CONSTRAINTS ON THE HIGGS SECTOR FROM RADIATIVE MASS GENERATION OF NEUTRINOS*

Thomas Gajdosik, Andrius Juodagalvis, Darius Jurčiukonis
Tomas Sabonis

Vilnius University, Universiteto 3, 01513, Vilnius, Lithuania

(Received October 20, 2015)

Adding a single gauge singlet and a second Higgs doublet to the original Standard Model allows an explanation of the observed smallness of the neutrino masses using the seesaw mechanism. This model predicts two neutral fermions with vanishing mass. But the one-loop contribution to the neutral fermion masses due to the second Higgs doublet lifts this degeneracy and allows to fit the model parameters to the observed neutrino mass differences. We present the determination of the additional Yukawa couplings that appear in our model by requiring that our model predicts the correct mass differences and mixings in the neutrino sector.

DOI:10.5506/APhysPolB.46.2323
PACS numbers: 11.30.Rd, 13.15.+g, 14.60.St

1. Lagrangian and parameters of the model

Adding to the Standard Model (SM) with 3 generations a single gauge singlet fermion N_{R} and a second Higgs doublet gives us our model, the 312-νSM, introduced in [1]. For the scalar sector, we get a general Two Higgs doublet model (2HDM), as described in [2]. We use the relations of [2] to express nine real parameters of the Higgs potential by four masses of the physical Higgs particles, and two mixing angles between the neutral Higgses. The remaining parameters are not physical [2]. In this basis, only the first Higgs doublet develops a vacuum expectation value (vev) v.

The seesaw mechanism includes a Majorana mass term

$$\mathcal{L}_{\mathrm{Maj}} = \tfrac{1}{2} M_{\mathrm{R}}^{*} \bar{N}' \boldsymbol{C} \bar{N}'^{\top} + \mathrm{h.c.} = -\tfrac{1}{2} M_{\mathrm{R}}^{*} \bar{N}' \widehat{N}' - \tfrac{1}{2} M_{\mathrm{R}} \overline{\widehat{N}}' N' \,, \tag{1}$$

where N' denotes the right projection of the Majorana fermion N, $\boldsymbol{C}$ is the charge conjugation matrix, M_{R} is the mass parameter of the singlet, and

* Presented by T. Gajdosik at the XXXIX International Conference of Theoretical Physics "Matter to the Deepest", Ustroń, Poland, September 13–18, 2015.

$\widehat{N} = \gamma^0 \boldsymbol{C} N^*$ denotes the Lorentz covariant conjugate [3] of the field N, that in our case also satisfies the Majorana condition $\widehat{N} = N$. To couple the gauge singlet to other fields in the most general way, we have to add three Yukawa couplings $(Y_{\rm L}^{(2)})_{jk}$ and $(Y_N^{(1,2)})_j$ to get the Yukawa Lagrangian

$$\mathcal{L}_{\rm Yuk} = -\bar{\ell}_{{\rm L}j}\phi_a \left(Y_{\rm L}^{(a)}\right)_{jk} e_{{\rm R}k} - \bar{\ell}_{{\rm L}j}\tilde{\phi}_a \left(Y_N^{(a)}\right)_j N + {\rm h.c.}\,, \tag{2}$$

where we write the generation indices j and k explicitly. Note, that the two Yukawa couplings to the gauge singlet define two (in general independent) directions in the three-dimensional lepton flavour space.

Inserting the vacuum expectation value v into the Yukawa Lagrangian Eq. (2), we get the neutrino mass terms written in the interaction eigenstates $\nu_{{\rm L}j}$ and N. These mass terms are diagonalised by the superpositions

$$\nu_{{\rm L}j} = U^*_{\alpha j} P_{\rm L}\zeta_\alpha\,, \qquad N' = P_{\rm R}N = U_{\alpha N}P_{\rm R}\zeta_\alpha\,, \tag{3}$$

where ζ_α are the physical Majorana mass eigenstates with masses m_α. The diagonalization can also be written in a matrix form $U_{(\nu)} M_\nu U_{(\nu)}^\top = {\rm diag}(m_\alpha)$ with the mass matrix

$$M_\nu = \begin{pmatrix} M_{\rm L} & M_{\rm D}^\top \\ M_{\rm D} & M_{\rm R} \end{pmatrix}, \qquad \text{with} \qquad \begin{array}{rcl} M_{\rm L} &=& 0_{3\times 3}\,, \\ M_{\rm D} &=& \frac{v}{\sqrt{2}} Y_N^{(1)}\,, \end{array} \tag{4}$$

and the neutrino mixing matrix

$$U_{(\nu)} = \begin{pmatrix} U_{\alpha j} \\ U_{\alpha N} \end{pmatrix} = \begin{pmatrix} \vec{a}^\top & 0 \\ \vec{b}^\top & 0 \\ ic\vec{c}^\top & -is \\ s\vec{c}^\top & c \end{pmatrix}, \qquad \text{where} \qquad \begin{array}{rcl} c^2 &=& \frac{m_4}{m_4+m_3}\,, \\ s^2 &=& \frac{m_3}{m_4+m_3}\,. \end{array} \tag{5}$$

The diagonalization gives the following conditions

$$M_{\rm D}\cdot\vec{a}^{\,*} = M_{\rm D}\cdot\vec{b}^{\,*} = 0 \qquad \text{and} \qquad M_{\rm D}\cdot M_{\rm D}^\dagger = m_{\rm D}^2 = m_3 m_4\,. \tag{6}$$

The unitarity of $U_{(\nu)}$ is guaranteed if $\vec{a}$, $\vec{b}$, $\vec{c}$ are orthonormal complex vectors and, in addition, the relation $M_{\rm D}\cdot\vec{c}^{\,*} = m_{\rm D}$, with the solution $\vec{c}^{\,\top} = m_{\rm D}^{-1}M_{\rm D}$ holds.

Using the idea of [1] and implementing it in a similar way as in [4], we define the overall phase of $\vec{b}$ to get the scalar quantity d as real and positive. We introduce also the overlap between the Yukawa couplings d':

$$d \;:=\; Y_N^{(2)}\cdot\vec{b}^{\,*} = \left(Y_N^{(2)}\right)_j \left(\vec{b}\right)_j^*\,, \tag{7}$$

$$d' \;:=\; Y_N^{(2)}\cdot Y_N^{(1)\dagger} = \frac{\sqrt{2}}{v} Y_N^{(2)}\cdot M_{\rm D}^\dagger = \frac{\sqrt{2}m_{\rm D}}{v} Y_N^{(2)}\cdot\vec{c}^{\,*}\,. \tag{8}$$

There is no freedom for d', as it is given by the input of the Yukawa couplings.

Using the diagonalization conditions, Eq. (6), the orthogonality of $\vec{a}$, $\vec{b}$, $\vec{c}$, and the definitions of d and d', we can determine the tree level mixing matrix $U_{(\nu)}$ by (1) taking $\vec{a}$ orthonormal to the two Yukawa couplings, $Y_N^{(1)}$ and $Y_N^{(2)}$, (2) $\vec{c} = Y_N^{(1)}/|Y_N^{(1)}|$, and (3) $\vec{b}$ orthonormal to $\vec{a}$ and $\vec{c}$:

$$(\vec{a}\,)_j \;:=\; \epsilon_{jk\ell} \left(Y_N^{(1)}\right)_k \left(Y_N^{(2)}\right)_\ell \Big/ \left(\left|Y_N^{(1)}\right| \left|Y_N^{(2)}\right|\right) , \tag{9}$$

$$\vec{b}^{\top} \;=\; d \left(\left|Y_N^{(1)}\right|^2 Y_N^{(2)} - d' Y_N^{(1)}\right) \Big/ \left(\left|Y_N^{(1)}\right|^2 \left|Y_N^{(2)}\right|^2 - \left|d'\right|^2\right) . \tag{10}$$

Assuming $s/c = \sqrt{m_3/m_4} \ll 1$, we can identify the matrix U made of $\vec{a}$, $\vec{b}$, and $\vec{c}$ with the PMNS-matrix. It allows us to determine the two Yukawa couplings to the singlet by the second and third row of the PMNS matrix and the three parameters d, d', and $m_{\mathrm{D}} = \sqrt{m_3 m_4}$:

$$Y_N^{(1)} = \frac{\sqrt{2} m_{\mathrm{D}}}{v} \vec{c}^{\top} \qquad \text{and} \qquad Y_N^{(2)} = d\, \vec{b}^{\top} - d' \frac{\sqrt{2} m_{\mathrm{D}}}{v} \vec{c}^{\top} . \tag{11}$$

2. Generating m_2

In an attempt to be more general than [4], we obtain the formula for the loop correction to the mass of a Majorana fermion, like in [5–8]. We do not include tadpoles in the contribution from scalars as the tadpole couplings to $\zeta_{1,2}$ vanish. The predicted mass $\delta m_1^{\mathrm{loop}} = 0$ vanishes for ζ_1, as expected. We get the approximate prediction for the mass of ζ_2, ignoring m_3 compared to m_4,

$$\delta m_2^{\mathrm{loop}} \approx d^2 \sum_{n=1}^{3} \mathrm{Re}\left[q_{n2}^2\right] m_4 B_0\left(p^2, m_{H_n}^2, m_4^2\right) , \tag{12}$$

where B_0 is the Passarino–Veltmann function, used in the convention of Denner [9]. The definition of q_{n2} contains the mixing of the neutral Higgses and is taken from [2]:

$$q_{12} = -s_{12} - i c_{12} s_{13} , \qquad q_{22} = c_{12} - i s_{12} s_{13} , \qquad q_{32} = i c_{13} , \tag{13}$$

with $s_{1j} := \sin\theta_{1j}$ and $c_{1j} := \cos\theta_{1j}$, the mixing angles of the neutral Higgses. We set $p^2 = m_2^2 = 0$. The formula Eq. (12) seems to indicate that the mass m_2 should be proportional to $m_4 \sim M_{\mathrm{R}}$, but since $\sum_{n=1}^{3}(\mathrm{Re}[q_{n2}])^2 = \sum_{n=1}^{3}(\mathrm{Im}[q_{n2}])^2 = 0$, the contributions with m_4 alone in the numerator cancel, leaving only terms $\sim m_{H_j}^2/m_4$, which amounts to the normal seesaw mass, multiplied by d^2 and some angles.

Taking the difference between the second lightest neutrino and the lightest neutrino, which is massless in our model, to be in the experimental 3σ range [10], we get the allowed band of Yukawa coupling values d as shown in Fig. 1. A narrow range of the allowed values means that d is basically directly determined by the measured $\Delta m^2_\odot$. The spikes in the plots come from cancellations between the contributions of the different Higgses for certain values of q_{n2} in connection with their chosen masses. The suppression of the sum in Eq. (12) requires d to increase in order to give the same value of m_2.

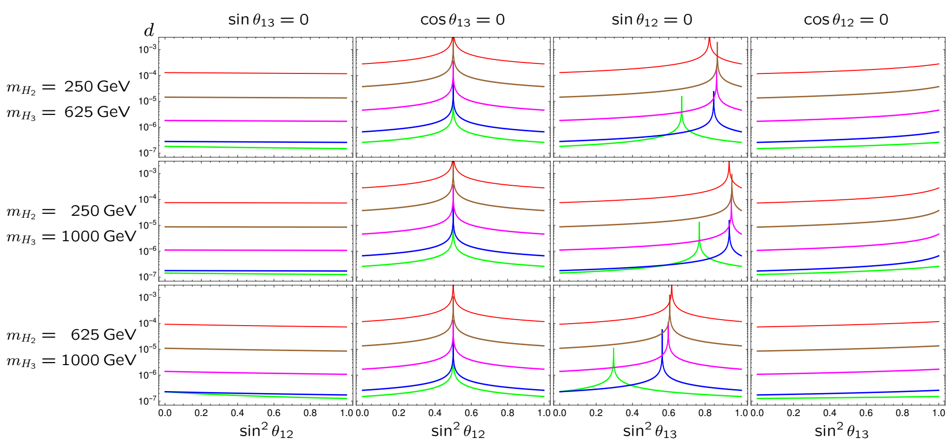

Fig. 1. (Colour on-line) Plot of the restriction on the Yukawa coupling strength d, coming from the requirement that the loop corrections give the correct mass for the second lightest neutrino, m_2. The allowed region is so thin that upper and lower bounds appear as a single line. The CP-conserving cases of the Higgs potential are listed on top, the values of the mixing angle of the neutral Higgses are displayed on the $\hat{x}$-axis. We take $m_{H_1} = 125$ GeV and display the masses of H_2, H_3 at the side. The lines in each panel correspond to increasing values of m_4 starting from the bottom: $m_4 = \{200, 2\times10^4, 2\times10^6, 2\times10^8, 2\times10^{10}\}$ GeV.

3. Using also m_3

The full renormalisation of our model requires knowledge of the mass of the heavy singlet state, which is not measured yet. But we can nevertheless attempt to restrict more model parameters by the additional information available from the neutrino sector, *i.e.* the second mass difference and the measured values of the PMNS-matrix (for the values see [10]). The needed additional assumption is formulated in [4] and allows us to calculate $\delta m_3^{\mathrm{loop}}$, while estimating the counter-terms from the variation of the parameters appearing in M_ν, Eq. (4). The requirement that also the third neutrino, ζ_3, gives the correct $\Delta m^2_{\rm atm}$ restricts the value of d', the other parameter in the second Yukawa coupling, irrespectively of the neutrino mass ordering. This possibility is shown in the plots presented at the conference [11].

For small values of the heavy singlet scale, the sub-dominant corrections due to charged particles in the loop can become of the same size as the mass m_3 itself. In this case, they have to be included in a numerical analysis.

4. Summary

We discussed the constraints of the 312-νSM model parameters. Assuming that the masses of the particles are known, the measured neutrino mass differences define the allowed Yukawa coupling strength between the neutrinos and the Higgs fields.

The authors thank the Lithuanian Academy of Sciences for the support (the project DaFi2015).

REFERENCES

[1] W. Grimus, H. Neufeld, *Nucl. Phys. B* **325**, 18 (1989).

[2] H.E. Haber, D. O'Neil, *Phys. Rev. D* **83**, 055017 (2011) [`arXiv:1011.6188 [hep-ph]`].

[3] P.B. Pal, *Am. J. Phys.* **79**, 485 (2011) [`arXiv:1006.1718 [hep-ph]`].

[4] W. Grimus, L. Lavoura, *Phys. Lett. B* **546**, 86 (2002) [`arXiv:hep-ph/0207229`].

[5] A. Pilaftsis, *Z. Phys. C* **55**, 275 (1992) [`arXiv:hep-ph/9901206`].

[6] M. Hirsch *et al.*, *Phys. Rev. D* **62**, 113008 (2000) [*Erratum ibid.* **65**, 119901 (2002)] [`arXiv:hep-ph/0004115`].

[7] A.A. Almasy, B.A. Kniehl, A. Sirlin, *Nucl. Phys. B* **818**, 115 (2009) [`arXiv:0902.3793 [hep-ph]`].

[8] T. Sabonis, Ph.D. Thesis — to be finished and published.

[9] A. Denner, *Fortsch. Phys.* **41**, 307 (1993) [`arXiv:0709.1075 [hep-ph]`].

[10] D.V. Forero, M. Tortola, J.W.F. Valle, *Phys. Rev. D* **90**, 093006 (2014) [`arXiv:1405.7540 [hep-ph]`].

[11] The presentation slides are available at: `http://indico.if.us.edu.pl/materialDisplay.py?contribId=46&sessionId=0&materialId=slides&confId=2`