PARTICLE-HOLE NATURE OF THE LIGHT HIGH-SPIN TOROIDAL ISOMERS*

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Nuclei under non-collective rotation with a large angular momentum above some threshold can assume a toroidal shape. In our previous work, we showed by using cranked Skyrme–Hartree–Fock approach that even–even, $N = Z$, high-K, toroidal isomeric states may have general occurrences for light nuclei with $28 \leq A \leq 52$. We present here some additional results and systematics on the particle-hole nature of these high-spin toroidal isomers.

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1. Introduction

A closed orientable surface has a topological invariant known as the Euler characteristic $\chi = 2 - 2g$, where the genus g is the number of holes in the surface. Nuclei, as we now know them, have the topology of a sphere with $\chi = 2$. Wheeler suggested that under appropriate conditions the nuclear fluid may assume a toroidal shape ($\chi = 0$). Using the liquid-drop model [1] and the rigid-body moment of inertia [2], it was shown that a toroidal nucleus, endowed with an angular momentum $I = I_z$ aligned about its symmetry z-axis beyond a threshold, is stable against the breathing deformation in which the major radius R contracts and expands. The rotating liquid-drop nuclei can also be stable against sausage instabilities (known also as Plateau–Rayleigh instabilities, in which the torus breaks into smaller fragments), when the same mass flow is maintained across the meridian to lead to high-I_z isomers within an angular momentum window [2].

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2. Recent investigations on rotating toroidal nuclei

Recently, Ichikawa *et al.* [3] found that toroidal high-spin isomer with $I = 60\hbar$ may be in a local energy minimum in the excited states of ^{40}Ca. They used a cranked Skyrme–Hartree–Fock (HF) method starting from the initial ring configuration of 10 alpha particles. In Ref. [4], it was found from the time-dependent HF and the random-phase approximation that a collective rotation about the axis perpendicular to symmetry axis of the toroidal isomer ^{40}Ca($I = 60\hbar$) can result in a pure collective precession motion. In our previous study [5], we found that rotating toroidal nuclei have general occurrences and we located 18 even–even, $N = Z$, high-spin toroidal isomeric states: ^{28}Si($I = 44\hbar$), ^{32}S($I = 48, 66\hbar$), ^{36}Ar($I = 56,\ 72,\ 92\hbar$), ^{40}Ca($I = 60, 82\hbar$), ^{44}Ti($I = 68, 88, 112\hbar$), ^{48}Cr($I = 72, 98, 120\hbar$), and ^{52}Fe($I = 52, 80, 104, 132\hbar$) in the region of $16 \leq A \leq 52$. Subsequent to the work of [5], Ichikawa *et al.* in Ref. [6] investigated the existence of toroidal isomers and their precession motions for nuclei with $28 \leq A \leq 52$. They also obtained high-spin toroidal isomers in ^{36}Ar, ^{40}Ca, ^{44}Ti, ^{48}Cr, and ^{52}Fe, confirming the general occurrence of high-spin toroidal isomers in this mass region in [5].

To study the occurrence of high-spin toroidal isomers we use a three-step method [5], (see also [7]). First, using the Skyrme–Hartree–Fock–Bogoliubov (HFB) model with the quadrupole moment constraints on Q_{20}, we look for those oblate configurations with toroidal nuclear density distributions, as shown in Fig. 1. The energies of axially-symmetric toroidal configurations as a function of Q_{20} lie on a slope. This indicates that the magnitudes of the shell corrections are not sufficient to stabilize the tori against the tendency to return to the topology of a sphere.

We next take these toroidal configurations as the initial configurations in Q_{20}-constrained cranking Skyrme–HF calculations. For a non-collectively rotating toroidal nucleus around the symmetry z-axis with aligned angular momentum, $I = I_z$, we use a Lagrange multiplier ω to describe the constraint $I_z = \langle \hat{J}_z \rangle = \sum_{i=1}^{N} \Omega_{zi}$, where Ω_{zi} is the z-component of i^{th} single-particle total angular momentum. When we locate the configurations which lie close to a local minimum for each quantized value of angular momentum, $I = I_z$, we repeat the cranked HF calculations without the Q_{20} constraint to find the high-spin toroidal isomeric states in free convergence in the last step. Results of this method in the case of ^{52}Fe are shown in Fig. 2.

From the quantum mechanical point of view, the non-collective rotation around the symmetry axis corresponds only to particle-hole (p-h) excitations in the axially-symmetric, $I = 0$, nucleus. We plot in Fig. 3 the $I = 0$ toroidal proton–quasiparticle state energies as a function of the quadrupole moment Q_{20} obtained in SkM*–HFB model for ^{52}Fe. The toroidal quasi-

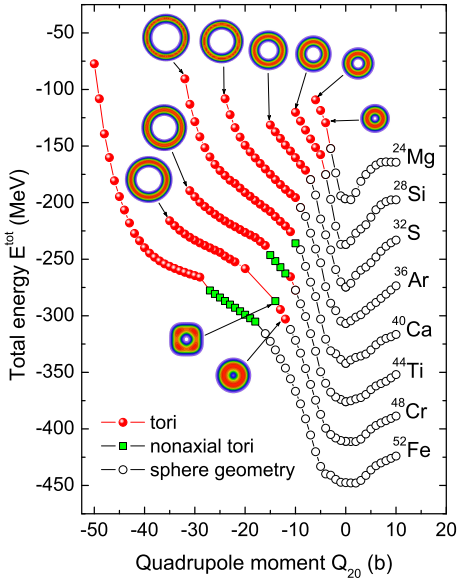


Fig. 1. (Colour on-line) The total HFB energy of ^{24}Mg, ^{28}Si, ^{32}S, ^{36}Ar, ^{40}Ca, ^{44}Ti, ^{48}Cr, and ^{52}Fe as a function of the quadrupole moment Q_{20} for the case of $I = 0$, taken from Ref. [5]. Axially-symmetric toroidal configurations are indicated by full circles, axially-asymmetric toroidal configurations by full squares, and configurations with a spherical topology by open circles. Some toroidal density distributions are displayed.

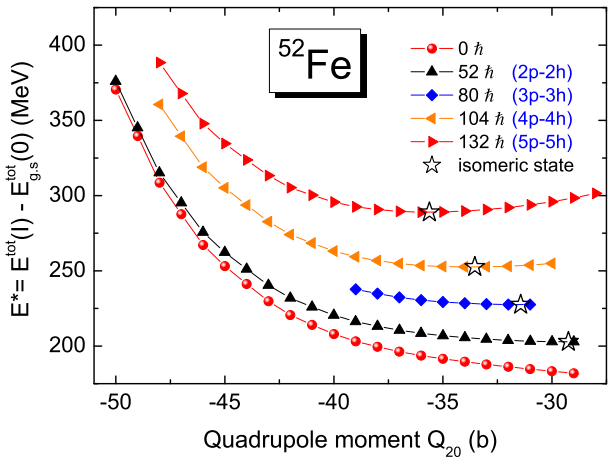


Fig. 2. (Color online) The excitation energy E^* of high-spin toroidal states of ^{52}Fe as a function of Q_{20} for different $I = I_z$ about the symmetry axis. The locations of isomeric toroidal energy minima are indicated by the star symbols.

particle states are labelled by asymptotic quantum numbers $[Nn_z\Lambda]\Omega$. It is clear from Fig. 3 and also Fig. 1 (a) in Ref. [5], that the low-lying states possess the nodal quantum number of $n_z = 0$. The occupation numbers of 14, 18, 22, and 26 in Fig. 3 indicate the toroidal shell-gaps, as in Fig. 1 (a) of [5]. The occupation of all levels below the Fermi energy of this even–even nucleus lead to the state with $I = 0$, while the 2p-2h and 3p-3h particle-hole excitation result to high-spin toroidal isomeric states with $I_z = 26$ and $40\hbar$, respectively.

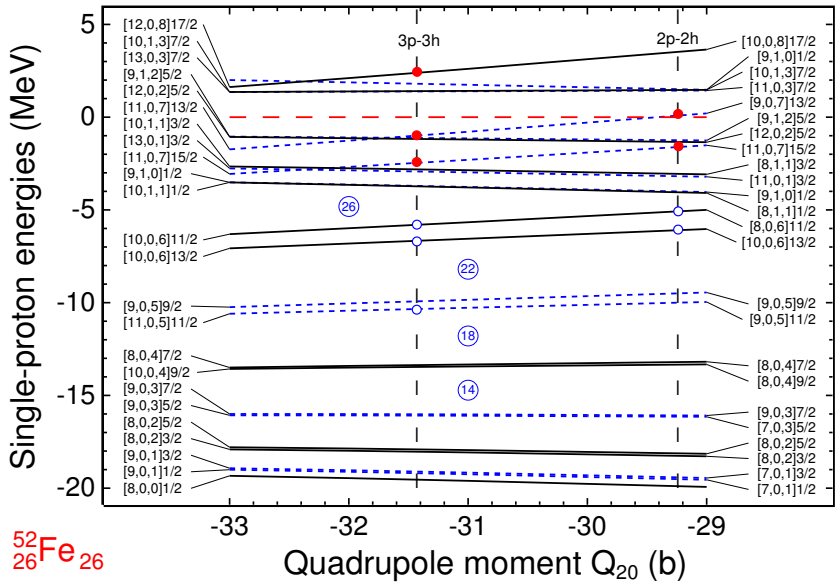


Fig. 3. (Color online) The toroidal proton–quasiparticle energies as a function of the quadrupole moment Q_{20} obtained in the SkM*–HFB model for ^{52}Fe with $I = 0$. They are labelled by $[Nn_z\Lambda]\Omega$, with even parity levels as solid lines, and odd parity levels as dashed lines. Starting from the $I = 0$ configuration, the 2p-2h and 3p-3h excitations shown in the plot (with holes as open circles and particles in solid circular points) lead to non-collective rotations with a total $I_z = 26$ and $40\hbar$, respectively. The vertical dashed lines coincide with the quadrupole deformations of two first toroidal isomers (Fig. 2). The horizontal (red) dashed line represents zero quasiparticle energy for $I = 0$.

Table I gives the simple rules to calculate the aligned angular momentum $I = I_z$ for different p-h excitations from states with $n_z = 0$, for neutron number N (or proton number Z), such that (N or Z mod 4)$= 0$ or 2. For nucleus ^{52}Fe with $N = Z = 26$ and a parameter $\Lambda_{\max} = 6$, one can find from Table I the aligned angular momentum $I_z = 2 \times 14$, 2×26, 2×40, 2×52, $2 \times 66\hbar$ for 1-, 2-, 3-, 4-, 5-p-h excitations, (see also Fig. 1 (b) in

Ref. [5]). With our method to study the occurrence of toroidal high-spin isomers described above, all the above p-h states turn out to be toroidal isomeric states as shown in Fig. 2, with the exception of the (1p-1h) $I_z = 28\hbar$ state that is apparently below the threshold for a non-collectively rotating toroidal nucleus.

TABLE I

The quantized values of aligned angular momentum $I = I_z$ for different particle-hole excitations of the states with $n_z = 0$, where parameter $\Lambda_{\max} = 0, 1, \ldots$

Excitation	N or $Z = 4\Lambda_{\max}$	N or $Z = 4\Lambda_{\max} + 2$
1p-1h	$I = 2\Lambda_{\max}$	$I = 2\Lambda_{\max} + 2$
2p-2h	$I = 4\Lambda_{\max} + 1$	$I = 4\Lambda_{\max} + 2$
3p-3h	$I = 6\Lambda_{\max}$	$I = 6\Lambda_{\max} + 4$
4p-4h	$I = 8\Lambda_{\max} + 1$	$I = 8\Lambda_{\max} + 4$
5p-5h	$I = 10\Lambda_{\max}$	$I = 10\Lambda_{\max} + 6$

We plot in Fig. 4 the density distributions of the isomeric toroidal states of ^{52}Fe (presented in Fig. 2) with $I = 52$, 80, 104, and $132\hbar$ as a cut in the radial direction x. One notes that when the aligned angular momentum I increases, the maximum toroidal density $\rho_{\max}$ decreases from 0.134 to 0.123 fm^{-3} and the major radius R increases from 7.39 to 8.20 fm. Only

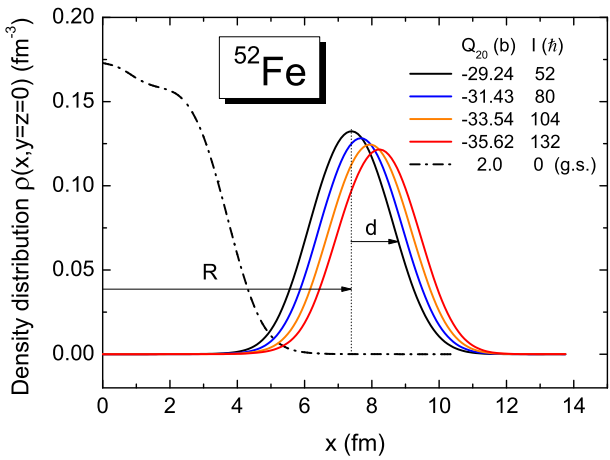


Fig. 4. (Colour on-line) The density distributions of the isomeric toroidal states of ^{52}Fe with $I = 52$, 80, 104, and $132\hbar$ as a cut in the radial direction x. The values of the isomeric quadrupole moments Q_{20} are indicated. For the first isomeric toroidal state with $I = 52\hbar$, we show the major radius R and the minor radius d (see the text). The dash-dotted curve shows the density distribution in the ground state of ^{52}Fe.

the minor radius d, defined as a half width at half maximum (HWHM) of the toroidal distribution, stays constant at $d \approx 1.37$ fm. For comparison, we also show the total density distribution of ^{52}Fe in its ground state (dash-dotted curve) with $\rho_{\max} = 0.173$ fm^{-3} which is distinctly larger than $\rho_{\max}$ of toroidal distributions. The above-mentioned features are typical for all high-spin toroidal isomeric states of even–even, $Z = N$ nuclei with $28 \leq A \leq 52$, (see Table 1 in Ref. [5]).

In Fig. 5, we plot the total energy of all found isomers as a function of R/d. One can see that the total energies of isomers obtained by the same p-h excitation show linear dependence on R/d. There appears to be a regular pattern on the systematics of high-spin toroidal isomers from which properties on the nuclear fluid in the exotic toroidal shape may be extracted.

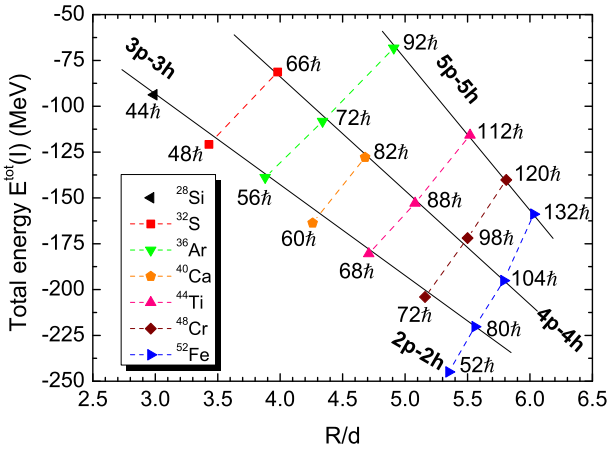


Fig. 5. (Colour on-line) The total energy of the isomeric toroidal states of ^{28}Si, ^{32}S, ^{36}Ar, ^{40}Ca, ^{44}Ti, ^{48}Cr, and ^{52}Fe of different I values as a function of R/d.

3. Conclusion

Non-collective nuclear rotations with a large angular momentum provide a favorable environment for the nuclear matter to redistribute itself. Our exploration of the density distribution of these nuclei with cranked self-consistent Skyrme–Hartree–Fock approach in the region of $28 \leq A \leq 52$ reveals the possibility of toroidal-shape high-spin isomers at their local energy minima, when the angular momenta are greater than some large thresholds. The particle-hole nature of these non-collective rotational states, the locations of these local energy minima, the magnitudes of the non-collective angular momenta, and the geometrical properties of these isomeric states have been systematically evaluated for further theoretical and experimental explorations.

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