BOUND MUON DECAY*

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We summarize the recent progress in evaluation of radiative corrections to bound muon decay spectrum. Calculated corrections reconcile the TWIST measurement with the theoretical prediction. Corrections in the endpoint region of the spectrum affect the sensitivity of the muon electron conversion searches.

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1. Introduction

Muonic atoms play an important role in experimental searches for the charged lepton flavour violation (CLFV). Muon bounded to a nucleus may undergo a coherent conversion into an electron. This process is a two-body decay. The final state consists of a mono-energetic electron and a recoiling nucleus.

Search for the conversion is the goal of the upcoming experiments: Mu2e [1] and COMET [2]. The Standard Model (SM) rate of the conversion is below the sensitivity of the planned experiments. However, many of the SM extensions predict CLFV effects that can be tested in the near future. A positive result from the searches will be a clear indication of a beyond the SM physics. Additional motivation for this searches comes from the current discrepancy between calculated and measured value of the muon anomalous magnetic moment [3, 4]. The physics responsible for the CLFV is likely to contribute also to the muon $g-2$.

Apart from the coherent conversion, there are two interesting SM processes that bound muons may undergo. The first one is the nuclear capture of a muon. Rate for this reaction strongly depends on the nucleus charge Z.

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Second reaction is a muon decay in orbit (DIO). This process is not only interesting as a background for the conversion searches; it has been measured in the TWIST experiment [5]. This measurement was precise enough to be sensitive to the quantum electrodynamics (QED) corrections.

The first order QED radiative corrections for a free muon spectrum were considered in [6, 7]. After more than 50 years we already know the full $\mathcal{O}(\alpha^2)$ corrections [8–11] to the muon lifetime and the spectrum. Corrections to the muon DIO spectrum were obtained recently [12, 13]. In this paper, we shortly summarize these results. First, we describe the central region of the spectrum, where the shape function formalism applies. Then, we discuss the endpoint region.

2. DIO spectrum

From the theoretical perspective, it is convenient to divide the DIO spectrum into two distinctive energy regions [14]: the central region and the endpoint region. Each region is characterized by a different scale of momentum that is transferred between the charged particles and the nucleus. Coincidentally, this division corresponds to different experiments. The central region was measured by the TWIST experiment. Endpoint region will contribute to the background for conversion process and will be measured precisely by the upcoming experiments.

2.1. Central region

The central region, also called the Michel region [15], is defined as an interval of electron energies (E_e) smaller than half of the muon mass m_μ and much larger than the average bound muon momentum $m_\mu Z\alpha$; $m_\mu Z\alpha \ll E_e \leq \frac{m_\mu}{2}$. This region is kinematically accessible also when the muon is free. Because of that, the typical momentum transfer in this region is of the order of the average bound muon momentum.

In this region, the dominant binding correction to the spectrum comes from a Doppler smearing caused by the muon motion in an atom. Description of this phenomena requires a resummation of multiple soft photons exchanged between the nucleus and the muon.

Proper field theoretical treatment of this effect requires introduction of the shape function and the factorization theorem. These methods were first developed in QCD for heavy quarks [16–21].

For muonic atom in the non-relativistic approximation, the shape function can be obtained analytically [22]. With subleading terms neglected, it is given by

$$S(\lambda) = \frac{8m_\mu^5 Z^5\alpha^5}{3\pi\left[\lambda^2 + m_\mu^2 Z^2\alpha^2\right]^3}\,. \tag{1}$$

Its form closely resembles the muon wave function in the momentum space. Formula (1) can be interpreted as a probability density distribution of the muon momentum along the electron direction of motion.

In practical applications, a finite nucleus size effects need to be taken into account. This can be obtained by a numerical solution of Dirac or Schrödinger equation. Once the shape function is known, the DIO spectrum is calculated as a convolution [23],

$$\frac{d\Gamma}{dE_e} = \int d\lambda\, S(\lambda) \frac{d\Gamma_{\mathrm{free}}}{dz} \frac{dz}{dE_e}\bigg|_{z\to z(\lambda)}, \tag{2}$$

where $\frac{d\Gamma_{\mathrm{free}}}{dz}$ is the electron energy spectrum in the free muon decay. Known radiative correction for a free muon [7, 9] can be included in this term to obtain the leading corrections to the muon DIO spectrum in the central region.

2.2. DIO endpoint

For a bound muon, the maximal electron energy is slightly smaller than the muon mass. Therefore, the endpoint region is located beyond the region of the phase space accessible for a free muon decay. This requires large momentum transfer between the nucleus and charged particles. In the leading order, only one virtual photon contributes to the decay amplitude. Quantitative description of the spectrum in the endpoint region requires non-trivial expansion of the muon and the electron wave functions; naive Born approximation does not give the correct result [24]. However, many qualitative features of the endpoint spectrum can be very easily understood without doing actual calculations.

In the presence of the Coulomb potential, that is time independent, the energy in the decay process is conserved. Hence, the neutrinos in the endpoint region are very soft. The leading energy dependence of the electron spectrum comes only from the phase space factor and the neutrino current. Denoting the sum of both neutrinos energies by Δ and counting the powers of the neutrino energy, we obtain

$$\frac{d\Gamma}{dE_e} \sim \frac{\Delta^5}{m_\mu^5} \approx \frac{(m_\mu - E_e)^5}{m_\mu^5}\,. \tag{3}$$

To understand leading $Z\alpha$ behaviour, we note that the amplitude is dominated by the short-distance interaction between muon and the nucleus. It is, therefore, justified to expand the muon wave function around the origin. The leading term behaves like $\psi(0) \sim (Z\alpha)^{\frac{3}{2}}$. Momentum conservation requires at least a single interaction with the binding potential; this introduces additional power of $Z\alpha$.

Combining all the factors together, we obtain

$$\frac{d\Gamma}{dE_e} \sim (Z\alpha)^5 \frac{\Delta^5}{m_\mu^5} \,. \tag{4}$$

Both factors suppress the number of events in the endpoint region. Numerical calculations [25] predict that only about 1.6×10^{-17} of electrons will have the energy different from the maximal energy by no more than 1 MeV.

Radiative corrections can be divided into several types. First one is the vacuum polarization. It effectively increases the QED coupling constant; the muon is more strongly bounded by the nucleus. Vacuum polarization rises the expected number of DIO event in the endpoint region by about 2.9%.

Large corrections are related to the emission of the real soft and collinear photons combined with corresponding virtual one-loop diagrams. Radiated photons decrease the electron energy, reducing the number of electrons with the highest possible energy. As a result of perturbative fixed order calculations, a singular terms proportional to $\ln \Delta$ appear. This problem is also known for the free muon spectrum [26]. It is a manifestation of breakdown of a fixed order perturbative approach. Near the endpoint, the soft real photons contribution needs to be resummed. This should not be confused with the central region where the soft Coulomb virtual photons were resummed. Following the standard prescription [27], we obtain that the power of Δ in Eq. (4) is changed to $5 + \frac{\alpha}{\pi}\delta_S$, where $\delta_S = 2\ln 2 - 2 + 2\ln\frac{m_\mu}{m_e}$. Remaining non-soft contribution still contains the large logarithm $\ln\frac{m_\mu}{m_e}$. This large term can be obtained as a convolution of the leading order spectrum with the electron structure function similarly as in the free muon case [28]. Combining these factors together, we obtain the leading correction

$$\frac{d\Gamma}{dE_e} \sim \frac{\Delta^5}{m_\mu^5}(Z\alpha)^5 \left[\left(\frac{\Delta}{m_\mu}\right)^{\frac{\alpha}{\pi}\delta_S} - \frac{46}{15}\frac{\alpha}{\pi}\ln\frac{m_\mu}{m_e} + \delta_{\mathrm{VP}}\right] + \mathcal{O}\left(Z^6\alpha^6\right) , \tag{5}$$

where δ_{VP} is the vacuum polarization correction. In addition, there are also corrections not enhanced by the large logarithm. In practical applications they can be neglected.

3. Conclusions

In this paper, we made a brief overview of the recent bound muon spectrum calculations. Corrections in the central region improve agreement between the TWIST data and the theoretical prediction. Corrections in the endpoint suppress the background for the conversion searches, enhancing the sensitivity of planned experiments. DIO calculations combine many different modern tools and methods. This makes them interesting also from a theoretical perspective. Description of the central region is closely related to the decays of mesons containing heavy quarks. However, for mesons, it is impossible to obtain a simple analytical formulas due to the non-perturbative nature of QCD at low energies. In the future, studies of the muon DIO may increase our understanding of various bound states from a quantum field theory perspective.

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