BFKL AMPLITUDE PARAMETRISATION FOR THE JET–GAP–JET EVENTS AT THE LHC ENERGIES* **

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The process of jet–gap–jet (JGJ) production is briefly described. The JGJ scattering amplitude parametrisation is discussed. On the basis of full amplitude calculations, the parametrisation formulas for the leading logarithmic (LL) and next-to-leading logarithmic (NLL) approximations are obtained. For each case, a sum over all conformal spins is considered. The obtained agreement is better than 0.25% for LL and 1% for NLL.

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1. Introduction

Hard diffractive processes have been an important part of the studies performed in high energy physics since their discovery in the UA8 experiment [1]. The data collected by HERA and Tevatron detectors allowed to deepen these studies. Nevertheless, now, in the LHC era, many questions still remain open.

The definition of diffraction is usually connected with an exchange of a colourless object. In the case of the electromagnetic fields, the exchange is mediated by a photon, whereas in the case of strong interactions, it is related

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to the Pomeron. A colourless exchange leads to one of the most prominent features of diffraction — the presence of a large rapidity gap. In theory, a gap is a space interval in rapidity in which no particles are produced. Moreover, since the colourless exchange does not influence the quantum numbers, the state of interacting objects is preserved.

In particular, jet events could be created as a result of the colourless exchange between interacting gluons (see Fig. 1). Such a process will be hereafter called the jet–gap–jet (JGJ) production.

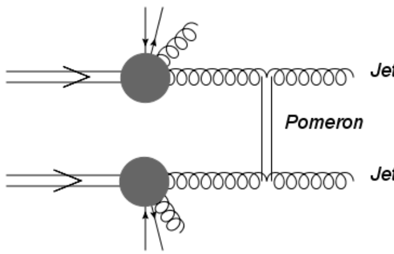

Fig. 1. Diagram of the jet–gap–jet production.

The presence of a hard scale in the jet–gap–jet events makes it possible to understand these events with perturbative methods. However, after many theoretical investigations, there is still no consensus on what is the relevant QCD mechanism. When the rapidity gap is sufficiently large, the perturbative QCD description of the jet–gap–jet events is usually performed in terms of the Balitsky–Fadin–Kuraev–Lipatov (BFKL) Pomeron [2]. As was shown in Ref. [3], this model can be tested experimentally by studying the behaviour of the ratio of dijet events with a gap to all dijet events as a function of the leading jet transverse momentum or the interval in rapidity between the two leading jets.

It is worth mentioning that the jet–gap–jet events can also be produced in single diffractive and double Pomeron exchange topologies [4]. Such events, so far never observed experimentally, are expected to be measured at the LHC.

In order to account for the experimental effects, the detector simulation is usually performed. The events used in such simulations should have properties similar to the ones that are to be measured. This requires a use of the Monte Carlo (MC) generators. The process of the jet–gap–jet production was implemented in several such tools, *e.g.* Herwig [5] or FPMC [6]. In order to speed up the calculations, the cross section formulas were parametrised [3]. These equations were fitted assuming that the transverse momentum of the leading jet did not exceed 120 GeV. However, as this assumption is no longer valid at the energies available at the LHC, a new parametrisation has to be carried out. In this paper, we present new fit formulas applicable for the LHC energies.

2. Jet–gap–jet formalism

The parton-level cross section for the jet–gap–jet production can be calculated as [3]

$$\frac{d^3\sigma^{pp\to XJJY}}{dx_1 dx_2 dp_{\mathrm{T}}^2} = S^2\, f_{\mathrm{eff}}\left(x_1, p_{\mathrm{T}}^2\right)\, f_{\mathrm{eff}}\left(x_2, p_{\mathrm{T}}^2\right) \frac{d\sigma^{gg\to gg}}{dp_{\mathrm{T}}^2}\,,$$

where S^2 is the gap survival probability, $f_{\mathrm{eff}}\left(x_{1,2}, p_{\mathrm{T}}^2\right)$ are the effective Parton Distribution Functions (PDFs)

$$f_{\mathrm{eff}}\left(x, p_{\mathrm{T}}^2\right) = g\left(x, p_{\mathrm{T}}^2\right) + \frac{C_{\mathrm{F}}^2}{N_{\mathrm{c}}^2}\left(q\left(x, p_{\mathrm{T}}^2\right) + \bar{q}\left(x, p_{\mathrm{T}}^2\right)\right)\,,$$

where $N_{\mathrm{c}} = 3$ is the number of colours, C_{F} — the QCD colour factor and g, q and $\bar{q}$ are the gluon, quark, antiquark distribution functions in the interacting hadrons. The $\frac{d\sigma^{gg\to gg}}{dp_{\mathrm{T}}^2}$ cross section is given by

$$\begin{aligned}\frac{d\sigma^{gg\to gg}}{dp_{\mathrm{T}}^2} &= \frac{1}{16\pi}\left|A\left(\Delta\eta, p_{\mathrm{T}}^2\right)\right|^2 \\ &= \frac{16N_{\mathrm{c}}^2\pi\alpha_{\mathrm{s}}^4}{C_{\mathrm{F}}^2 p_{\mathrm{T}}^4}\left|\sum_{p=-\infty}^{\infty}\int\frac{d\gamma}{2i\pi}\frac{\left[p^2-\left(\gamma-\frac{1}{2}\right)^2\right]\exp\left\{\bar{\alpha}\left(p_{\mathrm{T}}^2\right)\chi_{\mathrm{eff}}\,\Delta\eta\right\}}{\left[\left(\gamma-\frac{1}{2}\right)^2-\left(p-\frac{1}{2}\right)^2\right]\left[\left(\gamma-\frac{1}{2}\right)^2-\left(p+\frac{1}{2}\right)^2\right]}\right|^2,\end{aligned} \quad (1)$$

where χ_{eff} is the effective BFKL kernel [3], $\alpha_{\mathrm{s}}^2(p_{\mathrm{T}}^2) = \pi\bar{\alpha}_{\mathrm{s}}^2(p_{\mathrm{T}}^2)/N_{\mathrm{c}}$ — the running coupling constant and p denotes the conformal spin. The complex integral runs along an imaginary axis from $\frac{1}{2} - i\infty$ to $\frac{1}{2} + i\infty$. The necessity of summing up all the conformal spins was demonstrated in [7]. The BFKL kernel was calculated in the leading logarithmic (LL) [2] and next-to-leading logarithmic (NLL) [8] approximations.

3. Jet–gap–jet amplitude parametrisation

In principle, the JGJ amplitude can be directly implemented into the Monte Carlo generator. Unfortunately, due to the complexity of Eq. (1), the computation time is quite long. Since the usual number of events for a physics analysis is of the order of 10^5, the parametrisation procedure was postulated to speed up the generation process [3]. This parametrisation was done for the phase-space expected to be measurable using the Tevatron data [9]:

— transverse momentum of jets: $20 < p_{\mathrm{T}} < 120$ GeV,

— pseudorapidity distance between jets: $0 < \Delta\eta < 10$.

The conformal spins were summed from $p=-10$ to $p=10$, which is sufficient for the considered jet p_{T} range [3]. The parametrisation procedure resulted in the speed-up factors of 10 and 1000 for LL and NLL, respectively. In the following, we redo the parametrisation for the values expected to be observed at the LHC [10]: the range of jet transverse momentum extended to 1 TeV and the sum over conformal spins from -50 to 50.

3.1. Leading logarithmic approximation

The leading logarithmic approximation is known to be insufficient as the next-to-leading BFKL corrections are expected to be large [8]. However, for the completeness, we discuss it below. Denoting $z(p_{\mathrm{T}}^2)=\bar{\alpha}_{\mathrm{s}}^2(p_{\mathrm{T}}^2)\,\Delta\eta/2$, the LL cross section can be parametrised as:

— $A_{\mathrm{LL}}^{p=0}(z)=N\,\left[A+\exp\left(B+C\,z+D\,z^2+E\,z^3+F\,z^4\right)\right]$, for $p=0$,

— $A_{\mathrm{LL}}^{\mathrm{all}\;p}(z)=N\,\left[A+B\,z+\exp\left(C+D\,z+E\,z^2+F\,z^3\right)\right]$, for sum over conformal spins,

where the normalisation constant is equal to $N=\frac{\bar{\alpha}_{\mathrm{s}}^2}{4\pi}$ with α_{s}^2 fixed to 0.17.

The shape of the full amplitude as a function of pseudorapidity difference is shown in Fig. 2 (top). In the bottom, the comparison between the full amplitude calculations and the parametrisation results (fit) is presented. The obtained fit parameters are listed in Table I. For both considered cases, the differences are well below 2.5‰ for the whole pseudorapidity range.

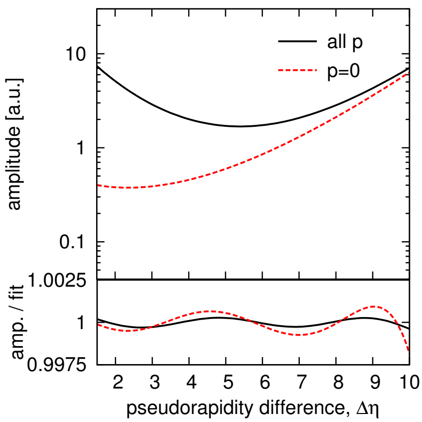

Fig. 2. Top: the shape of the full amplitude as a function of pseudorapidity difference. Bottom: comparison of the leading logarithmic full amplitude calculations and parametrisation results (fit). Solid (black) line is for the sum over all conformal spins and the dashed (red) is for $p=0$.

TABLE I

Fit parameters for leading logarithmic amplitude parametrisation for $p = 0$ and sum over conformal spins (all p).

Parameter	$p = 0$	All p
A	0.452 ± 0.023	-2.032 ± 0.022
B	2.2162 ± 0.0022	-1.135 ± 0.057
C	-6.436 ± 0.029	6.18035 ± 0.00012
D	21.16 ± 0.12	-14.3093 ± 0.0032
E	-16.46 ± 0.14	20.650 ± 0.011
F	5.586 ± 0.056	-6.4983 ± 0.0096

3.2. Next-to-leading logarithmic approximation

The parametrisation of NLL amplitude is more complex as it depends on both: jets transverse momenta (identical for both jets in collinear approach) and their pseudarapidity distance. In order to obtain the parametrisation formulas, the one-dimensional problem of finding the general dependence on the jet transverse momentum was addressed first. This formula was found to be

$$A_{\mathrm{NLL}}\left(p_{\mathrm{T}}, \Delta\eta = \text{fixed}\right) = N \left[A(\Delta\eta)\, p_{\mathrm{T}}^{B(\Delta\eta)} + C(\Delta\eta)\, p_{\mathrm{T}}^{D(\Delta\eta)}\right] . \qquad (2)$$

The exemplary results obtained for three different pseudorapidity distances ($\Delta\eta = 0.5$, $\Delta\eta = 5.5$ and $\Delta\eta = 9.5$) are shown in Fig. 3. In the top part of this figure, the shape of the distribution is presented, whereas in the bottom, the ratios of full amplitude calculation to fit are shown. The overall agreement is well below 1%, which means that Eq. (2) works for the whole considered $\Delta\eta$ range.

The parametrisation formulas for both cases ($p = 0$ and the sum over all conformal spins) were found to have the following forms:

— $A(z) = a_0 + a_1\, z + \exp\left(a_2 + a_3\, z + a_4\, z^2 + a_5\, z^3\right)$,

— $B(z) = b_0 + b_1\, z$,

— $C(z) = c_0 + c_1\, z + \exp\left(c_2 + c_3\, z + c_4\, z^2 + c_5\, z^3\right)$,

— $D(z) = d_0 + d_1\, z + d_2\, z^2 + d_3\, z^3$.

The shape of the full amplitude as a function of pseudorapidity difference for the NLL is shown in Fig. 4 (top). In the bottom, the comparison of the full amplitude calculations and the parametrisation results is shown. The obtained fit parameters are listed in Table II. For both considered cases, the differences are below 1% for the whole considered pseudorapidity range.

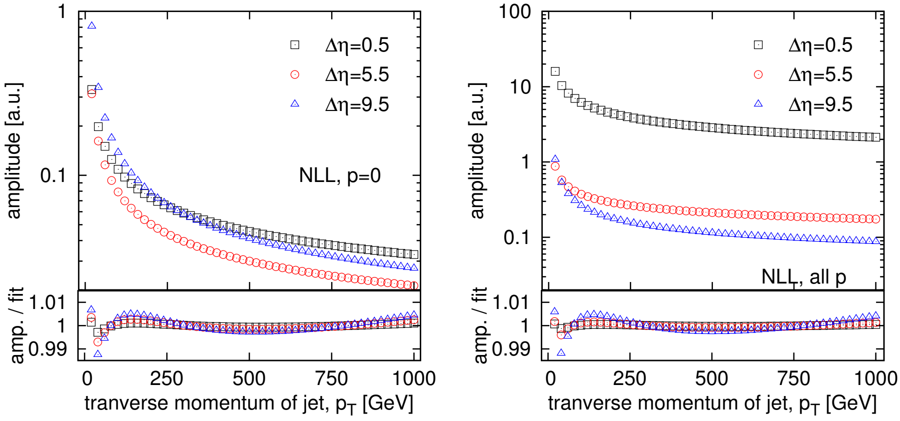

Fig. 3. Top: the shape of the full amplitude as a function of jet transverse momentum. Bottom: comparison between next-to-leading logarithmic full amplitude calculations and parametrisation results (fit) for $p = 0$ (left) and sum over all conformal spins (right). Rectangles (black) are for $\Delta\eta = 0.5$, circles (red) for $\Delta\eta = 5.5$ and triangles (blue) for $\Delta\eta = 9.5$.

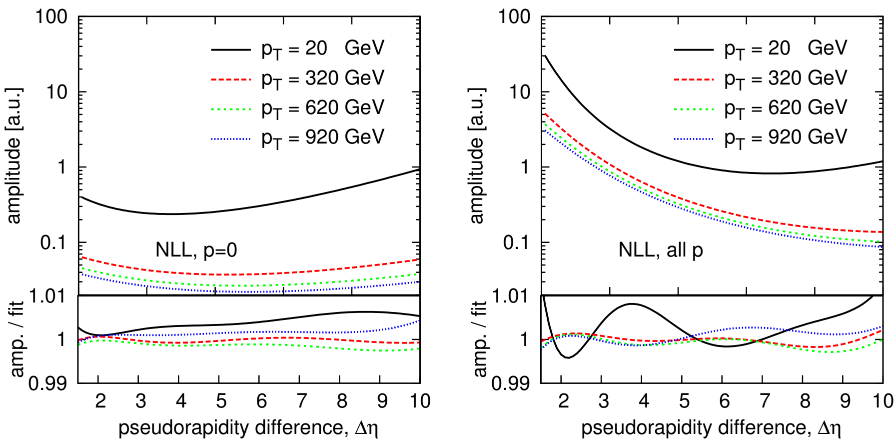

Fig. 4. Top: the shape of the full amplitude as a function of jet transverse momentum. Bottom: comparison between next-to-leading logarithmic full amplitude calculations and parametrisation results (fit) for $p = 0$ (left) and sum over all conformal spins (right) as a function of presudorapidity difference. Black (solid) lines are for jet transverse momentum of $p_{\mathrm{T}} = 20$ GeV, dashed (red) for $p_{\mathrm{T}} = 320$ GeV, fine-dashed (green) for $p_{\mathrm{T}} = 620$ GeV and dotted (blue) for $p_{\mathrm{T}} = 920$ GeV.

TABLE I

Fit parameters for next-to-leading logarithmic amplitude parametrisation for $p = 0$ and sum over conformal spins (all p).

Parameter	$p = 0$	All p
a_0	-51.3 ± 2.0	-24.7 ± 1.8
a_1	28.3 ± 9.5	235.6 ± 2.5
a_2	4.755 ± 0.019	7.606 ± 0.019
a_3	-2.13 ± 0.23	-19.97 ± 0.21
a_4	8.76 ± 0.52	36.60 ± 0.45
a_5	-3.14 ± 0.31	-16.56 ± 0.29
b_0	-0.9751 ± 0.0034	-0.6666 ± 0.0041
b_1	-0.7556 ± 0.0058	-0.9422 ± 0.0064
c_0	-0.58 ± 0.12	0.826 ± 0.045
c_1	0.300 ± 0.064	1.72 ± 0.12
c_2	2.031 ± 0.018	6.194 ± 0.014
c_3	-2.97 ± 0.13	-14.564 ± 0.088
c_4	6.87 ± 0.29	16.51 ± 0.21
c_5	-2.23 ± 0.18	-5.22 ± 0.15
d_0	-0.3880 ± 0.0018	-0.3681 ± 0.0015
d_1	0.096 ± 0.011	0.7878 ± 0.0093
d_2	-0.547 ± 0.025	-1.423 ± 0.020
d_3	0.216 ± 0.017	0.586 ± 0.014

4. Summary

The measurement of the jet–gap–jet production process would allow us not only to determine the cross section, but also to test the BFKL model.

The previous parametrisation, constructed to describe the Tevatron data, was extended to include phase-space regions available at the LHC energies. The jet transverse momentum was assumed to be within the 20 GeV to 1 TeV range and the conformal spins were summed from -50 to 50. The obtained agreement between the full amplitude calculations and the parametrisation results was found to be better than 0.25% for the leading logarithmic and 1% for the next-to-leading logarithmic approximations.

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