# $\chi_{c_1}$ AND $\chi_{c_2}$ PRODUCTION IN $e^+e^-$ ANNIHILATION\*

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Electromagnetic production of  $\chi_{c_1}$  and  $\chi_{c_2}$  states in  $e^+e^-$  annihilation is discussed in the frame of a quarkonium model.

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#### 1. Introduction

With the improving luminosity of  $e^+e^-$  colliders, searches for the direct resonant production of  $0^{++}$ ,  $1^{++}$ ,  $2^{++}$  states became possible. These states can be produced directly only through a neutral current or a higher order electromagnetic process. Here, a creation of these resonances through two virtual photons is considered. The amplitude for production of  $\chi_{c_0}$  is proportional to the electron mass and thus highly suppressed. All  $\chi_{c_i}$  states can be, however, produced in the processes  $e^+e^- \rightarrow e^+e^-\chi_{c_{1,2}} \rightarrow \gamma(J/\psi \rightarrow \mu^+\mu^-)$ will allow to measure the electronic widths of  $\chi_{c_1}$  and  $\chi_{c_2}$  resonances. The amplitudes for production of these states have been implemented [1] in the Monte Carlo generator PHOKHARA [2] and it can be used to extract the electronic widths from experimental data. In Section 2, we describe the adopted model for coupling of two photons to  $J^{++}$  states and determine the parameters of the model. In Section 3, we present the obtained results for the cross section and discuss the possibility of observing the potential signal.

#### 2. Quarkonium model

The amplitudes for coupling of two photons with momenta  $p_1$  and  $p_2$  to a fermion and an antifermion, which are in bound state, is described by

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two diagrams from figure 1, where f and  $\overline{f}$  are fermion and antifermion momenta. We have adopted model of coupling of  $\chi_{c_1}$  and  $\chi_{c_2}$  states to two photons developed in [3]. In the frame of this model, the fermion and the antifermion are treated in the non-relativistic approximation.



Fig. 1. Diagrams for coupling of fermion and antifermion, which are in bound state to two photons.

The  $\chi_{c_i} - \gamma^* - \gamma^*$  interaction is described in terms of a form factor c, which has the following form:

$$c \equiv c_i \left( M_{\chi_{c_i}}, p_1^2, p_2^2, m_i, a \right) = \frac{16\pi\alpha a}{\sqrt{m_i}} \frac{1}{\left( \left( p_1^2 + p_2^2 \right)/2 - M_{\chi_{c_i}} - b_i M_{\chi_{c_i}} - b_i^2/2 \right)^2},$$
(1)

where  $b_i = 2m_i - M_{\chi_{c_i}}$  is the binding energy,  $m_i$  the effective charm quark mass in the  $\chi_{c_i}$  state of mass  $M_{\chi_{c_i}}$ ,  $a = \sqrt{\frac{3}{4\pi}} 3Q_i^2 \phi'(0)$ , where  $\phi'(0)$  is the derivative of the wavefunction at the origin, and  $Q_i = 2/3$  is the charm quark electric charge. The parameters of this model  $(m_i \text{ and } a)$  have been extracted [1] from the measured values [4] of  $\Gamma(\chi_{c_i} \to \gamma\gamma, J/\psi\gamma)$  for i = 1, 2, where we have used the same form of the form factor for  $\chi_{c_i} - \gamma - \gamma$  and  $\chi_{c_i} - J/\psi - \gamma$  and assume the same parameter a and different effective mass of charm quark for  $\chi_{c_1}$  and  $\chi_{c_2}$ . The obtained values of the parameters are as follows:

$$a = 0.062(2) \left[ \text{GeV}^{5/2} \right],$$
  
 $m_1 = 1.463(3) \left[ \text{GeV} \right],$   
 $m_2 = 1.448(3) \left[ \text{GeV} \right].$ 

#### 3. Determination of the cross section

Determination of the electronic widths of  $\chi_{c_1}$  and  $\chi_{c_2}$  states can be achieved by measuring the cross section of the process  $e^+e^- \rightarrow \chi_{c_{1,2}} \rightarrow \gamma(J/\psi \rightarrow \mu^+\mu^-)$ , so we concentrate here on a selected final state which is easy to be observed experimentally. The diagram describing this process is presented in figure 2 (a). The same final state is produced through amplitudes shown in figures 2 (b) and 2 (c). The contribution from the one in figure 2 (b) is negligible if  $\mu^+\mu^-$  invariant mass is close to the mass of  $J/\psi$ , while the one in figure 2 (c) gives an irreducible background. The diagrams in figure 2 are just representative diagrams for classes of contributions and for example, the classes 2 (a) and 2 (b) include also diagrams with  $J/\psi^*-\gamma^*$  contribution in the loop.



Fig. 2. Diagrams for the: (a)  $e^+e^- \rightarrow \chi_{c_{1,2}} \rightarrow \gamma(J/\psi \rightarrow \mu^+\mu^-)$ , (b)  $e^+e^- \rightarrow \chi_{c_{1,2}} \rightarrow \gamma(\gamma \rightarrow \mu^+\mu^-)$ , (c) radiative return background.

All these amplitudes interfere. Two unknown relative phases appear, but as the contribution from diagram in figure 2 (b) is negligible, only one of them is relevant. Evaluation of the functions  $g_{1,2}$ , which come from loop integrals and formulae for complete amplitudes can be found in [1]. Functions  $g_1$ and  $g_2$  are related to electronic widths via formulae:

$$\Gamma(\chi_{c_1} \to e^+ e^-) = \frac{|g_1|^2}{12\pi} M_{\chi_{c_1}},$$
(2)

$$\Gamma(\chi_{c_2} \to e^+ e^-) = \frac{|g_2|^2}{40\pi} M_{\chi_{c_2}}.$$
(3)

Using results derived in [1], we have chosen the values of electronic widths for our calculations to be 0.04 eV for  $\chi_{c_1}$  and 0.09 eV for  $\chi_{c_2}$ .

In figure 3, we present the total cross section as a function of the phase between signal and background ( $\phi$ ) for  $\chi_{c_1}$  production and for  $\sqrt{s} = M_{\chi_{c_1}}$ . We have limited the value of the invariant mass of the muons (9.589157 GeV<sup>2</sup> <  $Q^2 < 9.592621$  GeV<sup>2</sup>) and set angular cuts on polar angles to be 20° <  $\theta_{\mu^+,\mu^-} < 160^\circ$ . As one can observe, in this plot the total cross section depends significantly on the phase ( $\phi$ ), so the possibility of observing the signal depends on the value of the phase between the signal and the background amplitudes. This plot was obtained for the  $\chi_{c_1}$ , but the same dependence can be observed for  $\chi_{c_2}$ . The only difference is the size of the cross section.

In the plot in figure 4, we present the cross section as a function of  $\sqrt{s}$  for production of  $\chi_{c_1}$  for different choices of the relative phase between the signal and the background amplitudes:  $\phi_{\text{max}}$  is the phase which maximizes the total cross section,  $\phi_{\text{min}}$  is the phase which minimizes the total cross section and  $\phi_{\text{inter}}$  is an intermediate choice of the phase. We show there also



Fig. 3. The total cross section for  $\chi_{c_1}$  production as a function of relative phase between signal and background ( $\phi$ ).



Fig. 4. The cross section for  $\chi_{c_1}$  production obtained for few different choices of the phase (see the text for details).

the cross section for the background (rad.ret) and the cross section without including the interference (without Int). The same plot for  $\chi_{c_2}$  is presented in figure 5. These cross sections were obtained for the following selection of events: 9.589157 GeV<sup>2</sup> <  $Q^2$  < 9.592621 GeV<sup>2</sup> and 20° <  $\theta_{\mu^+,\mu^-}$  < 160°. We have also assumed a beam spread equal to 1 MeV per beam distributed according to a Gaussian function. Despite the relative smallness of electronic widths (0.04 eV for  $\chi_{c_1}$  and 0.09 eV for  $\chi_{c_2}$ ), it is still possible to observe the signal over the radiative return background. The cross section for the  $\chi_{c_2}$  production is smaller than the one for  $\chi_{c_1}$  production, but the signal to the background ratio is bigger. In both cases, one can observe the signal a few percent above radiative return background. It is a value which can be measured at the BESIII experiment. As the model we use is not well tested experimentally, there is still room for contribution up to 10 times higher than the predicted ones [3]. The scan experiment in the vicinity of the mass of  $\chi_{c_1}$ ( $\chi_{c_2}$ ) provides also possibility of extraction of the phase between the radiative return amplitude and the  $\chi_{c_1}$  ( $\chi_{c_2}$ ) production amplitudes. Additionally, the tests of these charmonium bound state models can be performed with combined measurement of this cross section with measurement of differential cross section for the process  $e^+e^- \rightarrow e^+e^-(\chi_{c_{0,1,2}} \rightarrow \gamma(J/\psi \rightarrow \mu^+\mu^-))$ , which can be done by Belle II experiment.



Fig. 5. The cross section for  $\chi_{c_2}$  production obtained for few different choices of the phase (see the text for details).

### 4. Conclusions

We have adopted a model of coupling of  $\chi_{c_1}$  and  $\chi_{c_2}$  states to two photons, which reproduce the measured values of the  $\Gamma(\chi_{c_{1,2}} \to \gamma\gamma, \gamma J/\psi)$  decay widths. In the frame of this model, we have predicted cross sections for production of  $\chi_{c_1}$  and  $\chi_{c_2}$  in electron-positron annihilation, which allows for extraction of the electronic widths of these states. Despite the relative smallness of the electronic widths, it is still possible to observe the signal up to a few percent over the radiative return background, which is within reach of the BESIII Collaboration. The total cross section depends on the relative phase between the signal and the background amplitudes, which can be determined by performing a scan. The amplitudes for production of  $\chi_{c_{1,2}}$ have been implemented in the Monte Carlo generator PHOKHARA [2]. This work was supported in part by the Polish National Science Centre, Grant No. DEC-2012/07/B/ST2/03867 and the German Research Foundation DFG under Contract No. Collaborative Research Center CRC-1044.

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