004013].

[8] C. Bogner, S. Weinzierl, *Comput. Phys. Commun.* **178**, 596 (2008) [arXiv:0709.4092 [hep-ph]].

[9] J. Gluza, K. Kajda, T. Riemann, V. Yundin, *Eur. Phys. J.* **C71**, 1516 (2011) [arXiv:1010.1667 [hep-ph]].

[10] A.V. Smirnov, M. Tentyukov, *Comput. Phys. Commun.* **180**, 735 (2009) [arXiv:0807.4129 [hep-ph]].

[11] A.V. Smirnov, V.A. Smirnov, M. Tentyukov, *Comput. Phys. Commun.* **182**, 790 (2011) [arXiv:0912.0158 [hep-ph]].

[12] A.V. Smirnov, *Comput. Phys. Commun.* **185**, 2090 (2014) [arXiv:1312.3186 [hep-ph]].

[13] G. Heinrich, *Int. J. Mod. Phys. A* **23**, 1457 (2008) [arXiv:0803.4177 [hep-ph]].

[14] S.C. Borowka, arXiv:1410.7939 [hep-ph].

[15] T. Kaneko, T. Ueda, *Comput. Phys. Commun.* **181**, 1352 (2010) [arXiv:0908.2897 [hep-ph]].

[16] T. Kaneko, T. Ueda, *PoS* **ACAT2010**, 082 (2010) [arXiv:1004.5490 [hep-ph]].

[17] C. Bogner, S. Weinzierl, *J. Math. Phys.* **50**, 042302 (2009) [arXiv:0711.4863 [hep-th]].

[18] C. Bogner, S. Weinzierl, *Nucl. Phys. Proc. Suppl.* **183**, 256 (2008) [arXiv:0806.4307 [hep-ph]].

[19] A.V. Smirnov, V.A. Smirnov, *J. High Energy Phys.* **0905**, 004 (2009) [arXiv:0812.4700 [hep-ph]].

[20] C. Anastasiou, A. Lazopoulos, *J. High Energy Phys.* **0407**, 046 (2004) [hep-ph/0404258].

[21] A.V. Smirnov, *J. High Energy Phys.* **0810**, 107 (2008) [arXiv:0807.3243 [hep-ph]].

[22] A. von Manteuffel, C. Studerus, arXiv:1201.4330 [hep-ph].

[23] R.N. Lee, *J. Phys. Conf. Ser.* **523**, 012059 (2014) [arXiv:1310.1145 [hep-ph]].

[24] V. Papara, *Acta Phys. Pol. B* **46**, 2149 (2015), this issue.

[25] P. Gonnet, *ACM Trans. Math. Softw.* **37**, 26 (2010).

[26] Wolfram Research, Inc., Mathematica, Version 10.3, Champaign, IL (2015).

[27] T. Hahn, *Comput. Phys. Commun.* **168**, 78 (2005) [arXiv:hep-ph/0404043].

[28] S. Agrawal, T. Hahn, E. Mirabella, *J. Phys. Conf. Ser.* **368**, 012054 (2012) [arXiv:1112.0124 [hep-ph]].

[29] T. Hahn, *J. Phys. Conf. Ser.* **608**, 012066 (2015) [arXiv:1408.6373 [physics.comp-ph]].