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# ON SYSTEMATICS OF SPONTANEOUS FISSION HALF-LIVES[*]

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A simple phenomenological formula, based on the Świątecki idea for the evaluation of the spontaneous fission half-lives, is proposed. The model contains only one adjustable parameter fixed to the data for even–even nuclei and an additional hindrance factor for odd nuclei, which gives the effect of an odd particle. A good agreement with the experimental data for isotopes with $90 \leq Z \leq 103$ is achieved.

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## 1. Introduction

Close correlations between spontaneous fission half-lives and ground state masses of nuclei were noticed in 1955 by Świątecki [1]. He proposed simple formula, joining the observed fission half-lives with the difference between experimental and liquid drop masses ($\delta M$). A regular dependence of $\log_{10} T^{\mathrm{sf}}_{1/2}$ (after adding an empirical correction $k\delta M$) as a function of the fissility parameter $Z^2/A$ was obtained in Ref. [1].

The aim of the present paper is to check, whether the experimental data of spontaneous fission half-lives of heavy isotopes measured in last 60 years [2], fulfill conditions of the Świątecki's brilliant idea. To calculate macroscopic masses of nuclei, a modern version of the liquid drop model, derived in Ref. [3], is applied.

## 2. Calculation details

Following the method presented in Ref. [1], we have defined function $f(Z)$ as the logarithm of the spontaneous fission half-lives, corrected by a

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term proportional to the empirical microscopic energy

$$f(Z) = \log_{10}\left[T^{\rm sf}_{1/2}(Z,N)/y\right] + k\delta M(Z,N)\,, \tag{1}$$

where $k$ is an adjustable parameter. The empirical value of ground-state microscopic energy is defined as

$$\delta M^{\rm exp}_{\rm micr}(Z,N) = M_{\rm exp}(Z,N) - M_{\rm LSD}(Z,N,0)\,. \tag{2}$$

The macroscopic part of the mass ($M_{\rm LSD}$) for all considered nuclei were evaluated using the Lublin–Strasbourg Drop mass formula [3]

$$\begin{aligned} M_{\rm LSD}(Z,N,{\rm def}) &= 7.289034\,Z + 8.071431\,N - 0.00001433\,Z^{2.39} \\ &-15.4920(1-1.8601I^2)A + 16.9707(1-2.2938I^2)A^{2/3}\,B_{\rm surf}({\rm def}) \\ &+3.8602(1+2.3764I^2)A^{1/3}\,B_{\rm cur}({\rm def}) + 0.70978\,Z^2/A^{1/3}\,B_{\rm Coul}({\rm def}) \\ &-0.9181\,Z^2/A - 10\exp(-4.2|I|)\,B_{\rm cong}({\rm def}) + E_{\rm o-e}\,, \end{aligned} \tag{3}$$

where the odd–even energy term $E_{\rm o-e}$ is given in Ref. [4].

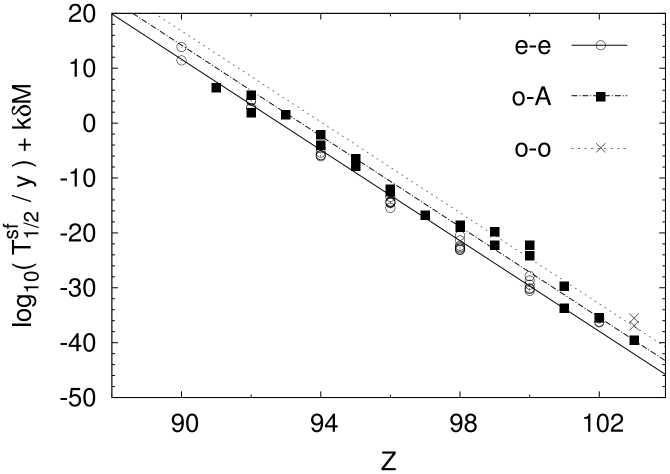

Fig. 1. Logarithms of the observed spontaneous fission half-lives [2] corrected with masses “shifts”, given in Eq. (1), as a function of proton number.

The function $f^{\rm FIT}(Z) = -4.1Z + 380.2 - k\delta M(Z,N)$ was fitted for 35 even–even fissioning isotopes with $Z \leq 102$. As one can see in Fig. 1, smooth dependence was achieved for $k = 7.7$/MeV. The slope of similar data for odd-$A$ and odd–odd nuclei is almost the same, but the corresponding lines are shifted by a constant. This constant, called *hindrance factor*, is taken $h = 2.5$ for odd-$A$ nuclei and doubled for odd–odd systems. Using this linear dependence between $f^{\mathrm{FIT}}$ and $Z$, one can approximate the spontaneous fission half-lives by the following formula:

$$\log_{10}\left(\frac{T^{\mathrm{sf}}_{1/2}(Z,N)}{y}\right) = -4.1Z + 380.2 - 7.7\delta M(Z,N) + \begin{cases} 0 & \text{for even} - \text{even}\,, \\ 2.5 & \text{for odd} - \mathrm{A}\,, \\ 5 & \text{for odd} - \text{odd}\,. \end{cases} \tag{4}$$

The half-lives ($T^{\mathrm{sf}}_{1/2}$) obtained in this way are compared with the experimental data taken from Ref. [2] in Fig. 2. Surprisingly good agreement of the model estimates with the observed spontaneous fission half-lives is achieved for all even–even isotopes.

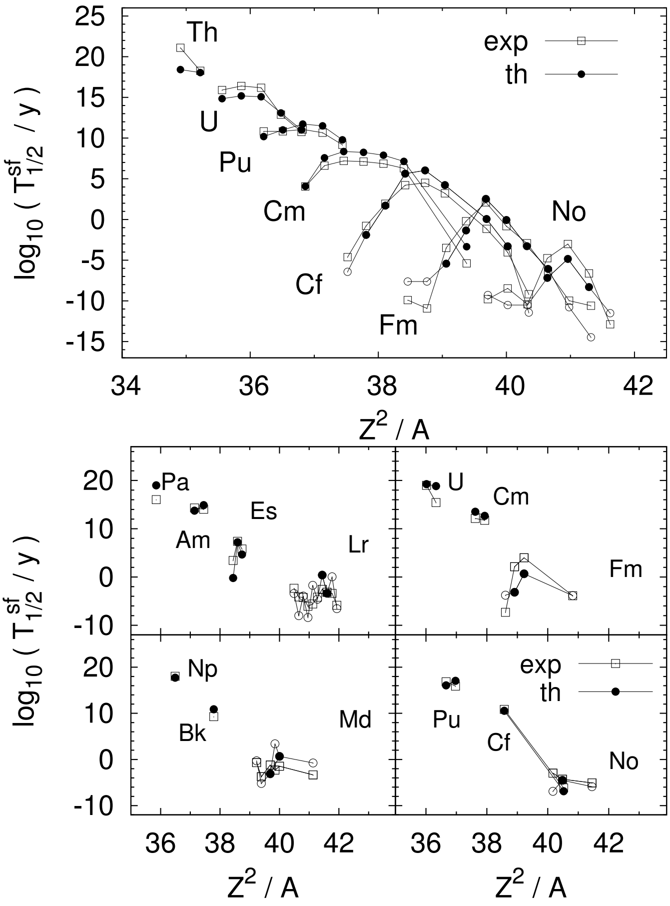

Fig. 2. Spontaneous fission half-lives of even–even (top) and odd (bottom) nuclei, calculated using formula (4) in comparison to the experimental values [2].

In order to understand this striking for the first sight result, one should remind the topographical theorem, proposed by Myers and Świątecki [4]: the mass of a nucleus in a saddle point is determined by the macroscopic part of the binding energy. The shell effects at the saddle are negligible, so the fission barrier height is approximately determined by the difference of the macroscopic (here LSD) mass at saddle and the ground state experimental mass: $V_B(Z,N) = M_{\mathrm{LSD}}(Z,N,\mathrm{saddle}) - M_{\mathrm{exp}}^{\mathrm{gs}}$. Liquid drop model correlates fission half-life with height of the macroscopic fission barrier. The $k$ parameter in Eq. (1) gives enhancement of the half-life due to the ground state shell effects. The constant $k$ plays a similar role as the hindrance factor, which origins from the barrier augmentation due to spin and parity conservation of an odd-$A$ nucleus along fission path [6].

## 3. Summary

The following conclusions can be drawn from our investigation:

- Simple phenomenological formula for the spontaneous fission half-lives depending on proton number and microscopic energy correction in the ground state reproduces experimental data for even–even nuclei with surprisingly good accuracy.
- Quality of evaluation for odd nuclei is worse, what is due to the fact, that the effect of an odd-particle on the barrier penetrability is described here by a single constant, independent of angular momentum or parity of the odd nucleon.

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## REFERENCES

[1] W.J. Świątecki, *Phys. Rev.* **100**, 937 (1955).

[2] `http://www.nndc.bnl.gov/nudat2/`

[3] K. Pomorski, J. Dudek, *Phys. Rev.* **C67**, 044316 (2003).

[4] W.D. Myers, W.J. Świątecki, *Nucl. Phys.* **A601**, 141 (1996).

[5] G. Audi *et al.*, *Chin. Phys.* **C36**, 1157 (2012).

[6] H.J. Krappe, K. Pomorski, *Theory of Nuclear Fission*, Springer Verlag, *Lect. Notes Phys.*, Vol. **838**, 2012, ISBN 978-3-642-23514-6.