

MULTIDIMENSIONAL  
SKYRME-DENSITY-FUNCTIONAL STUDY  
OF THE SPONTANEOUS FISSION OF  $^{238}\text{U}^*$

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We determined the spontaneous fission lifetime of  $^{238}\text{U}$  by a minimization of the action integral in a three-dimensional space of collective variables. Apart from the mass-distribution multipole moments  $Q_{20}$  (elongation) and  $Q_{30}$  (left–right asymmetry), we also considered the pairing-fluctuation parameter  $\lambda_2$  as a collective coordinate. The collective potential was obtained self-consistently using the Skyrme energy density functional SkM\*. The inertia tensor was obtained within the nonperturbative cranking approximation to the adiabatic time-dependent Hartree–Fock–Bogoliubov approach. The pairing-fluctuation parameter  $\lambda_2$  allowed us to control the pairing gap along the fission path, which significantly changed the spontaneous fission lifetime.

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This study of spontaneous-fission lifetimes is based on the energy-density-functional (EDF) theory and relies on the collective potential and inertia determined within the adiabatic time-dependent Hartree–Fock–Bogoliubov (ATDHFB) approach. In practical calculations, we use the Skyrme EDF parametrization SkM\* [1] and density-dependent pairing. The methodology adopted in this work strictly follows Refs. [2–4].

The ATDHFB inertia is calculated as

$$\mathcal{M}_{ij}^C = \frac{1}{2\dot{q}_i\dot{q}_j} \sum_{\alpha\beta} \frac{\left(F_{\alpha\beta}^{i*}F_{\alpha\beta}^j + F_{\alpha\beta}^iF_{\alpha\beta}^{j*}\right)}{E_\alpha + E_\beta}, \quad (1)$$

where  $\dot{q}_i$  and  $\dot{q}_j$  represent time derivatives of the collective coordinates. The sum is evaluated over all quasiparticle states and  $E_\alpha$  denotes the quasiparticle energy. Matrices  $F^i$  are obtained from

$$-F^{i*} = \left( B^T \frac{\partial \rho}{\partial q_i} A + B^T \frac{\partial \kappa}{\partial q_i} B - A^T \frac{\partial \kappa^*}{\partial q_i} A - A^T \frac{\partial \rho^*}{\partial q_i} B \right) \dot{q}_i, \quad (2)$$

where  $A$  and  $B$  are the Hartree–Fock–Bogoliubov (HFB) matrices, obtained self-consistently from the constrained HFB equations. The particle and pairing densities,  $\rho$  and  $\kappa$  respectively, are determined uniquely from  $A$  and  $B$ .

The total Routhian is

$$H'_{\text{HFB}} = \hat{H}_{\text{HFB}} - \sum_{l=2,3} q_l \hat{Q}_{l0} - \sum_{\tau=p,n} \left( \lambda_\tau \hat{N}_\tau - \lambda_{2\tau} \left( \hat{N}_\tau^2 - \langle N_\tau^2 \rangle \right) \right), \quad (3)$$

where  $\hat{H}_{\text{HFB}}$  is the HFB Hamiltonian,  $\hat{Q}_{20}$  and  $\hat{Q}_{30}$  are quadrupole and octupole moments, respectively, and  $\hat{N}_\tau$  is particle-number operator. The terms associated with  $\lambda_{2\tau}$  modify the pairing correlations of the system [2, 5] that can be assessed through the average pairing gaps

$$\Delta_\tau = \frac{\text{Tr}' \hat{\Delta}^\tau \rho_\tau}{\text{Tr} \rho_\tau}, \quad (4)$$

where  $\hat{\Delta}^\tau$  is the pairing field and  $\text{Tr}' A = \sum_n A_{n\bar{n}}$ , with bar over  $n$  indicating the time-reversed state.

Calculations presented in this work were performed in a three-dimensional (3D) collective space, where moments  $Q_{20}$  and  $Q_{30}$  control axial nuclear shapes and  $\lambda_2 = \lambda_{2p} = \lambda_{2n}$  allows for simultaneously changing proton and neutron pairing correlations. An early discussion of the effect of pairing fluctuations on fission dynamics was presented, for example, in Refs. [6, 7]

(see Ref. [2] for a comprehensive list of references). Although the potential energy  $V$  increases as the pairing gap deviates from the HFB value, the collective inertia behaves as  $\sim 1/\Delta^2$  and, therefore, the minimum-action path favors stronger pairing correlations [2].

In this contribution, we carry out a comparative study of  $^{238}\text{U}$ , assuming axial geometry. The role and importance of other degrees of freedom, such as triaxiality [2], will be discussed elsewhere.

Potential energy surfaces shown in Fig. 1 allow us to study competition between the deformation and pairing effects. It turns out that the pairing

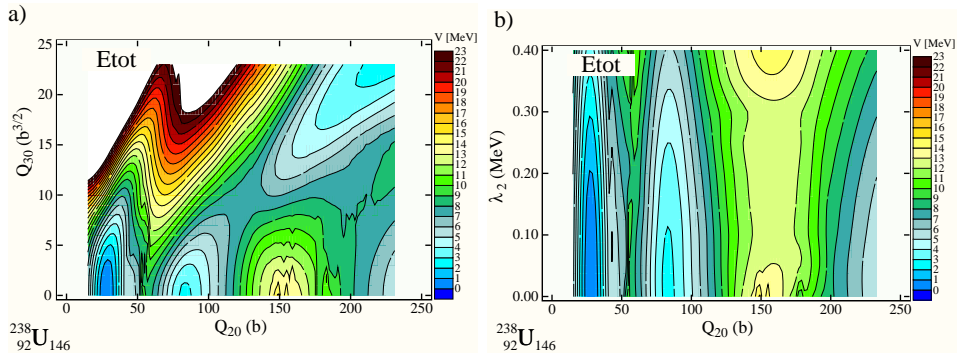


Fig. 1. Potential energy surfaces of  $^{238}\text{U}$  in the  $(Q_{20}-Q_{30})$  plane for  $\lambda_2 = 0.0$  (left) and in the  $(Q_{20}-\lambda_2)$  plane for  $Q_{30} = 0.0$  (right).

fluctuations are more important around the first saddle than in the ground-state energy minimum. As it is shown in Fig. 2, with increasing pairing, the potential energy increases, whereas the mass tensor, in general, decreases. Such a competition significantly affects the fission lifetimes. For example, our 2D calculations (along the  $\lambda_2 = 0$  path) yield  $T_{\text{SF}} = 2.34 \times 10^{21}$  y, while the 3D calculations including pairing predict  $T_{\text{SF}} = 3.63 \times 10^{17}$  y, which is closer to the experimental value of  $8.2 \times 10^{15}$  y. This is consistent with findings of recent Refs. [8, 9] based on Gogny-EDF framework.

In summary, we performed a preliminary axial-symmetry study of spontaneous fission of  $^{238}\text{U}$ , in which pairing fluctuations were treated dynamically by minimizing the collective action. Using the microscopic input based on the ATDHFB approach, we obtained a fair agreement with experiment.

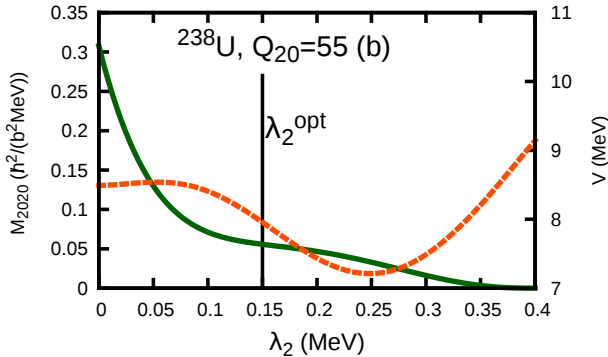


Fig. 2. The quadrupole diagonal inertia (solid line) and potential energy (dashed line) as functions of the pairing-fluctuation parameter  $\lambda_2$ . The multipole moments ( $Q_{20} = 55$  b and  $Q_{30} = 0$ ) correspond to the fission barrier. The vertical line marks the value of  $\lambda_2^{\text{opt}}$  that corresponds to the calculated dynamical fission path.

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