ANALYSIS OF THE $a_0(1450)$ AND $K_0^*(1430)$ WITH THE THERMAL QCD SUM RULES*

Zhi-Bin Wang

College of Electrical Engineering, Yanshan University
Qinhuangdao 066004, P.R. China

Zhi-Gang Wang†

Department of Physics, North China Electric Power University
Baoding 071003, P.R. China

(Received May 29, 2015; revised version received August 5, 2015)

In this article, we assume that the nonet scalar mesons above 1 GeV are $\bar{q}q$ states and study the temperature dependence of the masses and decay constants of the $a_0(1450)$ and $K_0^*(1430)$ using the thermal QCD sum rules. We fit the numerical values into analytical functions, which have applications in phenomenological analysis of the thermal QCD and in interpreting the heavy-ion collision experiments.

DOI:10.5506/APhysPolB.46.2467
PACS numbers: 12.38.Lg

1. Introduction

The fundamental properties of QCD such as (de)confinement and restoration (or breaking) of the chiral symmetry can be explored in the heavy-ion collisions. A quantitative understanding of these two phenomena is still lacking and hence poses a challenge for the future research. Heavy-ion collisions at FAIR energies permit the exploration of the QCD phase diagram in the region of high baryon densities, which is complementary to the investigations performed at the RHIC and LHC [1, 2]. In recent years, there has been an increasing interest in the modification of hadronic properties at finite temperature in order to interpret the heavy-ion collision experiments, and therefore in understanding of the restoration of the chiral symmetry. The QCD sum rules is one of the most attractive and applicable tools in this respect [3, 4].

† Corresponding author: zgwang@aliyun.com

Bochkarev and Shaposhnikov extend the QCD sum rules to study the ρ meson at finite temperature by assuming that both the operator product expansion and quark–hadron duality remain valid at finite temperature, but the vacuum condensates are replaced by their thermal expectation values [5]. The thermal QCD sum rules have several new features in exploring the deconfinement and restoration of the chiral symmetry, and have been applied to study the thermal properties of the light mesons, heavy mesons and heavy quarkonia [6, 7], and nucleons [8].

A key parameter signaling the deconfinement is the continuum threshold $s_0(T)$ [5] above which the hadronic spectral density is well described by perturbative QCD. At the critical temperature T_c for deconfinement, we expect that the hadrons disappear from the spectral functions and the quark condensates vanish. There exists a link between the deconfinement and chiral-symmetry restoration based on the QCD sum rules, $s_0(T)/s_0 = \langle\bar{q}q\rangle_\mathrm{T}/\langle\bar{q}q\rangle$ [9], which implies that the two phase transitions take place at roughly the same temperature.

The underlying structures of the scalar mesons are not well established theoretically, there are many candidates with $J^{PC} = 0^{++}$ below 2 GeV, which cannot be accommodated in one $\bar{q}q$ nonet. A prospective picture suggests that the scalar mesons $\{f_0(1370), a_0(1450), K_0^*(1430), f_0(1500)\}$ above 1 GeV can be assigned to be a conventional $\bar{q}q$ nonet with some possible glue components, while the scalar mesons $\{f_0(600), a_0(980), \kappa(800), f_0(980)\}$ below 1 GeV form an exotic $[qq]_{\bar{3}}[\bar{q}\bar{q}]_3$ nonet with substantial mixings with the $q\bar{q}$ states, meson–meson states and glueballs [10].

In this article, we assume that the nonet scalar mesons above 1 GeV are the $\bar{q}q$ states and we study their properties at finite temperature with the thermal QCD sum rules, and expect to obtain some new information about the nature of the scalar mesons from the thermal QCD analysis.

The article is arranged as follows: we derive the thermal QCD sum rules for the $a_0(1450)$ and $K_0^*(1430)$ in Sect. 2; in Sect. 3, we present the numerical results and discussions; Sect. 4 is reserved for our conclusions.

2. The thermal QCD sum rules for the $a_0(1450)$ and $K_0^*(1430)$

In the following, we write down the two-point correlation functions $\Pi(p)$ in the thermal QCD sum rules,

$$\Pi(p) = i\int d^4x e^{ip\cdot x}\langle T\left\{J(x)J(0)\right\}\rangle_\mathrm{T}\,, \tag{1}$$

where $J(x) = J_{a_0(1450)}(x)$, $J_{K_0^*(1430)}(x)$, $J_{f_0(1370)}(x)$, $J_{f_0(1500)}(x)$,

$$\begin{aligned} J_{a_0(1450)}(x) &= \frac{\bar{u}(x)u(x)-\bar{d}(x)d(x)}{\sqrt{2}}\,,\,\bar{u}(x)d(x)\,,\,\bar{d}(x)u(x)\,,\\ J_{K_0^*(1430)}(x) &= \bar{q}(x)s(x)\,,\,\bar{s}(x)q(x)\,,\\ J_{f_0(1370)}(x) &= \frac{\bar{u}(x)u(x)+\bar{d}(x)d(x)}{\sqrt{2}}\,,\\ J_{f_0(1500)}(x) &= \bar{s}(x)s(x) \end{aligned} \tag{2}$$

and $q = u$, d, the subscript index T denotes the thermal average of the correlation functions. The currents $J_{a_0(1450)}(x)$ interpolate the isospin triplet mesons $a_0(1450)$, the $\frac{\bar{u}(x)u(x)-\bar{d}(x)d(x)}{\sqrt{2}}$, $\bar{u}(x)d(x)$ and $\bar{d}(x)u(x)$ lead to the same QCD spectral density in the isospin limit. The currents $J_{K_0^*(1430)}(x)$ interpolate the isospin doublet mesons $K_0^*(1430)$, the $\bar{q}(x)s(x)$ and $\bar{s}(x)q(x)$ also lead to the same QCD spectral density in the isospin limit. In this article, we take the values $m_u = m_d$ and $\langle\bar{u}u\rangle = \langle\bar{d}d\rangle$. The currents $J_{f_0(1370)}(x)$ and $J_{f_0(1500)}(x)$ have zero isospin, and interpolate the $f_0(1370)$ and $f_0(1500)$, respectively. The $f_0(1370)$ and $f_0(1500)$ may have large glueball components [10], we neglect the currents $J_{f_0(1370)}(x)$ and $J_{f_0(1500)}(x)$ in this article. The thermal average of any operator $\mathcal{O}$ is defined as

$$\langle\mathcal{O}\rangle_{\mathrm{T}} = \frac{\mathrm{Tr}\left[\exp\left(-\beta H\right)\mathcal{O}\right]}{\mathrm{Tr}\left[\exp\left(-\beta H\right)\right]}\,, \tag{3}$$

the H is the QCD Hamiltonian, $\beta = \frac{1}{T}$, and the traces are carried out over the complete set of states.

We can insert a complete set of intermediate hadronic states with the same quantum numbers as the current operators $J(0)$ into the correlation functions $\Pi(p)$ to obtain the hadronic representation [3, 4]. After isolating the ground state scalar mesons and the pseudoscalar meson pairs, we get the following result

$$\begin{aligned} \Pi(p_0,\vec{p}) &= if_S^2m_S^2\left[\frac{i}{p^2-m_S^2+i\epsilon}+2\pi n_{\mathrm{B}}(p_0)\delta\left(p^2-m_S^2\right)\right]\\ &\quad -i\lambda_{SP_1P_2}^2g_{SP_1P_2}^2\int\frac{d^4k}{(2\pi)^4}\left[\frac{1+n_{\mathrm{B}}(\omega_1)}{k^2-m_1^2+i\epsilon}-\frac{n_{\mathrm{B}}(\omega_1)}{k^2-m_1^2-i\epsilon}\right]\\ &\quad \times\left[\frac{1+n_{\mathrm{B}}(\omega_2)}{(k-p)^2-m_2^2+i\epsilon}-\frac{n_{\mathrm{B}}(\omega_2)}{(k-p)^2-m_2^2-i\epsilon}\right]+\dots\,, \end{aligned} \tag{4}$$

where $\omega_1 = \sqrt{\vec{k}^2+m_1^2}$ and $\omega_2 = \sqrt{(\vec{k}-\vec{p})^2+m_2^2}$, the $n_{\mathrm{B}}(\omega) = [\exp(\beta\omega)-1]^{-1}$ is the Bose distribution function, the coupling constants f_S, $\lambda_{SP_1P_2}$,

$g_{SP_1P_2}$ are defined by

$$\begin{aligned}\langle 0|J(0)|S(p)\rangle &= f_S m_S\,, \\ \langle 0|J(0)|P_1P_2(p)\rangle &= \lambda_{SP_1P_2}\, g_{SP_1P_2}\,. \end{aligned} \tag{5}$$

In the soft limit $p \to 0$, $\lambda_{SP_1P_2} = \frac{f_S}{m_S}$. The revelent hadronic coupling constants in this article are:

$$\begin{aligned} g_{a_0^0\pi^0\eta} &= \sqrt{\frac{2}{3}}g\,, & g_{a_0^0K^+K^-} &= \frac{1}{\sqrt{2}}g\,, & g_{a_0^0K^0\bar{K}^0} &= -\frac{1}{\sqrt{2}}g\,, \\ g_{K_0^{*0}K^-\pi^+} &= g\,, & g_{K_0^{*0}\bar{K}^0\pi^0} &= -\frac{1}{\sqrt{2}}g\,, & g_{K_0^{*0}\bar{K}^0\eta} &= -\frac{1}{\sqrt{6}}g\,, \end{aligned} \tag{6}$$

where g can be obtained from the experimental data [11].

Now, we obtain the phenomenological retarded correlation functions $\Pi_H^R(p_0)$ through dispersion relation,

$$\begin{aligned}\Pi_H^R(p_0) &= \int_0^{s_0(T)} d\omega^2 \frac{1}{\omega^2 - \left(p_0^2 + i\epsilon\right)} \frac{\mathrm{Im}\Pi(\omega,0)}{\pi} \tanh\frac{\omega}{2T} \\ &= \frac{f_S^2 m_S^2}{\omega^2 - p_0^2} + \frac{\lambda_{SP_1P_2}^2 g_{SP_1P_2}^2}{16\pi^2} \\ &\quad\times \int_{(m_1+m_2)^2(T)}^{s_0(T)} d\omega^2 \frac{1}{\omega^2 - p_0^2} v \left[1 + 2n_{\mathrm{B}}\left(\frac{\omega}{2}\right)\right] \end{aligned} \tag{7}$$

and $v = \sqrt{1 - \frac{(m_1+m_2)^2}{\omega^2}}$.

We carry out the operator product expansion for the correlation functions $\Pi(p)$ at finite temperature by calculating the Feynman diagrams shown in Fig. 1, and obtain the retarded correlation functions at the quark level

$$\begin{aligned}\Pi^{\mathrm{R}}_{a_0(1450)}(p_0) &= \frac{3}{8\pi^2}\left(1 + \frac{11}{3}\frac{\alpha_{\mathrm{s}}}{\pi}\right) \int_0^{s_0(T)} d\omega^2 \frac{\omega^2}{\omega^2 - p_0^2}\left[1 - 2n_{\mathrm{F}}\left(\frac{\omega}{2}\right)\right] \\ &\quad - \frac{1}{8p_0^2}\left\langle \frac{\alpha_{\mathrm{s}}GG}{\pi}\right\rangle_{\mathrm{T}} - \frac{(m_u{+}2m_d)\langle\bar{u}u\rangle_{\mathrm{T}} + (m_d{+}2m_u)\left\langle\bar{d}d\right\rangle_{\mathrm{T}}}{2p_0^2} \\ &\quad - \frac{48\pi}{9}\frac{\alpha_{\mathrm{s}}\langle\bar{u}u\rangle_{\mathrm{T}}\left\langle\bar{d}d\right\rangle_{\mathrm{T}}}{p_0^4}\,, \end{aligned} \tag{8}$$

$$\Pi^{\mathrm{R}}_{K_0^*(1430)^S}(p_0) = \Pi^{\mathrm{R}}_{a_0(1450)}(p_0)\,\big|_{m_d\to m_s,\, \langle\bar{d}d\rangle\to\langle\bar{s}s\rangle}\,, \tag{9}$$

where the $n_{\rm F}(\omega) = [\exp(\beta\omega)+1]^{-1}$ is the Fermi distribution function. We obtain the $\Pi^{\rm R}_{a_0(1450)}(p_0)$ and $\Pi^{\rm R}_{K_0^*(1430)}(p_0)$ by assuming $J_{a_0(1450)}(x) = \bar{u}(x)d(x)$ and $J_{K_0^*(1430)}(x) = \bar{u}(x)s(x)$, respectively.

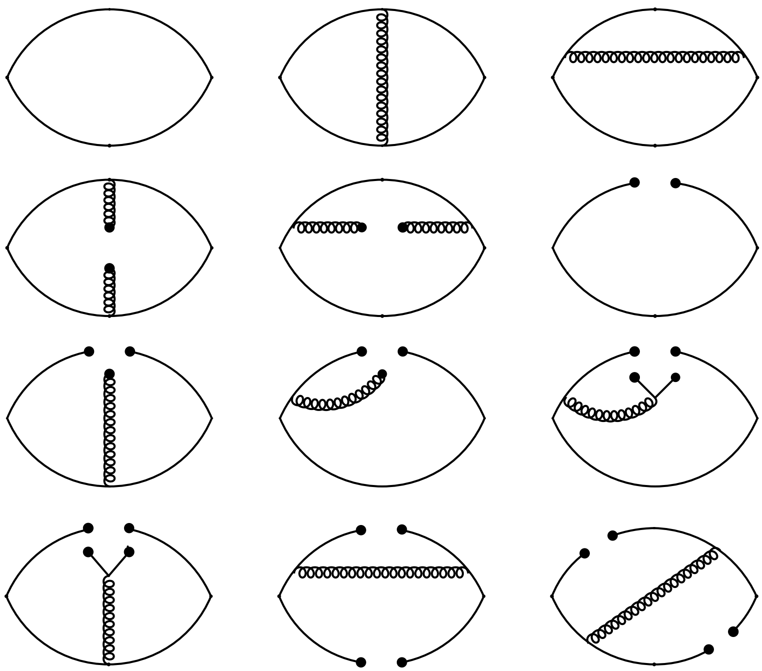

Fig. 1. Feynman diagrams that contribute to the correction functions; the diagrams obtained by exchanging the quark lines are implied.

We take the quark–hadron duality below the continuum thresholds $s_0(T)$, and perform the Borel transformation with respect to the variable $P^2 = -p_0^2$ to obtain the thermal QCD sum rules,

$$\begin{aligned} B_{M^2}\Pi^{\rm R}_{H,a_0(1450)}(p_0) &= B_{M^2}\Pi^{\rm R}_{a_0(1450)}(p_0)\,, \\ \frac{d}{d\,(1/M^2)}B_{M^2}\Pi^{\rm R}_{H,a_0(1450)}(p_0) &= \frac{d}{d\,(1/M^2)}B_{M^2}\Pi^{\rm R}_{a_0(1450)}(p_0)\,, \end{aligned} \tag{10}$$

where

$$\begin{aligned} B_{M^2}\Pi^{\rm R}_{H,S}(p_0) &= f_S^2 m_S^2 \exp\left(-\frac{m_S^2}{M^2}\right) + \frac{\lambda^2_{SP_1P_2} g^2_{SP_1P_2}}{16\pi^2} \\ &\quad \times \int_{(m_1+m_2)^2(T)}^{s_0(T)} d\omega^2 v \left[1 + 2n_{\rm B}\left(\frac{\omega}{2}\right)\right] \exp\left(-\frac{\omega^2}{M^2}\right)\,, \end{aligned} \tag{11}$$

$$
\begin{aligned}
B_{M^2}\Pi^{\rm R}_{a_0(1450)}(p_0) \;=\; & \frac{3}{8\pi^2}\left(1+\frac{11}{3}\frac{\alpha_{\rm s}}{\pi}\right)\int\limits_0^{s_0(T)} d\omega^2\omega^2\left[1-2n_{\rm F}\left(\frac{\omega}{2}\right)\right] \\
& \times\exp\left(-\frac{\omega^2}{M^2}\right)+\frac{1}{8}\left\langle\frac{\alpha_{\rm s}GG}{\pi}\right\rangle_{\rm T} \\
& +\frac{(m_u+2m_d)\langle\bar{u}u\rangle_{\rm T}+(m_d+2m_u)\left\langle\bar{d}d\right\rangle_{\rm T}}{2} \\
& -\frac{48\pi}{9}\frac{\alpha_{\rm s}\langle\bar{u}u\rangle_{\rm T}\left\langle\bar{d}d\right\rangle_{\rm T}}{M^2}\,. \qquad (12)
\end{aligned}
$$

The thermal QCD sum rules for the $K_0^*(1430)$ can be obtained by simple replacements.

3. Numerical results and discussions

The pion is the lowest excitation in QCD due to the dynamical breaking of chiral symmetry and the small u and d quark masses. When the temperature T is low and the thermal pion gas is dilute, the thermal average of an operator $\mathcal{O}$ is given by [6]

$$
\begin{aligned}
\langle\mathcal{O}\rangle_{\rm T} \;&=\; \langle 0|\mathcal{O}|0\rangle+\sum_{i=1,2,3}\int\frac{d^3\vec{k}}{(2\pi)^3\,2\omega_i}\left\langle\pi^i(k)|\mathcal{O}|\pi^i(k)\right\rangle n_{\rm B}(\omega_i)\,, \\
&=\; \langle 0|\mathcal{O}|0\rangle-\frac{1}{f_\pi^2}\sum_{i=1,2,3}\int\frac{d^3\vec{k}}{(2\pi)^3\,2\omega_i}\left\langle 0\right|\left[Q_5^i,\left[Q_5^i,\mathcal{O}\right]\right]\left|0\right\rangle n_{\rm B}(\omega_i)\,, \qquad (13)
\end{aligned}
$$

where $Q_5^i=\int d^3xA_0^i(x)$ is the axial-vector charge. The quark condensate

$$
\langle\bar{q}q\rangle_{\rm T} \;=\; \langle\bar{q}q\rangle\left(1-\frac{T^2}{4f_\pi^2}\right) \qquad (14)
$$

with the π meson decay constant $f_\pi=0.130$ GeV. At the temperature of chiral-symmetry restoration, $\langle\bar{q}q\rangle_{\rm T}=0$, then $T_{\rm c}=2f_\pi=260$ MeV, which is much larger than the critical temperature $T_{\rm c}\approx 200$ MeV from lattice calculations [12]. The thermal QCD sum rules for the ρ meson lead to results consistent with the critical temperature $T_{\rm c}=197$ MeV [13]. In this article, we take the value $T_{\rm c}=200$ MeV.

Here, we use the result based on chiral perturbation theory [14], and approximate temperature dependence of the quark condensate as

$$
\langle\bar{q}q\rangle_{\rm T}=\langle\bar{q}q\rangle\left[1-0.4\left(\frac{T}{T_{\rm c}}\right)^4-0.6\left(\frac{T}{T_{\rm c}}\right)^8\right]. \qquad (15)
$$

The values of the vacuum condensates and the light quark masses are taken as $\langle\bar{u}u\rangle = \langle\bar{d}d\rangle = -(0.24 \pm 0.01\,\text{GeV})^3$, $\langle\bar{s}s\rangle = 0.8\langle\bar{u}u\rangle$, $\langle\frac{\alpha_s GG}{\pi}\rangle = (0.33\,\text{GeV})^4$, $m_u = m_d = 6\,\text{MeV}$, $m_s = 140\,\text{MeV}$ at the energy scale $\mu = 1\,\text{GeV}$ [3, 4]. For the thermal gluon condensate, we take the function fitted to the data from lattice QCD [15]

$$\left\langle\frac{\alpha_s GG}{\pi}\right\rangle_T = \left\langle\frac{\alpha_s GG}{\pi}\right\rangle\left[1 - 1.015\left(\frac{T}{T_c}\right)^{3.078}\right]. \tag{16}$$

The threshold parameters are chosen as $s_0(T)/s_0 = \langle\bar{q}q\rangle_T/\langle\bar{q}q\rangle$ [9], $s_0^{a_0(1450)} = (2.0 \pm 0.1\,\text{GeV})^2$ and $s_0^{K_0^*(1430)} = (1.9 \pm 0.1\,\text{GeV})^2$. In the traditional QCD sum rules, the energy gap between the ground state and the first radial excited state is about (0.4–0.6) GeV.

From the experimental data $m_{K_0^*} = 1.425$ GeV, $\Gamma_{K_0^*} = 0.270$ GeV, $\text{Br}(K_0^*(1430)) \to K\pi) = 93\%$, $m_K = 0.495\,\text{GeV}$, $m_\pi = 0.140\,\text{GeV}$, we can obtain the value $g = 1.35\,\text{GeV}$ according to the partial decay width

$$\begin{aligned}&\Gamma\left(K_0^*(1430)\right) \to K\pi) \\ &= \frac{g^2}{16\pi m_{K_0^*}^3}\sqrt{\left[m_{K_0^*}^2 - (m_K + m_\pi)^2\right]\left[m_{K_0^*}^2 - (m_K - m_\pi)^2\right]}.\end{aligned} \tag{17}$$

The two-point QCD sum rules lead to the decay constants $f_{a_0(1450)} = 460\,\text{MeV}$ and $f_{K_0^*(1430)} = 445$ MeV at the energy scale $\mu = 1$ GeV [16], then we obtain the value $\lambda_{SP_1P_2} = 0.31$.

Firstly, let's set $T = 0$ and choose the Borel parameters as $M^2_{a_0(1450)} = (1.5\text{–}2.5)\ \text{GeV}^2$ and $M^2_{K_0^*(1430)} = (1.3\text{–}2.3)\ \text{GeV}^2$, then the pole contribution plus two-particle contribution are about (50–75)% and the main contributions come from the perturbative terms. The two criteria (pole dominance and convergence of the operator product expansion) of the QCD sum rules are fully satisfied, so we expect to make reasonable predictions. We take into account the uncertainties of the input parameters and obtain the results

$$\begin{aligned}m_{a_0(1450)} &= 1.48 \pm 0.09\ \text{GeV}, \\ m_{K_0^*(1430)} &= 1.42 \pm 0.10\ \text{GeV}, \\ f_{a_0(1450)} &= 0.44 \pm 0.02\ \text{GeV}, \\ f_{K_0^*(1430)} &= 0.41 \pm 0.02\ \text{GeV}\end{aligned} \tag{18}$$

at $T = 0$. The predictions for the masses $m_{a_0(1450)}$ and $m_{K_0^*(1430)}$ are consistent with the experimental data [11].

Now, we switch on the temperature T and choose the central values of the threshold parameters s_0 and the Borel parameters M^2, which lead to the experimental values of the masses at $T = 0$. In Eq. (11), the lower bound of the integral $(m_1 + m_2)^2(T) = (m_1 + m_2)^2\langle\bar{q}q\rangle_{\rm T}/\langle\bar{q}q\rangle$. In Fig. 2, we plot the masses and decay constants with variations of the T. From the figure, we can see that the masses and the decay constants decrease slowly at $T < 120$ MeV, then decrease quickly with the increase of T and reach zero near the critical temperature $T_{\rm c} = 200$ MeV. The hadrons undertake a phase transition from hadron states to quark–gluon plasma at sufficiently high temperature, the scalar mesons remain as hadronic states at $T < 120$ MeV as the mass and decay constant reductions are very small. At $T > 120$ MeV, the scalar mesons $a_0(1450)$ and $K_0^*(1430)$ disappear quickly.

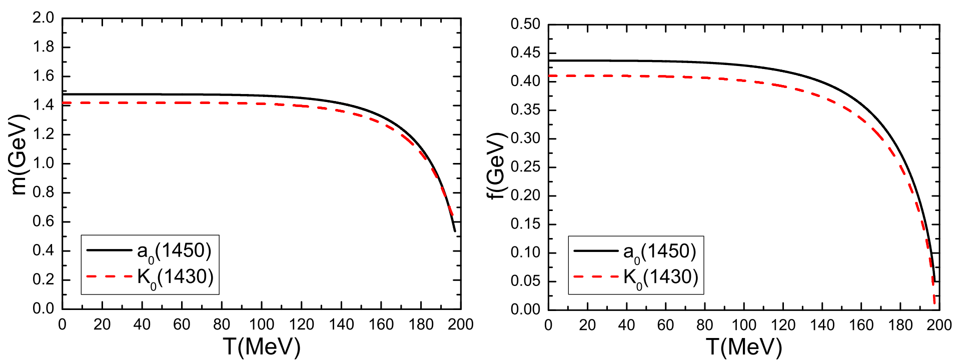

Fig. 2. The masses and decay constants with variations of the temperature T.

The numerical values of the masses and decay constants at finite temperature can be fitted into the following simple functions:

$$\begin{aligned}
m_{a_0(1450)}(T) &= 1.4861 - 3.5104 \times 10^{-4}\, t\, \exp\left(8.0147\, t\right)\ \text{GeV}\,, \\
m_{K_0^*(1430)}(T) &= 1.4226 - 3.9475 \times 10^{-4}\, t\, \exp\left(7.7755\, t\right)\ \text{GeV}\,, \\
f_{a_0(1450)}(T) &= 0.43987 - 5.3422 \times 10^{-4}\, t\, \exp\left(6.6279\, t\right)\ \text{GeV}\,, \\
f_{K_0^*(1430)}(T) &= 0.41664 - 7.4755 \times 10^{-4}\, t\, \exp\left(6.2985\, t\right)\ \text{GeV}\,, \qquad (19)
\end{aligned}$$

where $t = \frac{T}{T_{\rm c}}$. We can apply those functions in phenomenological analysis of the thermal QCD and in interpreting the heavy-ion collision experiments, which may shed light on the nature of the scalar mesons. The thermal properties of ρ, ω and J/ψ can be obtained by measuring the spectrum of the $\mu^+\mu^-$ pairs. In contrast to ρ, ω and J/ψ, it is difficult to study the thermal properties of the scalar mesons experimentally, more theoretical works are still needed.

4. Conclusion

In this article, we assume the nonet scalar mesons above 1 GeV are $\bar{q}q$ states and study the temperature dependence of the masses and decay constants of $a_0(1450)$ and $K_0^*(1430)$ using the thermal QCD sum rules, and fit the numerical values into analytical functions, which can be applied in phenomenological analysis of the thermal QCD and in interpreting heavy-ion collision experiments. In calculations, we assume the operator product expansion and quark–hadron duality remain valid at finite temperature, but replace the vacuum condensates by their thermal expectation values. Furthermore, we take into account both the ground state scalar mesons and the two-particle intermediate states at the phenomenological side. The numerical results indicate that the masses and decay constants decrease slowly at $T < 120$ MeV, then decrease quickly with the increase of T and reach zero near the critical temperature $T_{\mathrm{c}} = 200$ MeV. The phase transition takes place at about $T = (120–200)$ MeV.

This work is supported by the National Natural Science Foundation, Grant No. 11375063, and the Natural Science Foundation of Hebei province, Grant Number A2014502017.

REFERENCES

[1] B. Friman *et al.*, *The CBM Physics Book: Compressed Baryonic Matter in Laboratory Experiments*, Springer, Heidelberg 2011.

[2] M.F.M. Lutz *et al.*, `arXiv:0903.3905 [hep-ex]`; U. Wiedner, *Prog. Part. Nucl. Phys.* **66**, 477 (2011).

[3] M.A. Shifman, A.I. Vainshtein, V.I. Zakharov, *Nucl. Phys. B* **147**, 385 (1979); **147**, 448 (1979).

[4] L.J. Reinders, H. Rubinstein, S. Yazaki, *Phys. Rep.* **127**, 1 (1985).

[5] A.I. Bochkarev, M.E. Shaposhnikov, *Nucl. Phys. B* **268**, 220 (1986).

[6] T. Hatsuda, Y. Koike, S.H. Lee, *Nucl. Phys. B* **394**, 221 (1993).

[7] R.J. Furnstahl, T. Hatsuda, S.H. Lee, *Phys. Rev. D* **42**, 1744 (1990); T. Hatsuda, S.H. Lee, H. Shiomi, *Phys. Rev. C* **52**, 3364 (1995); C.A. Dominguez, M.S. Fetea, M. Loewe, *Phys. Lett. B* **387**, 151 (1996); S. Mallik, K. Mukherjee, *Phys. Rev. D* **58**, 096011 (1998); C.A. Dominguez, M. Loewe, J.C. Rojas, *J. High Energy Phys.* **0708**, 040 (2007); E.V. Veliev, T.M. Aliev, *J. Phys. G* **35**, 125002 (2008); E.V. Veliev, G. Kaya, *Acta Phys. Pol. B* **41**, 1905 (2010); C.A. Dominguez, M. Loewe, J.C. Rojas, Y. Zhang, *Phys. Rev. D* **83**, 034033 (2011); E.V. Veliev *et al.*, *Eur. Phys. J. A* **47**, 110 (2011).

[8] C. Adami, I. Zahed, *Phys. Rev. D* **45**, 4312 (1992); Y. Koike, *Phys. Rev. D* **48**, 2313 (1993); Y.J. Xu, Y.L. Liu, M.Q. Huang, *Commun. Theor. Phys.* **63**, 209 (2015).

[9] C.A. Dominguez, M. Loewe, *Phys. Lett. B* **233**, 201 (1989); A. Barducci *et al.*, *Phys. Lett. B* **244**, 311 (1990).

[10] C. Amsler, N.A. Tornqvist, *Phys. Rep.* **389**, 61 (2004).

[11] K.A. Olive *et al.*, *Chin. Phys. C* **38**, 090001 (2014).

[12] M. Cheng *et al.*, *Phys. Rev. D* **77**, 014511 (2008); A. Bazavov *et al.*, *Phys. Rev. D* **80**, 014504 (2009).

[13] A. Ayala, C.A. Dominguez, M. Loewe, Y. Zhang, *Phys. Rev. D* **86**, 114036 (2012).

[14] J. Gasser, H. Leutwyler, *Phys. Lett. B* **184**, 83 (1987); *Nucl. Phys. B* **321**, 387 (1989).

[15] C.A. Dominguez, M. Loewe, Y. Zhang, *Phys. Rev. D* **88**, 054015 (2013).

[16] H.Y. Cheng, C.K. Chua, K.C. Yang, *Phys. Rev. D* **73**, 014017 (2006).