S-MATRIX APPROACH TO THE Z RESONANCE*

Tord Riemann

Königs Wusterhausen, Germany
tordriemann@gmail.com

The proposed e^+e^--collider FCC-ee aims at an unprecedented accuracy for e^+e^- collisions into fermion pairs at the Z peak, based on about 10^{13} events. The S-matrix approach to the Z-boson line shape allows the model-independent quantitative description of the reaction $e^+e^- \to \bar{f}f$ around the Z peak in terms of few parameters, among them the mass M_Z and width Γ_Z of the Z boson. While weak and strong corrections remain "black", a careful theoretical description of the photonic interactions is mandatory. I introduce the method and describe applications and the analysis tool SMATASY/ZFITTER.

1. Introduction

The FCC-ee project aims at about 10^{13} events in the reaction

$$e^+e^- \to (\gamma, Z) \to f^+f^- \ + \ (n\,\gamma) \tag{1}$$

at the Z-boson resonance peak. The analysis in the Standard Model (SM) will deserve 2-loop accuracy; see [4] and many references therein. A promising model-independent alternative is the S-matrix approach [5–8], originally developed for the analysis of LEP data in 1991/1992 and first applied in 1992 by the L3 Collaboration [9, 10]. Later applications at LEP 1, Tristan and LEP 2 are described in [11–16]. The main problem for a model-independent approach is accuracy. Even if an ansatz is improper, the fit results may

---

* The summary of the material presented at the XXXIX International Conference of Theoretical Physics "Matter to the Deepest", Ustroń, Poland, September 13–18, 2015; also at the FCC-ee meeting in February 2015 at Pisa [1], at the CALC Conference in July 2015 at JINR, Dubna [2], and at the HEPKIT Workshop in October 2015 at the KIT, Karlsruhe [3].

nevertheless look precise: Compare the two ansatzes for the Z-boson propagator

$$\frac{1}{\left[s - M_Z^2 + iM_Z\,\Gamma_Z(s)\right]} \quad versus \quad \frac{1}{\left[s - \bar{M}_Z^2 + i\bar{M}_Z\,\bar{\Gamma}_Z\right]} . \tag{2}$$

To a very good accuracy, it holds: $\Gamma_Z(s) \approx s/M_Z^2 \times \Gamma_Z$. The different propagators lead, for one and the same given set of data, to a relative shift of the fitted Z mass [17]: $\bar{M}_Z \approx M_Z - 1/2\Gamma_Z^2/M_Z \approx M_Z - 34$ MeV. It is important to note that the "wrong fit" does not have enlarged error bars.

2. Total cross sections

The ansatz for the scattering amplitude in the complex energy plane comprises, in the case of massless fermion pair production, four non-interfering helicity matrix elements

$$\mathcal{M}^i(s) = \frac{R_\gamma^i}{s} + \frac{R_Z^i}{s - s_Z} + F^i(s), \qquad i = 1, \ldots, 4. \tag{3}$$

The pole terms have complex weights R_Z and R_γ, the latter corresponding to the photon, and the background $F(s)$ is an analytic function

$$F^i(s) = \sum_{n=0}^{\infty} F_n^i (s/s_0 - 1)^n . \tag{4}$$

Beware: Eq. (3) contains the photon pole R_γ^i/s, where R_γ^i will be assumed to have a (known) s-dependence. A second pole besides the Z is mathematically not consistent, because a Laurent series has one single pole only. In fact, one has to understand the term $R_\gamma^i(s)/s$ as a part of the background term $F(s)$. Such rewritten, it reads in fact

$$\frac{R_\gamma^i(s)}{s} = \sum_{n=0}^{\infty} R_n^i (s/s_0 - 1)^n \frac{1}{s_0} \left[1 + \frac{s_0 - s}{s_0} + \left(\frac{s_0 - s}{s_0}\right)^2 \ldots\right] . \tag{5}$$

The photon pole has to be understood as part of the background terms; but once it is known as a part of QED corrections, one may separate it from the rest of background and can take its knowledge explicitly into account.

An analysis of the Z-line shape will use the integrated effective Born cross section $\sigma_T(s)$

$$\sigma_T(s) = \sum_{i=1}^{4} \sigma^i(s) = \frac{1}{4} \sum_{i=1}^{4} s|\mathcal{M}^i(s)|^2 . \tag{6}$$

Further, the $\sigma_\mathrm{T}(s')$ has to be folded with a flux function in order to comprise in $\sigma_\mathrm{T}(s)$ also QED corrections [18–20]

$$\sigma_\mathrm{T}(s) = \frac{4}{3}\pi\alpha^2 \int \frac{ds'}{s} \left[\frac{r^\gamma}{s'} + \frac{s' R_\mathrm{T} + \left(s' - \bar{M}_Z^2\right) J_\mathrm{T}}{\left(s' - \bar{M}_Z^2\right)^2 + \bar{M}_Z^2 \Gamma_Z^2} + \ldots \right] \rho_\mathrm{ini}^\mathrm{T}\left(s'/s\right). \quad (7)$$

Here, two real parameters besides M_Z and Γ_Z appear, the R_T arising from the Z pole term and J_T from the γZ-interference.

The radiation from the final state may be absorbed into $\rho_\mathrm{ini}^\mathrm{T}$, and initial-final state interferences can be taken into account by an analogue formula to (7) with a slightly more complicated structure

$$\sigma_\mathrm{T,int}(s) = \int ds' \sigma\left(s, s'\right) \rho_\mathrm{int}^\mathrm{T}\left(s'/s\right). \quad (8)$$

A precise description of QED, not only in the running QED coupling, but also in the flux functions $\rho_\mathrm{ini}(s'/s)$ and $\rho_\mathrm{int}(s'/s)$ is mandatory. We mentioned here already that for the forward and backward cross section parts σ_F and σ_B different flux functions apply, and so the corrections to $\sigma_\mathrm{F-B}$ are different from those to σ_tot. Finally, it is recommended to use for the predictions of the QED corrections a sophisticated, flexible, well-tested tool like ZFITTER [21–24]. The recommended interface is the Fortran package SMATASY/ZFITTER [7]. The latest version of SMATASY runs with ZFITTER v.6.42 and is due to Grünewald [8]. Although it is not explicitly pronounced by the authors, the copyright conditions are the same as those for ZFITTER. Please notice that the valid CPC-license conditions [25] do not always guarantee that they are respected [24, 26–32] and thus we think that it is due to remind here the rules of Good Scientific Practice (Appendix) important for code developers for scientific community.

3. Asymmetries

Born asymmetries $A_\mathrm{FB} = \sigma_\mathrm{FB}/\sigma_\mathrm{T}$, $A_\mathrm{pol} = \sigma_\mathrm{pol}/\sigma_\mathrm{T}$, $A_\mathrm{LR} = \sigma_\mathrm{LR}/\sigma_\mathrm{T}$, as ratios of two Laurent series, are simple Taylor series. QED corrections lead to few simple correction factors; we reproduce two of them:

$$A_\mathrm{LR}^\mathrm{QED}(s) = A_{0,\mathrm{LR}}^\mathrm{Born} + c_{1,\mathrm{T}}(s) A_{1,\mathrm{LR}}^\mathrm{Born} \left(\frac{s}{M_Z^2} - 1 \right) + \ldots, \quad (9)$$

$$A_\mathrm{FB}^\mathrm{QED}(s) = c_{0,\mathrm{FB}}(s) A_{0,\mathrm{FB}}^\mathrm{Born} + c_{1,\mathrm{FB}}(s) A_{1,\mathrm{FB}}^\mathrm{Born} \left(\frac{s}{M_Z^2} - 1 \right) + \ldots \quad (10)$$

The real constants A_0 and A_1 are of experimental relevance

$$A_{0,\mathrm{A}} = \frac{R_\mathrm{A}}{R_\mathrm{T}}, \qquad A_{1,\mathrm{A}} = \left[\frac{J_\mathrm{A}}{R_\mathrm{A}} - \frac{J_\mathrm{T}}{R_\mathrm{T}} \right] A_0, \qquad \mathrm{A} = \mathrm{FB}, \mathrm{pol}, \mathrm{LR}. \quad (11)$$

The QED corrections are contained in smooth set-up dependent, *model-independent* factors $C(s)$:

$$c_{0,\mathrm{FB}}(s) = \frac{C_{\mathrm{FB}}^R}{C_{\mathrm{T}}^R}, \quad c_{0,\mathrm{T}}(s) = 1, \quad c_{1,\mathrm{A}}(s) = c_{0,\mathrm{A}} \; \frac{C_{\mathrm{T}}^J}{C_{\mathrm{T}}^R}, \quad a = \mathrm{T}, \mathrm{FB}. \quad (12)$$

As examples, we reproduce $C_{\mathrm{T}}^R(s)$ and $C_{\mathrm{FB}}^R(s)$:

$$C_{\mathrm{A}}^R(s) = \int dk \; \rho_{\mathrm{ini}}^{\mathrm{A}}(s'/s) \frac{s' R_{\mathrm{A}}}{s R_{\mathrm{A}}} \frac{\left(s - \bar{M}_Z^2\right)^2 + \bar{M}_Z^2 \Gamma_Z^2}{\left(s' - \bar{M}_Z^2\right)^2 + \bar{M}_Z^2 \Gamma_Z^2}, \qquad a = \mathrm{T}, \mathrm{FB}. \quad (13)$$

4. Applications

The described model-independent approach allows experimental fits to the mass and width of the Z boson which are, in accuracy, competitive to the Standard Model approach. The minimal number of data points (in s) will be five, as may be seen from (7). There is a correlation of Z-peak position s_{peak}, the Z-mass value, and the γZ-interference term J_{T}

$$\Delta \sqrt{s_{\mathrm{peak}}} = \Delta \bar{M}_Z + \frac{1}{4} \frac{\Gamma_Z^2}{\bar{M}_Z} \; \Delta \left(\frac{J_{\mathrm{T}}}{R_{\mathrm{T}}}\right). \quad (14)$$

Any misidentification of J_{T} leads to a correlated systematic shift of $\bar{M}_Z$. In the Standard Model, the J_{T} is a derived quantity and thus fixed, while here it is a floating quantity — if we do not decide to fix it "by hand" or by other data. As a consequence, the error bars in the strict S-matrix approach for $\bar{M}_Z$ will be systematically a bit bigger than in the SM approach. For further discussions, we refer to the literature quoted and to [2, 9, 33, 34]. Whether the S-matrix approach can be useful at the FCC-ee deserves a detailed investigation on the approximations made in the SMATASY/ZFITTER toolkit which have not been discussed here.

Appendix

Good Scientific Practice and software

As quoted above, it happens that the rules of Good Scientific Practice are violated when software is concerned. The reasons root deeply in history. For decades, software was not considered as a genuine result of scientific work, but mere as an auxiliary work with no value by itself. This changed with its rising complexity. But until today, in almost all of the official documents on Good Scientific Practice, there is mentioning of texts, figures, ideas, data, but no mentioning of software. And a need of its protection against misuse — by lack of attribution or by improper use — is questioned, especially by non-experts.

International academic fundamental research relies on universal principles and ethical rules, and on national legal regulations. We are seekers of the truth. One of our most important principles is honesty. Society has to trust us in our doings because an independent control is, due to the complexity of our work, impossible. We researchers carry the responsibility to prevent and, in case it happens, sanction fraud in science. Fraud destroys the balance of competition and cooperation. It also destroys the trust, and it finally destroys the contract of society and science.

Academic research, and we are part of that, is distinguished from other research by the need of correct attribution:

— *Attribution* of a scientific achievement to those who made it.
  Often by a proper citation. Other proper attributions are possible.

There is no need to explain why proper attribution is substantial. Violations are called plagiarism. A proper attribution informs on:

— *What was done?*
— *Who did it?*
— *How important is it?*

In the case of ethical or legal problems, seek cooperations and not confrontations. Questions have to be answered, in this order:

— *Facts:* What are the initial facts? Investigate carefully.
— *Rules:* Are there violated rules? Seek a healing by negotiations.
— *Sanctions:* In case. Are there sanctions foreseen? By whom?

A round table discussion on "Open-source, knowledge sharing and scientific collaboration" at ACAT2013 is summarized in [32]. An important point is the authors' right to set "conditions of use", often formalized in license agreements. In scientific practice, they replace law — if they are respected. We use here the CPC-license [25], which is unfortunately questioned by a variety of colleagues and institutions. If this will become a common practice, no scientist will be willing to develop a software to be shared by the community. Other well-known license model families are GPL licenses (Gnu public licenses, not ideal for academic attributions, although often recommended for software) and CC licenses (Creative Commons licenses). To respect licenses as valid "conditions of use" is essential because there is no international copyright law. With no doubt — it would be extremely difficult to authors to defend rights at a court. The big international scientific centers and all the national science organizations are asked not only to set the rules, but also to defend them.

REFERENCES

[1] T. Riemann, "S-matrix Approach to the Z Line Shape — a Reminiscence. Prospects?", talk at the 2015 Pisa FCC-ee meeting, `https://agenda.infn.it/getFile.py/access?contribId=6&sessionId=7&resId=0&materialId=slides&confId=8830`

[2] T. Riemann, "S-matrix Approach to the Z Line Shape", talk at CALC2015, JINR, Dubna, Russia, July 2015, `https://indico.cern.ch/event/368497/session/7/contribution/59/attachments/1134075/1622013/02_riemann-tord-calc2015.pdf`

[3] T. Riemann, "Studying the Z Boson Resonance — Not with Weak Loops — But with the SMATASY/ZFITTER-kit", talk at HEPKIT2015, Oct 5–8, 2015, KIT, Karlsruhe, Germany, `https://indico.cern.ch/event/369827/timetable/#all.detailed`

[4] A. Freitas, *J. High Energy Phys.* **1404**, 070 (2014).

[5] A. Leike, T. Riemann, J. Rose, *Phys. Lett. B* **273**, 513 (1991) [`arXiv:hep-ph/9508390`].

[6] T. Riemann, *Phys. Lett. B* **293**, 451 (1992) [`arXiv:hep-ph/9506382`].

[7] S. Kirsch, T. Riemann, *Comput. Phys. Commun.* **88**, 89 (1995) [`arXiv:hep-ph/9408365`].

[8] M. Grünewald, S. Kirsch, T. Riemann, Fortran package SMATASY v.6.42, June 2, 2005, `http://www.cern.ch/Martin.Grunewald/afs/public/smatasy/smata6_42.fortran`

[9] S. Kirsch, S. Riemann, L3 note 1233, 1992, `http://l3.web.cern.ch/l3/note/notes1992.html`

[10] O. Adriani *et al.* [L3 Collaboration], *Phys. Lett. B* **315**, 494 (1993).

[11] K. Miyabayashi *et al.* [TOPAZ Collaboration], *Phys. Lett. B* **347**, 171 (1995).

[12] K. Yusa *et al.* [VENUS Collaboration], *Phys. Lett. B* **447**, 167 (1999).

[13] P. Holt, *PoS* **HEP2001**, 115 (2001),
http://pos.sissa.it/archive/conferences/007/115/hep2001_115.pdf

[14] K. Sachs, Proc. of XXXVIII Rencontres de Moriond: Electroweak
Interactions and Unified Theories, Les Arcs, March 15–22, 2003,
[arXiv:hep-ex/0307009].

[15] S. Schael *et al.* [ALEPH, DELPHI, L3, OPAL, SLD, LEP Electroweak
Working Group, SLD Electroweak Group, SLD Heavy Flavour Group],
Phys. Rep. **427**, 257 (2006) [arXiv:hep-ex/0509008].

[16] S. Schael *et al.* [ALEPH, DELPHI, L3, OPAL, LEP Electroweak Working
Group], *Phys. Rep.* **532**, 119 (2013) [arXiv:1302.3415 [hep-ex]].

[17] D.Y. Bardin, A. Leike, T. Riemann, M. Sachwitz, *Phys. Lett. B* **206**, 539
(1988).

[18] D.Y. Bardin *et al.*, *Phys. Lett. B* **229**, 405 (1989).

[19] D.Y. Bardin *et al.*, *Nucl. Phys. B* **351**, 1 (1991) [arXiv:hep-ph/9801208].

[20] A. Akhundov, A. Arbuzov, S. Riemann, T. Riemann, *Phys. Part. Nucl.* **45**,
529 (2014) [arXiv:1302.1395 [hep-ph]].

[21] D.Y. Bardin *et al.*, *Comput. Phys. Commun.* **59**, 303 (1990).

[22] D.Y. Bardin *et al.*, *Comput. Phys. Commun.* **133**, 229 (2001)
[arXiv:hep-ph/9908433].

[23] A. Arbuzov *et al.*, *Comput. Phys. Commun.* **174**, 728 (2006)
[arXiv:hep-ph/0507146].

[24] The ZFITTER homepage: http://sanc.jinr.ru/users/zfitter, Sep 2015,
in preparation.

[25] The CPC non-profit use license agreement of the Computer Physics
Communications Program Library,
http://cpc.cs.qub.ac.uk/licence/licence.html

[26] The ZFITTER homepage: http://zfitter.education, since April 5, 2014.

[27] Former ZFITTER homepage: http://zfitter.com, June 20, 2013 till March
2014, documentation.

[28] W. Löwer, "Schiedsspruch" of the German Ombudsman for Science in the
Gfitter Case, July 3, 2012, English translation by G. Schuckelt, Intertext
January 2013, http://zfitter.education/Schiedsspruch_Prof_Loewer-
en-03Jul2012.pdf

[29] ZFITTER Collaboration, Memorandum on ZFITTER/Gfitter, November 4,
2012, http://zfitter.com/2012-11-04-memo-zfitter-gfitter-v24.pdf

[30] DESY, webpage "Gfitter01: Previous Code Version", June 25, 2013,
http://fh.desy.de/projekte/gfitter01/Gfitter01.htm

[31] H. Dosch, C. Scherf, webpage "DESY — Fall ZFITTER/GFITTER —
Stellungnahme DESY Direktorium zum Fall ZFITTER/Gfitter", March 22,
2014, in German, http://zfitter-gfitter.desy.de

[32] F. Carminati, D. Perret-Gallix, T. Riemann, *J. Phys. Conf. Ser.* **523**, 012066 (2014) [arXiv:1407.0540 [hep-ph]].

[33] A. Leike, S. Riemann, T. Riemann, *Phys. Lett. B* **291**, 187 (1992) [arXiv:hep-ph/9507436].

[34] M. Grünewald, S. Kirsch, submitted to *Z. Phys. C*, 1993, http://www-lib.kek.jp/cgi-bin/img_index?9402241