CP INVARIANCE STUDY OF $J/\psi \to \Lambda\bar{\Lambda}$ AND Λ NONLEPTONIC DECAYS IN HELICITY FRAME*

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We present the joint helicity amplitudes for $J/\psi \to \Lambda\bar{\Lambda}$, $\Lambda(\bar{\Lambda})$ decays to different final states in the helicity frame. Two observables to search for CP violation in $J/\psi \to \Lambda\bar{\Lambda}$ can be expressed with the information of helicity angles of baryon and anti-baryon. Four decay parameters of Λ and $\bar{\Lambda}$, namely, α_-, α_+, α_0 and $\bar{\alpha}_0$, can be obtained with the joint helicity amplitude equations by the likelihood fit method. With the data sample of 10^{10} J/ψ decays accumulated by BESIII, the precision of the measurements is estimated to be about 10^{-3}.

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1. Introduction

$\Lambda\bar{\Lambda}$ decay is one of the octet baryon–anti-baryon pairs decays of J/ψ and other charmonium states. The branching ratio of $J/\psi \to \Lambda\bar{\Lambda}$ has been measured by BES [1] and CLEO [2] collaborations. The world average value is $(1.61\pm0.15)\times10^{-3}$ [3]. It is noted that this decay channel is very special to study the CP invariance not only in $J/\psi \to \Lambda\bar{\Lambda}$ but also in the nonleptonic decay of $\Lambda(\bar{\Lambda})$.

CP violation in $J/\psi \to \Lambda\bar{\Lambda}$ decay is studied in Ref. [4]. Two observables are suggested to test CP invariance,

$$A_{J/\psi} = \theta\left(\hat{\boldsymbol{p}}\cdot(\hat{\boldsymbol{q}}_1\times\hat{\boldsymbol{q}}_2)\right) - \theta\left(-\hat{\boldsymbol{p}}\cdot(\hat{\boldsymbol{q}}_1\times\hat{\boldsymbol{q}}_2)\right) , \tag{1}$$

$$B = \hat{\boldsymbol{p}}\cdot(\hat{\boldsymbol{q}}_1\times\hat{\boldsymbol{q}}_2) , \tag{2}$$

where $\theta(x)$ is 1 if $x > 0$ and is zero if $x < 0$. $\hat{\boldsymbol{p}}$, $\hat{\boldsymbol{q}}_1$ and $\hat{\boldsymbol{q}}_2$ are the momentum unit vectors of Λ, proton and anti-proton. Any nonzero values for them signal CP violation. Beside $J/\psi \to \Lambda\bar{\Lambda}$, the measurement can be carried in other charmonium states decay to $\Lambda\bar{\Lambda}$ experimentally.

CP violation can also be studied in nonleptonic decays of Λ. Nonleptonic hyperon decays have been long known as an ideal laboratory to study the parity violation [5]. Considering a nonleptonic decay of the hyperon Y, the angular distribution of the baryon in the center-of-mass (CM) system of Y takes the form of $\frac{dN}{d\Omega} \propto 1 + \alpha_Y \boldsymbol{P} \cdot \hat{\boldsymbol{p}}$, where $\boldsymbol{P}$ is the polarization vector of the hyperon, $\hat{\boldsymbol{p}}$ is the momentum unit vector of the baryon, and α_Y is the hyperon decay parameter, which characterizes the parity violation in the decays. Taking $\Lambda \to p\pi^-$ as an example, a CP-odd observable, A_Λ, can be defined as

$$A_\Lambda = \frac{\alpha_- + \alpha_+}{\alpha_- - \alpha_+}\,, \tag{3}$$

where α_- is the decay parameter of $\Lambda \to p\pi^-$, α_+ is the decay parameter of $\bar{\Lambda} \to \bar{p}\pi^+$. If Λ decays to $n\pi^0$ and $\bar{\Lambda}$ decays to $\bar{n}\pi^0$, α_- and α_+ should be replaced by α_0 and $\bar{\alpha}_0$. If CP is conserved, this observable vanishes for $\alpha_- = -\alpha_+$ or $\alpha_0 = -\bar{\alpha}_0$. Any nonzero value implies evidence for CP asymmetry in Λ decays. This asymmetry has been previously performed at $p\bar{p}$ colliders by the R608 [6] and PS185 [7] collaborations, and at an e^+e^- collider by the DM2 Collaboration [8]. The latest result is measured by the BES Collaboration [9], although the precision has been much improved, it is insufficient to observe CP violation at the level predicted by the Standard Model.

The precise measurement of the Λ decay parameter also plays an important role in the determination of Ω^- and Ξ^- decay parameters. Note that the non-polarized Ω^- or Ξ^- decays can produce polarized Λ particle. Namely, Λ is the daughter particle in the decays of Ω^- and Ξ^-. In the Λ rest frame, the angular distribution of the proton takes the form of $\frac{dN}{d\cos\theta} \propto 1 + \alpha_{\Omega(\Xi)}\alpha_- \cos\theta$. Experimentally, the extraction of $\alpha_{\Omega(\Xi)}$ from the product $\alpha_{\Omega(\Xi)}\alpha_-$ is dependent on the value of α_-. The accuracy of $\alpha_{\Omega(\Xi)}$ measurement is dependent on the accuracy of α_-. The situation is the same for $\bar{\Omega}^+$ and $\bar{\Xi}^+$ which can produce polarized $\bar{\Lambda}$. So, the measurement of α_- and α_+ plays a unique role in other hyperon decay parameters measurement.

In this paper, based on the study in Refs. [4, 10], we detail the information on the CP observables in $J/\psi \to \Lambda\bar{\Lambda}$, and the decay parameters of Λ and $\bar{\Lambda}$ with helicity amplitude analysis. The similar study is applied for $\psi(2S) \to \gamma\chi_{cJ} \to \gamma\Lambda\bar{\Lambda}$ [11]. Nowadays, 10^{10} J/ψ decays have been accumulated by BESIII detector, the advantage of this work is obvious and high accuracy can be achieved for the statistics.

2. Helicity amplitudes analysis of $J/\psi \to \Lambda\bar{\Lambda}$, $\Lambda \to B\pi$, $\bar{\Lambda} \to \bar{B}\pi$

The helicity amplitudes for $J/\psi \to \Lambda\bar{\Lambda}$, $\Lambda \to B\pi$, $\bar{\Lambda} \to \bar{B}\pi$ decays are constructed in the helicity frame defined as:

1. In $J/\psi \to \Lambda\bar{\Lambda}$, the z-axis of the J/ψ rest frame is along Λ outgoing direction, which changes from event to event. The e^+ beam is along the direction of the solid angle (θ, ϕ).
2. For Λ decay $\Lambda \to B\pi$, the solid angle of the daughter particle $\Omega_1(\theta_1, \phi_1)$ is referred to the Λ rest frame, where the z-axis is taken along the outgoing direction of Λ in its mother particle rest frame. The helicity frame of $\bar{\Lambda}$ has a similar definition which is described by the solid angle $\bar{\Omega}_1(\bar{\theta}_1, \bar{\phi}_1)$.

Figure 1 shows the definition of the helicity frame for $J/\psi \to \Lambda\bar{\Lambda}$, $\Lambda \to B\pi$, $\bar{\Lambda} \to \bar{B}\pi$.

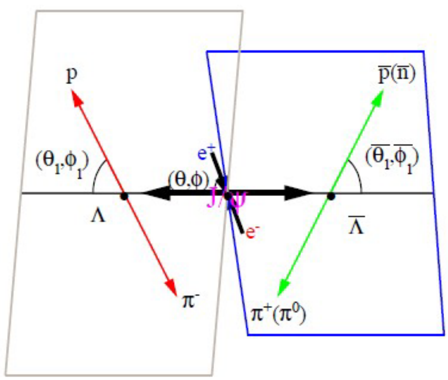

Fig. 1. Definition of the helicity frame for $J/\psi \to \Lambda\bar{\Lambda}$, $\Lambda \to B\pi$, $\bar{\Lambda} \to \bar{B}\pi$.

The joint helicity amplitudes of $J/\psi \to \Lambda\bar{\Lambda}$, $\Lambda \to B\pi$, $\bar{\Lambda} \to \bar{B}\pi$ can be expressed by $A_{\lambda,\bar{\lambda}}$, B_{λ_B} and $\bar{B}_{\lambda_{\bar{B}}}$ as

$$|\mathcal{M}|^2 \propto \sum_{\lambda,\bar{\lambda},\lambda',\bar{\lambda}',\lambda_B,\bar{\lambda}_{\bar{B}}} \rho^{\lambda-\bar{\lambda},\lambda'-\bar{\lambda}'}(\theta,\phi) A_{\lambda,\bar{\lambda}} A^*_{\lambda',\bar{\lambda}'} B_{\lambda_B} B^*_{\lambda_B} \bar{B}_{\lambda_{\bar{B}}} \bar{B}^*_{\lambda_{\bar{B}}} \times D^{1/2*}_{\lambda,\lambda_B}(\Omega_1) D^{1/2}_{\lambda',\lambda_B}(\Omega_1) D^{1/2*}_{\bar{\lambda},\bar{\lambda}_B}\left(\bar{\Omega}_1\right) D^{1/2}_{\bar{\lambda}',\bar{\lambda}_B}\left(\bar{\Omega}_1\right) , \quad (4)$$

where $\lambda, \bar{\lambda}$, $\lambda_B, \bar{\lambda}_B$ are the helicity values for $\Lambda, \bar{\Lambda}$, baryon and anti-baryon and $\rho^{(i,j)}(\theta,\phi) = \sum_{k=\pm 1} D^1_{i,k}(\Omega) D^{1*}_{j,k}(\Omega)$ is the density matrix for the J/ψ produced in e^+e^- annihilation. The element $\rho^{(i,j)}$ is equal to $\rho^{(j,i)*}$ and equal to $(-1)^{(i+j)}\rho^{(-i,-j)}$. The density matrix elements are shown in Table I.

TABLE I

J/ψ density matrix elements.

$\rho^{(1,1)}(\theta,\phi) = \frac{1+\cos^2\theta}{2}$	$\rho^{(0,0)}(\theta,\phi) = \sin^2\theta$
$\rho^{(1,0)}(\theta,\phi) = \frac{\sin\theta\cos\theta}{\sqrt{2}} e^{-i\phi}$	$\rho^{(1,-1)}(\theta,\phi) = \frac{\sin^2\theta}{2} e^{-2i\phi}$

$D^J_{\lambda,\lambda'}(\Omega) \equiv D^J_{\lambda,\lambda'}(\phi,\theta,0)$ is the Wigner D-function. The standard Wigner D-function is defined as:

$$D^j_{m,m'}(\alpha\beta\gamma) = \sum_{k=0}^{j+m}(-1)^{m'-m+k}\frac{\sqrt{(j-m)!(j+m)!(j-m')!(j+m')!}}{k!(j+m-k)!(j-m'-k)!(k+m'-m)!} \times e^{-im'\alpha}e^{-im\gamma}\left(\cos\frac{\beta}{2}\right)^{2j+m-m'-2k}\left(\sin\frac{\beta}{2}\right)^{m'-m+2k}. \quad (5)$$

The final baryons in Λ and $\bar{\Lambda}$ decays are $p, \bar{p}, n, \bar{n}$. The possible helicity values for $p, \bar{p}, n, \bar{n}$ are $1/2$ or $-1/2$, so the density matrix elements of $D^J_{\lambda,\lambda'}(\Omega)$ with $J = 1/2$ are shown in Table II.

TABLE II

Density matrix elements of $D^J_{\lambda,\lambda'}(\Omega)$ with $J = 1/2$.

$D^{\frac{1}{2}}_{\frac{1}{2},\frac{1}{2}}(\Omega) = e^{-i\frac{\phi}{2}}\cos\frac{\theta}{2}$	$D^{\frac{1}{2}}_{\frac{1}{2},-\frac{1}{2}}(\Omega) = -e^{-i\frac{\phi}{2}}\sin\frac{\theta}{2}$
$D^{\frac{1}{2}}_{-\frac{1}{2},\frac{1}{2}}(\Omega) = e^{i\frac{\phi}{2}}\sin\frac{\theta}{2}$	$D^{\frac{1}{2}}_{-\frac{1}{2},-\frac{1}{2}}(\Omega) = e^{i\frac{\phi}{2}}\cos\frac{\theta}{2}$

From parity invariance, $A_{-\lambda,-\lambda'} = A_{\lambda,\lambda'}$, so the decay parameters in Λ and $\bar{\Lambda}$ decays are defined as

$$\alpha_- = \alpha\left(\Lambda \to p\pi^-\right) = \frac{|B_{1/2}|^2 - |B_{-1/2}|^2}{|B_{1/2}|^2 + |B_{-1/2}|^2}, \quad (6)$$

$$\alpha_+ = \alpha\left(\bar{\Lambda} \to \bar{p}\pi^+\right) = \frac{|\bar{B}_{1/2}|^2 - |\bar{B}_{-1/2}|^2}{|\bar{B}_{1/2}|^2 + |\bar{B}_{-1/2}|^2}, \quad (7)$$

$$\alpha_0 = \alpha\left(\Lambda \to n\pi^0\right) = \frac{|B_{1/2}|^2 - |B_{-1/2}|^2}{|B_{1/2}|^2 + |B_{-1/2}|^2}, \quad (8)$$

$$\bar{\alpha}_0 = \alpha\left(\bar{\Lambda} \to \bar{n}\pi^0\right) = \frac{|\bar{B}_{1/2}|^2 - |\bar{B}_{-1/2}|^2}{|\bar{B}_{1/2}|^2 + |\bar{B}_{-1/2}|^2}. \quad (9)$$

If CP invariance is conserved, one has $B_\lambda = -\bar{B}_{-\lambda}$, $\alpha_- = -\alpha_+$ and $\alpha_0 = -\bar{\alpha}_0$ can be gotten. The angular distribution of baryons(p, n) and anti-baryons$(\bar{p}, \bar{n})$ in $\Lambda(\bar{\Lambda})$ rest frame can be written as:

$$\frac{dN}{d\Omega} \propto 1 + \alpha_- |\boldsymbol{P}| \cos\theta_1 \,, \qquad \Lambda \to p\pi^- \,, \tag{10}$$

$$\frac{dN}{d\Omega} \propto 1 + \alpha_+ |\boldsymbol{P}| \cos\bar{\theta}_1 \,, \qquad \bar{\Lambda} \to \bar{p}\pi^+ \,, \tag{11}$$

$$\frac{dN}{d\Omega} \propto 1 + \alpha_0 |\boldsymbol{P}| \cos\theta_1 \,, \qquad \Lambda \to n\pi^0 \,, \tag{12}$$

$$\frac{dN}{d\Omega} \propto 1 + \bar{\alpha}_0 |\boldsymbol{P}| \cos\bar{\theta}_1 \,, \qquad \bar{\Lambda} \to \bar{n}\pi^0 \,. \tag{13}$$

The product of decay parameter and polarization of $\Lambda(\bar{\Lambda})$ can be gotten by fitting the angular distribution of the baryons(anti-baryons). If the polarization of $\Lambda(\bar{\Lambda})$ could be measured, the decay parameter could be extracted, but in e^+e^- collision, the average value of polarization for the produced $\Lambda(\bar{\Lambda})$ is zero, so, experimentally, the decay parameter cannot be measured in this way.

From Eq. (4), the angular distribution parameter of $\Lambda(\bar{\Lambda})$ can be defined as

$$\alpha = \frac{\left|A_{1/2,-1/2}\right|^2 - 2\left|A_{1/2,1/2}\right|^2}{\left|A_{1/2,-1/2}\right|^2 + 2\left|A_{1/2,1/2}\right|^2} \,, \tag{14}$$

if the normalization condition is selected as $\| A_{1/2,-1/2}|^2 + 2|A_{1/2,1/2}|^2 = 1$, one has

$$\left|A_{1/2,-1/2}\right|^2 = \frac{1+\alpha}{2} \qquad \text{and} \qquad \left|A_{1/2,1/2}\right|^2 = \frac{1-\alpha}{4} \,. \tag{15}$$

Combining Eqs. (4), (6), (7), (14), (15) and integrating over ϕ, Eq. (4) is simplified to

$$\begin{aligned} \frac{d|\mathcal{M}|^2}{d(\cos\theta)d\Omega_1 d\bar{\Omega}_1} \propto\ & (1-\alpha)\sin^2\theta \\ & \times \left[1 + \alpha_-\alpha_+ \left(\cos\theta_1\cos\bar{\theta}_1 + \sin\theta_1\sin\bar{\theta}_1\cos\left(\phi_1 + \bar{\phi}_1\right)\right)\right] \\ & -(1+\alpha)\left(1+\cos^2\theta\right)\left(\alpha_-\alpha_+\cos\theta_1\cos\bar{\theta}_1 - 1\right) \,. \end{aligned} \tag{16}$$

Equation (16) is the helicity amplitude equation for $J/\psi \to \Lambda\bar{\Lambda} \to p\pi^-\bar{p}\pi^+$. With different final states of Λ and $\bar{\Lambda}$ decays, according to Eqs. (6)–(9), one can also get helicity amplitude equations for $J/\psi \to \Lambda\bar{\Lambda}$ with different final states:

— $J/\psi \to \Lambda\bar{\Lambda} \to p\pi^-\bar{n}\pi^0$

$$\begin{aligned}\frac{d|\mathcal{M}|^2}{d(\cos\theta)d\Omega_1 d\bar{\Omega}_1} &\propto (1-\alpha)\sin^2\theta \\ &\times \left[1+\alpha_-\bar{\alpha}_0\left(\cos\theta_1\cos\bar{\theta}_1+\sin\theta_1\sin\bar{\theta}_1\cos\left(\phi_1+\bar{\phi}_1\right)\right)\right] \\ &-(1+\alpha)\left(1+\cos^2\theta\right)\left(\alpha_-\bar{\alpha}_0\cos\theta_1\cos\bar{\theta}_1-1\right)\,, \end{aligned} \tag{17}$$

— $J/\psi \to \Lambda\bar{\Lambda} \to n\pi^0\bar{p}\pi^+$

$$\begin{aligned}\frac{d|\mathcal{M}|^2}{d(\cos\theta)d\Omega_1 d\bar{\Omega}_1} &\propto (1-\alpha)\sin^2\theta \\ &\times \left[1+\alpha_0\alpha_+\left(\cos\theta_1\cos\bar{\theta}_1+\sin\theta_1\sin\bar{\theta}_1\cos\left(\phi_1+\bar{\phi}_1\right)\right)\right] \\ &-(1+\alpha)\left(1+\cos^2\theta\right)\left(\alpha_0\alpha_+\cos\theta_1\cos\bar{\theta}_1-1\right)\,, \end{aligned} \tag{18}$$

— $J/\psi \to \Lambda\bar{\Lambda} \to n\pi^0\bar{n}\pi^0$

$$\begin{aligned}\frac{d|\mathcal{M}|^2}{d(\cos\theta)d\Omega_1 d\bar{\Omega}_1} &\propto (1-\alpha)\sin^2\theta \\ &\times \left[1+\alpha_0\bar{\alpha}_0\left(\cos\theta_1\cos\bar{\theta}_1+\sin\theta_1\sin\bar{\theta}_1\cos\left(\phi_1+\bar{\phi}_1\right)\right)\right] \\ &-(1+\alpha)\left(1+\cos^2\theta\right)\left(\alpha_0\bar{\alpha}_0\cos\theta_1\cos\bar{\theta}_1-1\right)\,. \end{aligned} \tag{19}$$

3. CP violation of $J/\psi \to \Lambda\bar{\Lambda}$, $\Lambda \to B\pi$, $\bar{\Lambda} \to \bar{B}\pi$

In the helicity frame of $J/\psi \to \Lambda\bar{\Lambda} \to p\pi^-\bar{p}\pi^+$, by using the momenta of $p, \pi^-, \bar{p}$ and π^+, Eqs. (1) and (2) can be written as:

$$A_{J/\psi} = \theta\left(\sin\theta_1\sin\bar{\theta}_1\sin\left(\phi_1-\bar{\phi}_1\right)\right)-\theta\left(\sin\theta_1\sin\bar{\theta}_1\sin\left(\bar{\phi}_1-\phi_1\right)\right)\,, \tag{20}$$

$$B = \sin\theta_1\sin\bar{\theta}_1\sin\left(\phi_1-\bar{\phi}_1\right)\,. \tag{21}$$

With Eq. (20), one can try to search for the CP violation in $J/\psi \to \Lambda\bar{\Lambda}$. Reference [12] has shown the result which is consistent with the expectation of CP conservation.

According to Refs. [9, 10], the Λ angular distribution parameter α and the product of Λ and $\bar{\Lambda}$ decay parameters with different final states can be determined by using the unbinned maximum likelihood method with Eqs. (16)–(19). Table III shows the undetermined parameters with different final states. In each channel, one can get the product of one Λ decay parameter and one $\bar{\Lambda}$ decay parameter.

TABLE III

Λ angular distribution parameter α and the product of Λ and $\bar{\Lambda}$ decay parameters with different final states.

Final states	Λ angular distribution	$\Lambda, \bar{\Lambda}$ decay parameters
$p\pi^-\bar{p}\pi^+$	α	$\alpha_-\alpha_+$
$p\pi^-\bar{n}\pi^0$	α	$\alpha_-\bar{\alpha}_0$
$n\pi^0\bar{p}\pi^+$	α	$\alpha_0\alpha_+$
$n\pi^0\bar{n}\pi^0$	α	$\alpha_0\bar{\alpha}_0$

The advantage of the helicity amplitude analysis for the four channels in Table III is that four products of Λ and $\bar{\Lambda}$ decay parameters can be determined. With these four products, Λ decay parameters α_-, α_+ and $\bar{\Lambda}$ decay parameters $\alpha_0, \bar{\alpha}_0$ can be obtained respectively. BESIII has already accumulated 10^{10} J/ψ decays, the detection efficiency is simulated at least 5% for pure neutral channels and 30% for pure charged channels with Monte Carlo, the measurements can be done with high precision. The precision of the measurements is shown in Table IV.

TABLE IV

The precision of the measurements.

Final states	Detection efficiency	$\Lambda, \bar{\Lambda}$ decay parameters	Precision
$p\pi^-\bar{p}\pi^+$	30%	$\alpha_-\alpha_+$	10^{-3}
$p\pi^-\bar{n}\pi^0$	15%	$\alpha_-\bar{\alpha}_0$	10^{-3}
$n\pi^0\bar{p}\pi^+$	15%	$\alpha_0\alpha_+$	10^{-3}
$n\pi^0\bar{n}\pi^0$	5%	$\alpha_0\bar{\alpha}_0$	10^{-2}

From Table IV, the precision of four products of Λ and $\bar{\Lambda}$ decay parameters could be improved by two orders of magnitude compared with the current values [3]. Combining these four products, one can get α_-, α_+, α_0 and $\bar{\alpha}_0$ respectively. The CP invariance could be obtained by Eq. (3) with α_-, α_+ or α_0, $\bar{\alpha}_0$, the precision would be also improved.

4. Summary

The helicity amplitudes for $J/\psi \to \Lambda\bar{\Lambda}$, $\Lambda(\bar{\Lambda})$ decays to different final states are presented. Two observables to search for CP violation in $J/\psi \to \Lambda\bar{\Lambda}$ can be expressed in the helicity frame with the information of helicity angles of baryon and anti-baryon. Four decay parameters of Λ and $\bar{\Lambda}$ can be obtained with the joint helicity amplitude equations by the likelihood fit method which has been already used in Ref. [9]. With the data sample of 10^{10} J/ψ decays accumulated by BESIII, the precision for α_-, α_+, α_0 and $\bar{\alpha}_0$, compared with the current value, could be improved by two orders of magnitude. It would be helpful for the further study of the CP invariance both in $J/\psi \to \Lambda\bar{\Lambda}$ and the nonleptonic decays of $\Lambda(\bar{\Lambda})$.

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