

RADIUS STABILIZATION AND DARK MATTER WITH A BULK HIGGS IN WARPED EXTRA DIMENSION*

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We employ an $SU(2)$ bulk Higgs doublet as the stabilization field in the Randall–Sundrum model with appropriate bulk and brane-localized potentials. The gauge hierarchy problem can be solved for an exponentially IR-localized Higgs background field with mild values of fundamental parameters of the 5D theory. We consider an IR–UV–IR background geometry with the 5D SM fields in the bulk such that all the fields have even and odd towers of KK-modes. The zero-mode 4D effective theory contains all the SM fields plus a stable scalar, which serves as a dark matter candidate.

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1. Introduction

The 5D warped model of Randall and Sundrum (RS) with two D3-branes (RS1) provides an elegant solution to the *hierarchy problem* [1]. The two D3-branes are localized on the fixed points of the orbifold S_1/\mathbb{Z}_2 , a “UV-brane” at $y = 0$ and an “IR-brane” at $y = L$ — the UV–IR model, see Fig. 1. The solution for the RS geometry is [1]

$$ds^2 = e^{-2k|y|} \eta_{\mu\nu} dx^\mu dx^\nu + dy^2, \quad (1)$$

where k is a constant of the order of 5D Planck mass M_* . Randall and Sundrum showed that if the 5D theory involves only one mass scale M_*

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then, due to the presence of non-trivial warping along the extra-dimension, the effective mass scale on the IR-brane is rescaled to $m_{KK} \equiv ke^{-kL} \sim \mathcal{O}[\text{TeV}]$ for $kL \sim \mathcal{O}(37)$. However, the RS proposal [1] lacked the mechanism of stabilizing the separation L between the two branes. A stabilization mechanism for the RS1 geometry was proposed by Goldberger and Wise (GW) [2]; which employs a real scalar field in the bulk of RS geometry with localized potentials on both branes.

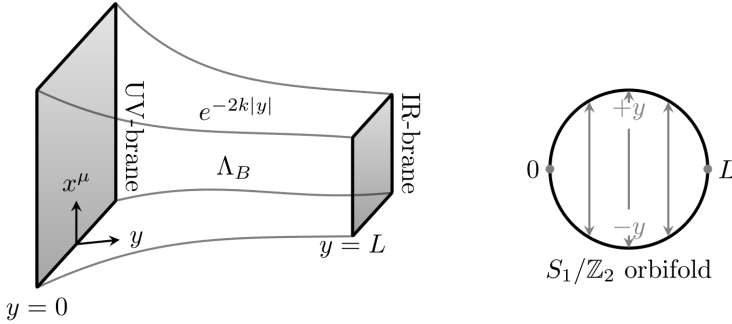


Fig. 1. Cartoon of RS1 geometry.

The aim of this work is twofold: (i) to analyse the GW stabilization mechanism with an $SU(2)$ bulk Higgs doublet¹ and, (ii) to investigate the lowest odd KK-mode of the bulk Higgs in a 5D \mathbb{Z}_2 -symmetric model with *warped KK-parity* [4, 5]. In order to achieve the first goal, we consider an $SU(2)$ Higgs doublet in the bulk of RS1 geometry in Sec. 2 and use the *superpotential method* to solve the Higgs-gravity coupled Einstein equations [4, 6]. The second goal is achieved in Sec. 3, where we introduce the IR–UV–IR model with a warped KK-parity and consider the 5D SM bosonic sector in the bulk of this model. In the zero-mode effective theory, we have a stable scalar, which is the dark matter candidate, along with all the SM fields.

2. Radius stabilization with a bulk Higgs doublet

We employ an $SU(2)$ Higgs doublet H in the bulk of a 5D geometric interval $y \in [0, L]$ with the following scalar-gravity action,

$$S = \int d^5x \sqrt{-g} \left\{ 2M_*^3 R - |D_M H|^2 - V(H) - \sum_i V_i(H) \delta(y - y_i) \right\}, \quad (2)$$

¹ Recently, Geller *et al.* [3] also treated an $SU(2)$ bulk Higgs doublet as a stabilization field, however, they assumed a weak backreaction and have not solved the full Higgs-gravity coupled Einstein equations.

where $V(H)$ and $V_i(H)$ (for $i = 1, 2$) are the bulk and brane potentials, respectively, whereas $y_{1(2)} \equiv 0(L)$ are the UV(IR)-brane locations. We employ the following metric ansatz

$$ds^2 = e^{2\sigma(y)} \eta_{\mu\nu} dx^\mu dx^\nu + dy^2, \quad (3)$$

where $\sigma(y)$ is a y -dependent warp-function and $\eta_{\mu\nu} \equiv \text{diag}(-1, 1, 1, 1)$. We write the y -dependent vacuum expectation value (vev) of the Higgs field as

$$\langle H \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ h_v(y) \end{pmatrix}. \quad (4)$$

The background scalar-gravity coupled Einstein equations following from the action (2) and the metric ansatz (3) can be solved analytically by using the *superpotential method* [7]. We assume the scalar potential $V(h_v)$ can be written in the following form:

$$V(h_v) = \frac{1}{8} \left(\frac{\partial \mathcal{W}(h_v)}{\partial h_v} \right)^2 - \frac{1}{24M_*^3} \mathcal{W}(h_v)^2, \quad (5)$$

where the superpotential $\mathcal{W}(h_v)$ satisfies

$$\begin{aligned} h'_v &= \frac{1}{2} \frac{\partial \mathcal{W}(h_v)}{\partial h_v}, & \sigma' &= -\frac{\mathcal{W}(h_v)}{24M_*^3}, \\ \frac{1}{2} \mathcal{W}(h_v) \Big|_{y_i-\epsilon}^{y_i+\epsilon} &= V_i(h_v) \Big|_{h_v=h_v(y_i)}, & \frac{1}{2} \frac{\partial \mathcal{W}(h_v)}{\partial h_v} \Big|_{y_i-\epsilon}^{y_i+\epsilon} &= \frac{\partial V_i(h_v)}{\partial h_v} \Big|_{h_v=h_v(y_i)}, \end{aligned} \quad (6)$$

(7)

where a *prime* denotes a derivative w.r.t. the y -coordinate. The last line above follows from the *jump* conditions across the branes.

We consider the following form of superpotential $\mathcal{W}(h_v)$

$$\mathcal{W}(h_v) = 24M_*^3 k + \Delta k h_v^2, \quad \Delta \equiv 2 + \sqrt{4 + \mu_B^2/k^2}, \quad (8)$$

where Δ parametrises the bulk Higgs mass μ_B and in the dual CFT it corresponds to the scaling dimension of the composite operator \mathcal{O}_H . The scalar potential $V(h_v)$ following from Eq. (5) takes the form

$$V(h_v) = -24M_*^3 k^2 + \frac{1}{2} \mu_B^2 h_v^2 - \frac{\Delta^2 k^2}{24M_*^3} h_v^4. \quad (9)$$

We employ the following forms of the brane-localized potentials,

$$V_{1(2)}(h_v) = \pm \mathcal{W}(h_v) + \frac{\lambda_{1(2)}}{4k^2} \left(h_v^2 - v_{1(2)}^2 \right)^2, \quad (10)$$

where $v_{1(2)}$ and $\lambda_{1(2)}$ are the values of background vevs and quartic couplings at the UV(IR) branes, respectively. The background vev $h_v(y)$ and warp function $\sigma(y)$ are obtained by integrating Eq. (6) (see Fig. 2)

$$h_v(y) = v_2 e^{\Delta k(|y|-L)}, \quad \sigma(y) = -k|y| - \frac{v_2^2 e^{-2\Delta kL}}{48M_*^3} [e^{2\Delta k|y|} - 1]. \quad (11)$$

The background vev profile $h_v(y)$ satisfies following normalization condition:

$$\int_0^L dy e^{2\sigma(y)} h_v^2(y) = v_{\text{SM}}^2, \quad (12)$$

where $v_{\text{SM}} \simeq 246$ GeV is the SM Higgs vev. The brane separation L is fixed by Eq. (11) as

$$kL = \frac{1}{\Delta} \ln \left(\frac{v_2}{v_1} \right). \quad (13)$$

For $\Delta \geq 2$, in order to solve the gauge hierarchy problem one needs to have $kL \simeq \mathcal{O}(37)$, which implies that $v_1 \ll v_2$ ². For instance, Eq. (12) gives

$$v_{\text{SM}} \simeq \frac{v_2 e^{-kL}}{\sqrt{k(\Delta-1)}} \sim \mathcal{O} [\text{TeV}], \quad (14)$$

for $v_2 \sim \mathcal{O}(M_{\text{Pl}})$, $\Delta \geq 2$ and $kL \sim \mathcal{O}(37)$.

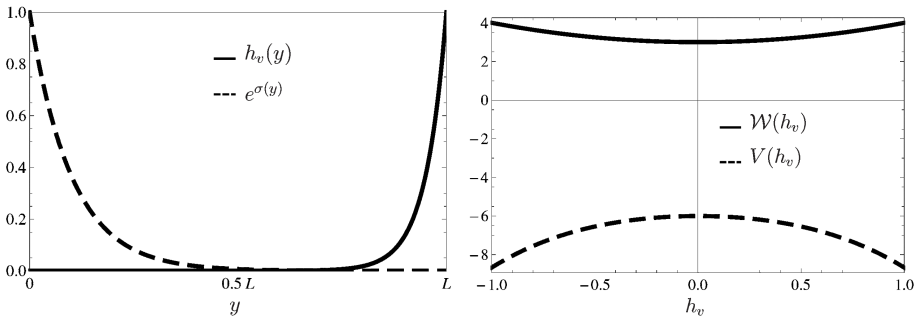


Fig. 2. The left graph illustrates shapes of the background vev $h_v(y)$ and the warp factor $e^{\sigma(y)}$ as a function of y , whereas, the right graph shows shapes of superpotential $\mathcal{W}(h_v)$ and the bulk potential $V(h_v)$ as a function of background vev h_v . The parameter choice adopted for the graphs is: $\Delta = 2$ and $k = v_2 = M_*$.

² Note that the requirement $v_1 \ll v_2$ is not an extra fine-tuning other than the one required to set 4D cosmological constant zero, see also [3].

3. Warped Higgs dark matter

In this section, we consider a \mathbb{Z}_2 -symmetric, IR–UV–IR, warped extra dimensional model presented in Ref. [4]. The geometric \mathbb{Z}_2 -symmetry leads to a *warped KK-parity*, under which all the bulk fields have towers of even and odd KK-modes. The lowest odd KK-particle (LKP) is stable and hence can serve as a dark matter candidate.

As it is shown in Ref. [5], the “IR–UV–IR” geometry is equivalent to a single copy of “UV–IR” RS1 geometry, where all the bulk fields satisfy Neumann (or mixed) and Dirichlet boundary conditions (b.c.) at $y = 0$ corresponding to the even and odd fields. Hence, we discuss merely UV–IR geometry with two sets of b.c. at $y = 0$. We consider the bosonic sector of the SM gauge group in the bulk of RS1 geometry³

$$S = - \int d^5x \sqrt{-g} \left\{ \frac{1}{4} F_{MN}^a F^{aMN} + \frac{1}{4} B_{MN} B^{MN} + |D_M H|^2 + \mu_B^2 |H|^2 + V_1(H) \delta(y) + V_2(H) \delta(y - L) \right\}, \quad (15)$$

where the brane-localized potentials are

$$V_1(H) = \frac{m_{\text{UV}}^2}{k} |H|^2, \quad V_2(H) = -\frac{m_{\text{IR}}^2}{k} |H|^2 + \frac{\lambda_{\text{IR}}}{k^2} |H|^4, \quad (16)$$

and $F_{MN}^a [B_{MN}]$ is the 5D field strength tensor for $\text{SU}(2)[\text{U}(1)_Y]$.

It is a straightforward exercise to obtain an effective 4D theory by integrating the 5th dimension after KK-decomposition of the bulk fields. After obtaining the 4D effective theory, one can get a low-energy 4D effective theory by assuming the KK-scale, $m_{KK} \equiv k e^{-kL} \sim \mathcal{O}(\text{few})$ TeV, is much heavier than all the zero-modes of the theory [4, 5]. It turns out, after applying the appropriate b.c., that the odd zero-mode wave functions for the gauge fields (the same for the odd Goldstone zero-modes) are zero. Hence, the odd zero-mode gauge fields and the odd Goldstone modes of the zero-mode odd Higgs doublet are not present in the effective theory. With these observations, we can write the 4D effective Lagrangian for the zero-modes as

³ In this section, we consider $\sigma(y) \simeq -k|y|$, *i.e.* the weak backreaction. Moreover, we neglect the quartic terms in the bulk and UV-brane Higgs potentials, as these terms are suppressed, and introduce some convenient notations for brane potential parameters.

$$\begin{aligned}
\mathcal{L}_{\text{eff}}^{4\text{D}} = & -\frac{1}{2}\mathcal{W}_{\mu\nu}^+\mathcal{W}^{-\mu\nu} - \frac{1}{4}\mathcal{Z}_{\mu\nu}\mathcal{Z}^{\mu\nu} - \frac{1}{4}\mathcal{F}_{\mu\nu}\mathcal{F}^{\mu\nu} - m_W^2 W_\mu^+ W^{-\mu} - \frac{1}{2}m_Z^2 Z_\mu Z^\mu \\
& -\frac{1}{2}\partial_\mu h\partial^\mu h - \frac{1}{2}m_h^2 h^2 - \frac{1}{2}\partial_\mu\chi\partial^\mu\chi - \frac{1}{2}m_\chi^2\chi^2 - \lambda v h^3 - \frac{\lambda}{4}h^4 \\
& -\frac{\lambda}{4}\chi^4 - 3\lambda v h\chi^2 - \frac{3}{2}\lambda h^2\chi^2 - \frac{g_4^2}{2}vW_\mu^+W^{-\mu}h - \frac{g_4^2}{4}W_\mu^+W^{-\mu}(h^2+\chi^2) \\
& -\frac{1}{4}(g_4^2+g_4'^2)vhZ_\mu Z^\mu - \frac{1}{8}(g_4^2+g_4'^2)Z_\mu Z^\mu(h^2+\chi^2), \quad (17)
\end{aligned}$$

where $\mathcal{V}_{\mu\nu}$ is the field strength tensor of the SM gauge bosons V_μ . Notice that the above zero-mode 4D effective Lagrangian contains all the SM fields (including the Higgs boson h) plus a scalar χ , component of the odd zero-mode Higgs field. The above Lagrangian has a \mathbb{Z}_2 -symmetry⁴, under which all the SM fields are even, whereas the scalar χ is odd, *i.e.* $\chi \rightarrow -\chi$. Hence, the scalar χ is our dark matter candidate. The masses of the SM Higgs boson h , dark-Higgs χ and gauge bosons are

$$m_h^2 = 2\mu^2, \quad m_\chi^2 = m_h^2 + \delta m^2, \quad m_W^2 = \frac{g_4^2 m_Z^2}{g_4^2 + g_4'^2} = \frac{1}{4}g_4^2 \frac{\mu^2}{\lambda}, \quad (18)$$

where $m_h = 125$ GeV and $\delta m^2 \equiv 3m_{\text{KK}}^2 m_t^2 / (4\pi^2 v_{\text{SM}}^2)$ is the shift in the dark-Higgs mass square due to the quantum corrections (quadratically divergent) below the cut-off scale m_{KK} [4]. The parameters of the above effective Lagrangian $\mu, v_{\text{SM}}, \lambda, g_4$ and g_4' are determined in terms of the 5D fundamental parameters of the theory, which are then fixed in such a way that we recover all the SM values of these parameters within our low-energy effective theory.

Now, we calculate the annihilation cross section and the relic abundance of the dark matter. The Feynman diagrams contributing to dark matter annihilation are shown in Fig. 3.

In Fig. 4 (left panel), we have plotted the annihilation cross section for the contributing channels as a function of m_χ . As shown in the graph, the total cross section is dominated by WW and ZZ final states. The main contributions for these final states are those generated by contact interactions $\chi\chi WW(ZZ)$, whereas, all the other final states that include the Higgs boson h or the top quark is very small in comparison to $\chi\chi \rightarrow WW(ZZ)$. The dark matter relic abundance $\Omega_\chi h^2$ is shown in Fig. 4 (right panel). We observe that $\Omega_\chi h^2 \lesssim 10^{-4}$ once the electroweak precision bound on the KK-mass scale m_{KK} is imposed [4].

⁴ This \mathbb{Z}_2 symmetry is a manifestation of the warped KK-parity, see *e.g.* [4, 5].

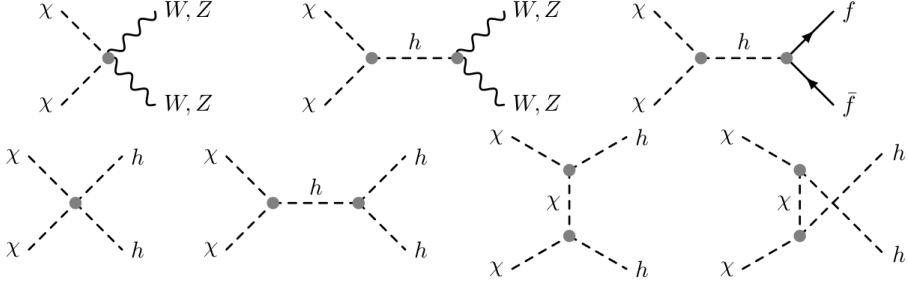
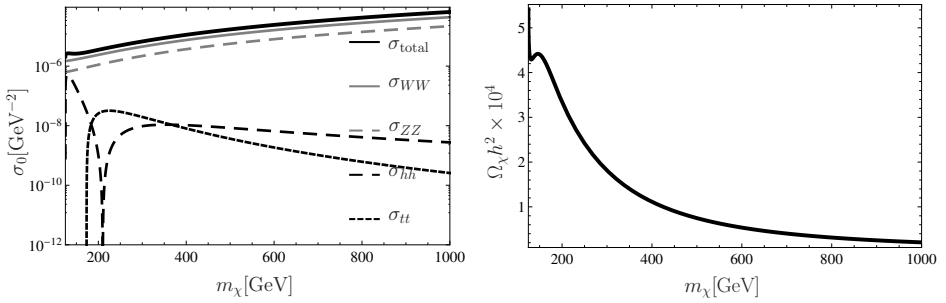


Fig. 3. Dark matter annihilation diagrams.


 Fig. 4. The above graphs show the annihilation cross-section σ_0 for different final states (left) and the χ abundance $\Omega_\chi h^2 \times 10^4$ (right) as a function of dark matter mass m_χ .

4. Conclusions

In this work, we have investigated two implications of the bulk Higgs doublet in warped extra dimensions.

- We present the $SU(2)$ bulk Higgs doublet in warped extra dimension as the GW stabilizing field, where the superpotential method is employed to solve the Higgs-gravity coupled Einstein equations. We show that an IR-localized Higgs background can fix the size of extra dimension such that it solves the hierarchy problem.
- We consider a geometric \mathbb{Z}_2 symmetric model of warped extra dimension, the IR–UV–IR, which has a warped KK-parity. Within this model, we investigated the zero-mode effective theory for the bulk Standard Model (SM) bosonic sector. This effective theory contains all the SM fields and a stable scalar particle which is a dark matter candidate. It is found that this dark matter candidate can provide only a small fraction of the observed dark matter abundance.

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