HIGH-PRECISION LUMINOSITY AT e^+e^- COLLIDERS: THEORY STATUS AND CHALLENGES*

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We review the state of the art of the theory predictions available for the Bhabha process in QED, paying particular attention to the implementation of the theoretical ingredients into Monte Carlo generators used for high-precision luminosity measurements at present and future e^+e^- colliders. The challenges posed by per mille normalization at present flavor factories, as well as by the precision tag at the 10^{-4} level for future e^+e^- facilities, are also discussed.

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1. Introduction

In accelerator physics, the luminosity is the machine parameter from which any cross section can be derived from the relation $\sigma = N_{\rm obs}/\mathcal{L}$, where $N_{\rm obs}$ is the observed number of events of the process under consideration and $\mathcal{L}$ is the (instantaneous) luminosity.

Since, in general, the luminosity can be only poorly determined in terms of its dependence on the accelerator parameters, it is much more convenient, especially for precision measurements at e^+e^- colliders, to measure the luminosity by inverting the above relation and using an appropriate reference process to obtain the luminosity as $\mathcal{L} = N_{\rm obs}/\sigma_{\rm theory}$. In this way, the luminosity is measured by counting the number of events of the chosen reference process and normalizing this number to the corresponding theoretical cross section. It follows that the normalization process must be a process with clean topology and high statistics, and calculable with high accuracy, in

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order to maintain the total luminosity uncertainty as small as possible. At e^+e^- colliders, the best luminosity monitoring process with these features is Bhabha scattering.

At present e^+e^- accelerators with c.m. energy between 1 and 10 GeV (flavor factories) [1], the final-state electrons and positrons are detected at large polar angles with the same detectors used for the measurement of other processes, due to the absence of specific luminosity counters. At LEP during the 1990s, the luminosity was measured in terms of small-angle Bhabha scattering [2, 3] by means of dedicated, high-quality luminometers put in the very forward region close to the beams. Also for future high-energy facilities at the TeV scale, such as TLEP [4] or ILC [5], it is proposed to use the small-angle Bhabha (SABH) process to monitor the luminosity. Both for large-angle Bhabha scattering (LABH) at low energies and SABH at high energies, the process is only marginally affected by Z-exchange contributions and behaves as a pure QED process, dominated by t-channel photon exchange. This makes the process ideal for luminosity measurements.

2. The quest for luminosity precision

There are well-known examples of physics measurements which stress the rôle played by a precise luminosity measurement at e^+e^- colliders.

A first example comes from flavor factories, where the luminosity is measured with a quite good precision, between a few per mille and one per cent, depending on the experiments. One of the main goals of meson factories is the measurement of the hadronic cross section at low energies, with an accuracy at the per cent level or better. On the other hand, the hadronic cross section is a crucial experimental ingredient for the calculation, via dispersion relations, of the leading order hadronic contribution to the $g-2$ and of the light quark contribution to the running of α. At present, the experimental value and the theoretical prediction for the muon anomaly differ by more than 3σ, with a theory error dominated by the leading order hadronic contribution. Since the luminosity is a crucial parameter for the measurement of the low-energy hadronic cross section, this example emphasizes the importance of a precise luminosity measurement for precision tests of the SM or possible hints of New Physics.

A similar example comes from LEP, where the luminosity was measured with sub-per mille precision and the Bhabha cross section computed with comparable uncertainty. One of the greatest achievements of LEP is the measurement of the number of light neutrinos which, in turn, depends on the measurement of the hadronic peak cross section. Again, this measurement crucially depends on luminosity which, therefore, plays a rôle in the 2σ disagreement between the experimental value and the model expectation, with a luminosity error dominated by the theory contribution.

3. Luminosity, radiative corrections and generators

As remarked in Section 1, precision luminosity measurements require very precise calculations, including all the relevant radiative corrections, which stem from the QED sector of the SM.

QED corrections to Bhabha scattering are enhanced by large logarithms due to photon collinear emission of the kind of $\ln(Q^2/m_e^2)$, where Q^2 is the relevant squared energy scale and m_e the electron mass. These logarithms enter order-by-order the perturbative expansion with their canonical coefficients at NLO, NNLO and in higher-order (h.o.) contributions, and with a typical size that lowers the convergence of the perturbative expansion, being of the order of 15 for LABH at meson factories and of 17 for SABH at LEP and TLEP/ILC at a c.m. energy around the Z resonance.

The precise calculations for the Bhabha scattering cross section need to be available in the form of Monte Carlo (MC) generators, to allow for realistic simulations of the process and a data–theory comparison under complex event selection criteria. To meet the precision requirements, the typical theoretical ingredients implemented in the MC codes are the complete set of QED corrections at NLO and the leading logarithmic (LL) contributions due to multiple photon emission, which are taken into account using methods like collinear Structure Functions, QED Parton Shower (PS) or YFS exclusive exponentiation. These corrections are supplemented by the effect of vacuum polarization and, for SABH at high energies, the contribution of Z exchange.

It is worth noting that, although none of the present generators includes the full set of NNLO corrections, the bulk of the most important sub-leading $O(\alpha^2)$ corrections, *i.e.* $O(\alpha^2 L)$ photonic corrections enhanced by infrared logarithms, is effectively incorporated by means of factorization of NLO contributions with LL corrections. The partial inclusion of these corrections sets the theoretical accuracy of the generators, as discussed in the following.

4. Luminosity at flavor factories

At flavor factories, the luminosity is measured with good precision, mainly using LABH scattering but also two-photon production as a cross check or to obtain the luminosity as an average of the measurements obtained with the two processes. Typically, two independent generators are used to avoid loss of precision or introduce a bias in the measurement.

The reference code adopted by most of the experimental collaborations is the generator `BabaYaga` [6, 7]. In its most precise version, `BabaYaga@NLO`, the generator includes the exact NLO corrections matched to a QED PS for the simulation of exclusive multiple photon emission and has an estimated accuracy of 0.1%. Other programs used at meson factories for simulations of the LABH process with a formulation and precision similar to that of `BabaYaga@NLO` are `BHWIDE` [8], developed during the LEP time, and `MCGPJ` [9], a code realized by a Dubna–Novosibirsk collaboration.

Because of the precision requirements, a relevant question in the context of luminosity measurements is related to the accuracy of the theoretical calculations and corresponding codes. According to a universally accepted classification introduced in the LEP era, the generators are affected by two sources of uncertainties: *(i)* a technical precision, which stems from possible bugs, imperfections or approximations in the numerical algorithms; *(ii)* a theoretical precision, coming from physics sources. Concerning the main sources of uncertainties affecting the theoretical precision, one of them is related to the hadronic contribution to the vacuum polarization, which is treated using dispersion relations and is therefore parametric, driven by the experimental uncertainty of the hadron cross section measurements. The second source of uncertainty is purely perturbative, due to the incomplete inclusion of QED corrections at NNLO. Fortunately, all the pieces of the Bhabha cross section at NNLO in QED have been computed over the last decade or so (see *e.g.* Ref. [1]) and, therefore, these calculations represent an important benchmark to assess the theoretical precision of the luminosity MC tools.

Within the Working Group (WG) on Radiative Corrections and MC for low energies [1], a particular effort was devoted to reach a reliable estimate of the theoretical uncertainty of the luminosity cross section calculation. Following similar work done during the LEP workshops in the 1990s, detailed comparisons between the predictions of the available NNLO calculations and the corresponding approximations present in `BabaYaga@NLO` led to the estimate of the total theoretical uncertainty summarized in Table I. It quotes

TABLE I

The total theoretical uncertainty of the LABH scattering cross section at flavor factories.

Source of unc. [%]	1–2 GeV	BESIII	BaBar/Belle
$\|\delta_{\rm VP}\|$ [Jegerlehner]	—	0.01	0.03
$\|\delta_{\rm VP}\|$ [HMNT]	0.02	0.01	0.02
$\|\delta^{\alpha^2}_{\rm photonic}\|$	0.02	0.02	0.02
$\|\delta^{\alpha^2}_{\rm pairs}\|$	0.03	0.02	$0.03 \div 0.07$
$\|\delta^{\alpha^2}_{\rm SV,H}\|$	0.05/0.03	0.05/0.03	0.05/0.03
$\|\delta^{\alpha^2}_{\rm HH}\|$	—	—	—
$\|\delta_{\rm total}\|$	0.07/0.05	0.06/0.04	$\sim 0.07 \div 0.09$

the official WG evaluation updated by the conclusions of the work on leptonic and hadronic pair corrections of Ref. [10] and by a less conservative estimate of the soft plus virtual (SV) corrections to hard bremsstrahlung according to Refs. [11, 12].

Note that the estimate of the total theoretical uncertainty in the luminosity measurement at flavor factories, between 0.05% and 0.1%, is comparable to that achieved at LEP in the 1990s. However, it slightly deteriorates in the proximity of the very narrow resonances because of the uncertainty induced by the hadronic contribution to the vacuum polarization.

5. Luminosity at LEP and future e^+e^- colliders

At future high-energy e^+e^- accelerators, the luminosity will be monitored using SABH scattering, with an expected experimental precision similar to that obtained at LEP, *i.e.* a few times 10^{-4}, or even better for data taking at the Z peak. Hence, for these facilities LEP represents the benchmark for future theoretical work, with a challenging target accuracy at the 10^{-4} level.

At the end of LEP experimentation, the total theoretical uncertainty associated to the luminosity cross section calculation was quoted in the range of 0.05–0.06%. The leading tool which allowed to meet the goal was `BHLUMI` [13], a generator based on the matching of complete $\mathcal{O}(\alpha)$ corrections with YFS exclusive exponentiation and used by all the four experiments during the whole LEP period. The main steps in the SABH theory error reduction at LEP, excluding the components due to technical precision, are summarized in Table II.

TABLE II

Main steps in the reduction of the total theoretical uncertainty of the SABH scattering cross section at LEP.

Type of correction/uncert.	Ref. [15]	Refs. [3, 14]	Ref. [11]	Ref. [19]
Missing photonic $\mathcal{O}(\alpha^2 L)$	0.15%	0.10%	0.027%	0.027%
Missing photonic $\mathcal{O}(\alpha^3 L^3)$	0.008%	0.015%	0.015%	0.015%
Vacuum polarization	0.04%	0.04%	0.04%	0.04%
Light pairs	0.03%	0.03%	0.03%	0.01%
Z-exchange	0.015%	0.015%	0.015%	0.015%
Total	0.16%	0.11%	0.061%	0.054%

The first major contribution was obtained during the LEP2 workshop at CERN in 1995 [3, 14], which allowed to reduce the uncertainty associated to the sub-leading $\mathcal{O}(\alpha^2)$ photonic corrections from 0.15% [15] to 0.1%, through an extensive work of comparison between the results of various independent codes [16–18], differing in the treatment of $\mathcal{O}(\alpha^2 L)$ contributions, and the BHLUMI predictions. A second milestone was the work of Ref. [11], where a new analysis of $\mathcal{O}(\alpha^2)$ sub-leading contributions based on the comparison between the BHLUMI approximations with the NNLO calculation of the SV corrections to hard bremsstrahlung and of two hard photon emissions, allowed to reduce this uncertainty by a large factor, down to the 0.03% level. By combining the latter error estimate with the other sources of approximation, one gets the total 0.061% uncertainty, which represents the ultimate precision of BHLUMI and was quoted by ALEPH, DELPHI and L3 at the end of LEP.

On the other hand, OPAL cited a more aggressive 0.054% theoretical accuracy, using the results of Ref. [19], where the exact NNLO contribution due to virtual and real lepton pair corrections was computed. This led to a reduction of the light pair correction from 0.03% to 0.01%, and the conclusions of these study were used by OPAL to apply a correction factor to the BHLUMI predictions.

In view of the experiments at future facilities, the SABH theoretical uncertainty can be presumably reduced by a factor of two or three by the inclusion of missing sub-leading photonic and light pair corrections in BHLUMI. Also the inclusion of new, updated parameterizations with smaller hadronic uncertainties to the vacuum polarization contribution will be required. A further interesting option would be the realization of new Bhabha generators including the full set of NNLO corrections matched to h.o. effects. However, in spite of these possible improvements, the theoretical precision will be in any case limited by the uncertainty due to vacuum polarization, as presently available parameterizations give rise to an uncertainty between 0.02% and 0.03% for the SABH cross section, depending on the considered c.m. energy of future accelerators.

6. Summary

High-precision luminosity measurements at e^+e^- colliders require precision calculations of the Bhabha process encoded into accurate MC generators. The accuracy of the presently available predictions for LABH at flavor factories and SABH at LEP is at the sub-per mille level and robust.

For next-generation accelerators (TLEP and ILC), the LEP theoretical uncertainty can be presumably reduced by a factor of two or three with improvements in the existing codes or the development of new generators including NNLO plus h.o. corrections. New tests of the physical (in particular, of the impact of pure weak corrections) plus technical precision will be also required. However, for the challenging 10^{-4} precision, the accuracy is presently limited by the hadronic uncertainty to the vacuum polarization, which will be only reduced by new measurements of the hadron production cross section at low energies.

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