RECENT RESULTS AT ULTRAHIGH SPIN: TERMINATING STATES AND BEYOND IN MASS 160 RARE-EARTH NUCLEI*

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A classic region of band termination at high spin occurs in rare-earth nuclei with around ten valence nucleons above the ^{146}Gd closed core. Results are presented here for such non-collective oblate ($\gamma = 60^{\circ}$) terminating states in odd-Z ^{155}Ho, odd–odd ^{156}Ho, and even–even ^{156}Er, where they are compared with neighbouring nuclei. In addition to these particularly favoured states, the occurrence of collective triaxial strongly deformed (TSD) bands, bypassing the terminating states and extending to over $65\hbar$, is reviewed.

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1. Introduction

Rare-earth nuclei with a limited number of neutrons above the $N = 82$ shell closure, and protons above $Z = 64$, lie in a transitional region where collective behaviour varies rapidly with changing particle number [1, 2]. They tend to be susceptible to shape changes in the triaxial γ plane and exhibit energetically favoured non-collective band-terminating states at spins around 40–50$\hbar$ dependent on the limited number of valence particles available outside the ^{146}Gd closed core [3].

Above the terminating states, particle-hole excitations across the $Z = 64$ shell gap of the spherical nuclear core can generate states of higher angular momentum [4], but these core-excited configurations are energetically expensive, characterized by high energy transitions ($\sim$ 1.0–2.5 MeV) feeding the favoured terminating states. Moreover, these excitations can only increase the available angular momentum by a few units ($\sim 4\hbar$) before terminating in additional non-collective oblate states. To go to higher spin, the nucleus must adopt a more collective shape, most likely a triaxial strongly deformed (TSD) shape with a large moment of inertia as first seen in ^{157,158}Er [5]. Particle-hole excitations across the $N = 82$ shell gap are required to produce such collective shapes.

2. New terminating states in Ho and Er isotopes

The $^{155-157}$Ho isotopes were studied using the ^{124}Sn(^{37}Cl,xn)$^{161-x}$Ho fusion-evaporation reaction at a bombarding energy of 180 MeV. The experiment was performed at the ATLAS accelerator facility of the Argonne National Laboratory, USA, using the Gammasphere γ-ray spectrometer [6] equipped with 101 Compton-suppressed HPGe detectors. Events were recorded when at least five detectors fired in prompt time coincidence, and over the course of six days of beam time a total of approximately 10^{10} events were recorded. In the off-line analysis, approximately 10^{11} quadruple-coincident γ-ray events (γ^4) were unfolded from the raw data and replayed into a four-dimensional hypercube for subsequent analysis. In addition, γ-ray spectra were generated according to the angle, θ, of the HPGe detectors relative to the beam direction in order to determine the multipolarities of the emitted γ rays through an angular-correlation analysis.

A comprehensive level scheme has been built for odd-Z ^{155}Ho [7, 8]. This nucleus appears collective up to spin $\sim 30\hbar$ but irregular structures emerge at higher spin, culminating in an energetically favoured 'valence-space' terminating state at $I^\pi = 79/2^-$. Possible one-particle–one-hole 'core-excited' non-collective states at $I^\pi = 87/2^-$ and $89/2^+$ have also been identified. A representative γ^4 coincidence spectrum is shown in Fig. 1, which includes several high-energy ($E_\gamma > 1$ MeV) transitions that feed the maximally aligned, valence-space $I^\pi = 79/2^-$ state. In particular, the 1883 and 1927 keV stretched E2 transitions that directly feed this state demonstrate the high energy required to form particle-hole excitations across the $Z = 64$ gap in order to generate higher-spin states beyond $79/2\hbar$; a similar situation was found in ^{157}Er [4].

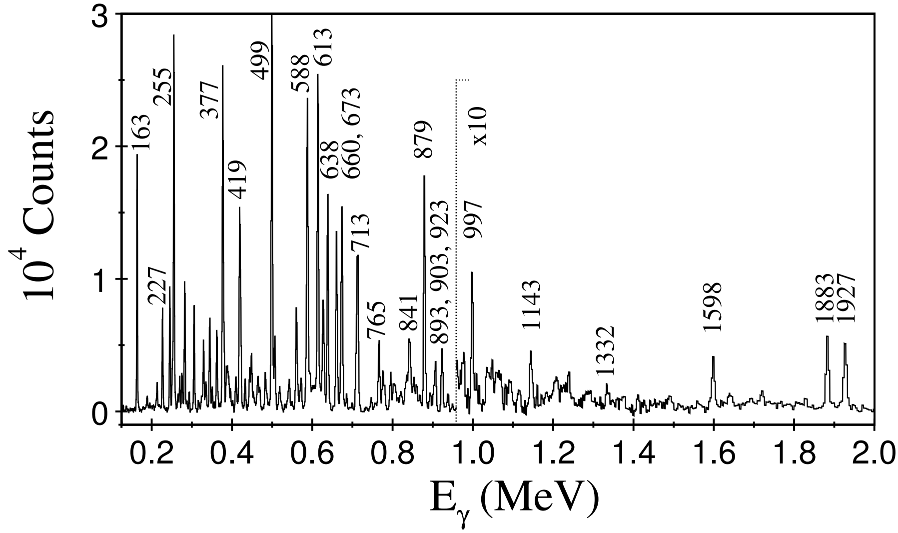

Fig. 1. Triple-gated quadruple-coincidence spectrum for ^{155}Ho [7, 8], with transition energies labelled in keV. The y-scale is expanded by a factor of ten above $E_\gamma \sim 0.95$ MeV in order to show the weak high-energy transitions that feed the maximally aligned $I^\pi = 79/2^-$ band-terminating state.

The level scheme of the odd–odd ^{156}Ho isotope has also been considerably expanded from this work, with favoured valence-space terminating states established at $I^\pi = 41^+$ and 42^+ [9]. Finally, analysis of a previous Gammasphere experiment [7, 8, 10] has identified a new core-excited terminating state in ^{156}Er at $I^\pi = 46^+$, above the valence-space termination at 42^+ [11].

3. $N = 88$ systematics

In Fig. 2, the energies relative to a rigid-rotation reference [12] of the highest-spin states in ^{155}Ho are plotted together with levels in the $N = 88$ isotones ^{154}Dy [13] and ^{156}Er. The non-collective oblate states are encircled. The $I^\pi = 79/2^-$ terminating state of ^{155}Ho represents a fully aligned configuration where all nine particles outside the ^{146}Gd closed core are maximally aligned and generate the full nuclear spin. Similarly, the 36^+ state in ^{154}Gd and the 42^+ state in ^{156}Er correspond to maximally aligned valence-space configurations of these nuclei. However, the $65/2^-$ state of ^{155}Ho represents a non-collective state where two $f_{7/2}$ neutrons are still coupled to $I^\pi = 0^+$, *i.e.* this state is not one in which all valence particles are maximally aligned. In order to generate states of higher angular momentum than the valence-space configurations, energetically expensive particle-hole excitations of the ^{146}Gd closed core are required. In this manner, the 42^- and 48^+ states in ^{154}Dy, together with the new states at $87/2^-$ and $89/2^+$ states in ^{155}Ho, and the 46^+ state in ^{156}Er, are formed by proton particle-hole excitations across $Z = 64$. The configurations of the non-collective states in these $N = 88$ isotones, relative to the doubly magic ^{146}Gd core, are given in full in Table I. The aligned neutron configuration, $\nu\{(i_{13/2})^2_{12}(f_{7/2}/h_{9/2})^4_{14}\}_{26^+}$, is particularly favourable for these $N = 88$ isotones.

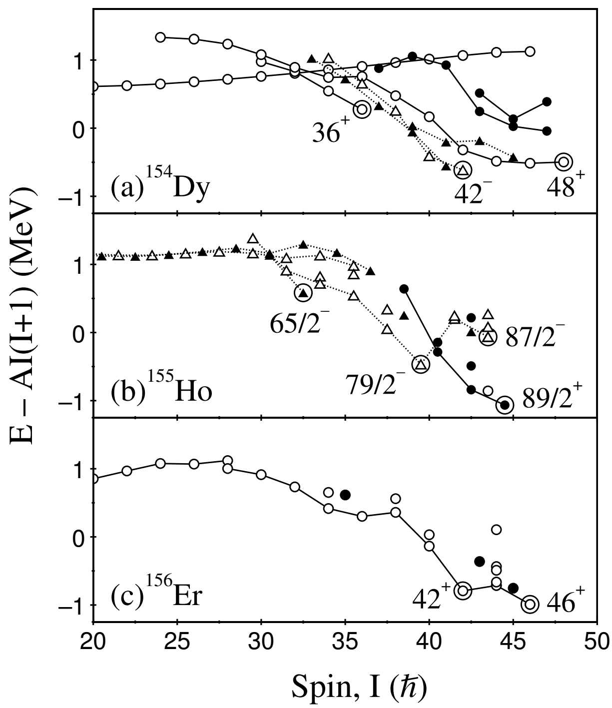

Fig. 2. Energy plotted relative to a rigid-rotation reference for the $N = 88$ isotones (a) ^{154}Dy, (b) ^{155}Ho, and (c) ^{156}Er, respectively. Non-collective oblate states are encircled and labelled by their spin and parity.

TABLE I

Experimentally observed non-collective states in $N = 88$ isotones, relative to the ^{146}Gd core, including the new states in ^{155}Ho and ^{156}Er.

Nuclide	I^π	Ref.	Aligned configuration
$^{154}_{66}$Dy	36^+	[13]	$\pi\{(h_{11/2})^2_{10}\}_{10^+}$ $\nu\{(i_{13/2})^2_{12}(f_{7/2}/h_{9/2})^4_{14}\}_{26^+}$
$^{154}_{66}$Dy	42^-	[13]	$\pi\{(d_{5/2}/g_{7/2})^{-1}_{5/2}(h_{11/2})^3_{27/2}\}_{16^-}$ $\nu\{(i_{13/2})^2_{12}(f_{7/2}/h_{9/2})^4_{14}\}_{26^+}$
$^{154}_{66}$Dy	48^+	[13]	$\pi\{(d_{5/2}/g_{7/2})^{-2}_{6}(h_{11/2})^4_{16}\}_{22^+}$ $\nu\{(i_{13/2})^2_{12}(f_{7/2}/h_{9/2})^4_{14}\}_{26^+}$
$^{155}_{67}$Ho	$65/2^-$		$\pi\{(h_{11/2})^3_{27/2}\}_{27/2^-}$ $\nu\{(i_{13/2})^2_{12}(f_{7/2}/h_{9/2})^4_{7}\}_{19^+}$
$^{155}_{67}$Ho	$79/2^-$		$\pi\{(h_{11/2})^3_{27/2}\}_{27/2^-}$ $\nu\{(i_{13/2})^2_{12}(f_{7/2}/h_{9/2})^4_{14}\}_{26^+}$
$^{155}_{67}$Ho	$87/2^-$		$\pi\{(d_{5/2}/g_{7/2})^{-1}_{5/2}(h_{11/2})^3_{27/2}(d_{3/2})^1_{3/2}\}_{35/2^-}$ $\nu\{(i_{13/2})^2_{12}(f_{7/2}/h_{9/2})^4_{14}\}_{26^+}$
$^{155}_{67}$Ho	$89/2^+$		$\pi\{(d_{5/2}/g_{7/2})^{-1}_{5/2}(h_{11/2})^4_{16}\}_{37/2^+}$ $\nu\{(i_{13/2})^2_{12}(f_{7/2}/h_{9/2})^4_{14}\}_{26^+}$
$^{156}_{68}$Er	42^+	[11]	$\pi\{(h_{11/2})^4_{16}\}_{16^+}$ $\nu\{(i_{13/2})^2_{12}(f_{7/2}/h_{9/2})^4_{14}\}_{26^+}$
$^{156}_{68}$Er	46^+		$\pi\{(d_{5/2}/g_{7/2})^{-1}_{5/2}(h_{11/2})^4_{16}(d_{3/2})^1_{3/2}\}_{20^+}$ $\nu\{(i_{13/2})^2_{12}(f_{7/2}/h_{9/2})^4_{14}\}_{26^+}$

4. $Z = 67$ systematics

The energies of high-spin states in the holmium isotopes $^{155-157}$Ho are plotted in Fig. 3 relative to a rigid-rotation reference [12]. The valence-space terminating configurations consist of the $79/2^-$ state in ^{155}Ho, the 41^+ and 42^+ states in ^{156}Ho, and the $87/2^-$ state in ^{157}Ho. Similar to the $65/2^-$ state in ^{155}Ho, the $75/2^-$ state in ^{157}Ho represents a non-collective configuration in which one pair of $f_{7/2}$ valence neutrons is still coupled to $I^\pi = 0^+$. It can be seen that the terminating states in ^{155}Ho are prominent and energetically favoured by 1.0–1.5 MeV over the lower-spin collective states. However, for ^{157}Ho, with two more neutrons, the terminating states are only just yrast by ~ 0.1 MeV, and, indeed, no evidence for terminating states has been found for heavier Ho isotopes. The configurations of the non-collective states in these particular $Z = 67$ isotopes are given in full in Table II. The holmium

($Z = 67$) terminating states contain the $\pi\{(h_{11/2})^3\}_{27/2^-}$ configuration, while the erbium ($Z = 68$) terminating states contain the $\pi\{(h_{11/2})^4\}_{16^+}$ configuration.

TABLE II

Experimentally observed non-collective states in $N = 89$ and 90 isotones, relative to the ^{146}Gd core, including the new states in ^{156}Ho.

Nuclide	I^π	Ref.	Aligned configuration
$^{156}_{67}\mathrm{Ho}_{89}$	41^+		$\pi\{(h_{11/2})^3_{27/2}\}_{27/2^-}$ $\nu\{(i_{13/2})^2_{12}(f_{7/2}/h_{9/2})^5_{31/2}\}_{55/2^-}$
$^{156}_{67}\mathrm{Ho}_{89}$	42^+		$\pi\{(h_{11/2})^3_{27/2}\}_{27/2^-}$ $\nu\{(i_{13/2})^2_{12}(f_{7/2}/h_{9/2})^5_{33/2}\}_{57/2^-}$
$^{157}_{68}\mathrm{Er}_{89}$	$87/2^-$	[14]	$\pi\{(h_{11/2})^4_{16}\}_{16+}$ $\nu\{(i_{13/2})^2_{12}(f_{7/2}/h_{9/2})^5_{31/2}\}_{55/2^-}$
$^{157}_{68}\mathrm{Er}_{89}$	$89/2^-$	[14]	$\pi\{(h_{11/2})^4_{16}\}_{16+}$ $\nu\{(i_{13/2})^2_{12}(f_{7/2}/h_{9/2})^5_{33/2}\}_{57/2^-}$
$^{157}_{68}\mathrm{Er}_{89}$	$93/2^+$	[4]	$\pi\{(h_{11/2})^4_{16}\}_{16+}$ $\nu\{(i_{13/2})^3_{33/2}(f_{7/2}/h_{9/2})^4_{14}\}_{61/2^+}$
$^{157}_{67}\mathrm{Ho}_{90}$	$87/2^-$	[15]	$\pi\{(h_{11/2})^3_{27/2}\}_{27/2^-}$ $\nu\{(i_{13/2})^2_{12}(f_{7/2}/h_{9/2})^6_{18}\}_{30+}$
$^{158}_{68}\mathrm{Er}_{90}$	46^+	[16]	$\pi\{(h_{11/2})^4_{16}\}_{16+}$ $\nu\{(i_{13/2})^2_{12}(f_{7/2}/h_{9/2})^6_{18}\}_{30+}$
$^{158}_{68}\mathrm{Er}_{90}$	48^-	[16]	$\pi\{(h_{11/2})^4_{16}\}_{16+}$ $\nu\{(i_{13/2})^3_{33/2}(f_{7/2}/h_{9/2})^5_{31/2}\}_{32-}$
$^{158}_{68}\mathrm{Er}_{90}$	49^-	[16]	$\pi\{(h_{11/2})^4_{16}\}_{16+}$ $\nu\{(i_{13/2})^3_{33/2}(f_{7/2}/h_{9/2})^5_{33/2}\}_{33-}$

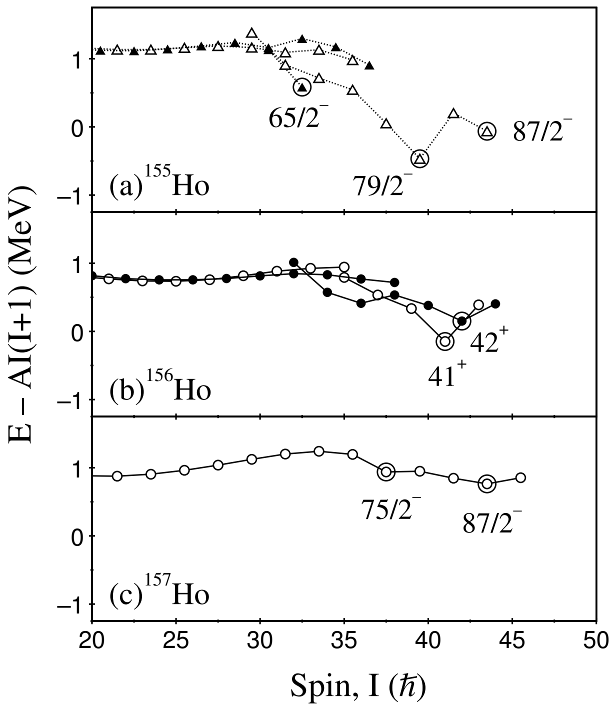

Fig. 3. Energy plotted relative to a rigid-rotation reference for the $^{155-157}$Ho isotopes. Non-collective oblate states are encircled and labelled by their spin and parity.

5. Collective bands beyond the terminating states

Although it appears energetically expensive to generate non-collective angular momentum beyond the valence-space terminating states, strongly deformed nuclear configurations with large moments of inertia become an efficient means of generating the very highest values of angular momentum. Such bands were originally established in the ^{157,158}Er isotopes [5] and interpreted as triaxial strongly deformed (TSD) structures, extending to over $60\hbar$. Three collective bands with similar high moments of inertia, tentatively assigned to ^{157}Ho, have also been observed in the present experiment [17].

Calculations based on the cranked Nilsson–Strutinsky (CNS) procedure [12] and also the Skyrme–Hartree–Fock method with tilted-axis cranking [18] have been performed to investigate the triaxial shapes of these ultrahigh-spin collective structures. The CNS calculations suggest that two-particle–two-hole neutron excitations across $N = 82$, incurring $N_{\rm osc} = 5$ holes ($\nu h_{11/2}$), are an important ingredient in creating collectivity, but it is the $N_{\rm osc} = 4$ holes in the proton core which induce the triaxial shape [5].

Mean level lifetime measurements of states within the high-spin TSD bands of the erbium isotopes ^{154}Er, ^{157}Er, and ^{158}Er have been undertaken in order to extract transition quadrupole moments, Q_{t}, and hence give an insight into the deformation (shape) of the bands. A summary of the transition quadrupole moments extracted, using the Doppler-shift-attenuation method (DSAM), is listed in Table III. These measurements were all made under similar experimental conditions using the Gammasphere spectrometer. Furthermore, the known axially symmetric superdeformed (SD) bands in ^{151}Dy [20] and ^{154}Er [21] have been used to calibrate the values. It can be seen in Table III that the erbium TSD bands all have measured quadrupole moments of 10–11 eb. This value is consistent with a triaxial nuclear shape with deformation parameters $\varepsilon_2 \sim 0.4$ and $\gamma \sim 20°$.

TABLE III

Transition quadrupole moments extracted for TSD bands in erbium isotopes. The values are calibrated by the measurements of axial SD bands in ^{154}Er and ^{151}Dy.

Nuclide	Band	Q_{t} (eb)	Ref.
^{154}Er	1	11.1 ± 1.0	[19]
^{154}Er	3	9.9 ± 2.2	[19]
^{157}Er	1	$10.9^{+0.6}_{-0.5}$	[10]
^{157}Er	2	$11.1^{+1.2}_{-0.9}$	[10]
^{158}Er	1	$11.7^{+0.7}_{-0.6}$	[10]
^{158}Er	2	$11.1^{+1.3}_{-1.0}$	[10]
^{158}Er	3	$9.6^{+1.5}_{-1.1}$	[17]
^{151}Dy	SD	17 ± 2	[10]
^{154}Er	2 (SD)	19.5 ± 3.2	[19]

Recent cranking calculations [22–24], looking at moment-of-inertia properties of TSD band 1 in ^{158}Er, suggest that the spins of this band are several units of spin higher than first proposed in Ref. [5]. If this is true, the experimental TSD band 1 of ^{158}Er would extend to $\sim 75\hbar$ and represent the highest spin structure ever observed. The implied triaxial shape of the erbium TSD bands also allows the possibility of observing wobbling motion (nuclear precession, as seen in Lu isotopes [25]) at ultrahigh spin approaching the fission limit. Indeed, the erbium TSD bands may represent a more-deformed, higher-spin version of the wobbling bands.

In conclusion, the differing ways in which nuclei generate angular momentum, as exemplified by the structural evolution of yrast states in these $N \sim 90$, $Z \sim 68$ rare-earth nuclei, from the ground state up to spin $\sim 70\hbar$, reveal the fascinating behaviour of a finite, mesoscopic quantum system. Indeed, both single-particle and collective modes of spin generation are competative at high spin.

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