LOW-|t| STRUCTURES IN ELASTIC SCATTERING AT THE LHC*

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Possible low-|t| structures in the differential cross section of pp elastic scattering at the LHC are predicted. It is argued that the change of the slope of the elastic cross section near $t = -0.1$ GeV^2 has the same origin as that observed in 1972 at the ISR, both related to the $4m_\pi^2$ branch point in the |t|-channel of the scattering amplitude. Apart from that structure, tiny oscillations at small |t| may be present on the cone at low |t|.

1. Introduction

Followed by the first publications on pp elastic scattering at $\sqrt{s} = 7$ TeV in the broad |t| range of 5×10^{-3} GeV$^2 < |t| < 2.5$ GeV^2 [1, 2], the TOTEM Collaboration recently made public [3] their new results at still lower values of |t| at $\sqrt{s} = 8$ TeV.

Contrary to earlier statements [2], considerable deviation from the linear exponential cone was found. Namely, a change of the local slope $B(t) = \frac{d}{dt}(\ln \frac{d\sigma(s,t)}{dt})$ at 8 TeV by about 0.5 GeV^{-2} around $|t| \approx 0.1$ GeV^2 was observed as well. As emphasized in Ref. [4], a single exponential is excluded by 7σ.

In the present paper, we argue that this structure is a recurrence of the similar phenomenon observed in 1972 at the ISR, both related to t-channel unitarity effects of the scattering amplitude. Anticipating the relevant TOTEM publication, below, we present a method to handle possible structures in the diffraction cone and make predictions based on a Regge-pole model extrapolating from the ISR energy region to that of the LHC[1].

2. The 'break' phenomenon; preliminaries

The change of the local slope $B(t)$ around $|t| \approx 0.1$ by about $\approx$ GeV^{-2}, called the 'break' (in fact, a smooth curvature in $B(t)$, at $\sqrt{s} = 21.5$, 30.8, 44.9, 53.0 GeV) was first observed and discussed in Ref. [6] (see Table I in [6] quoted below and illustrated in Fig. 1). The change of the slope is smooth, the word 'break' being used merely for brevity. Although there is little doubt about the universality of this phenomena, the position and the size of the effects and its dependence on energy is still disputable.

TABLE I

Slope $B(t)$ calculated with different exponentials for the ISR data [6].

$\sqrt{s}$ [GeV]	$|t|$-range, [GeV^2]	B [GeV^{-2}]	Err. [GeV^{-2}]	ΔB [GeV^{-2}]	Err. [GeV^{-2}]
21.5	0.05–0.094	11.57	0.030		
	0.138–0.2380	10.42	0.17	1.15	0.20
30.8	0.046–0.090	11.87	0.28		
	0.138–0.240	10.91	0.22	0.96	0.50
44.9	0.046–0.089	12.87	0.20		
	0.136–0.239	10.83	0.20	2.04	0.40
53.0	0.060–0.112	12.40	0.30		
	0.168–0.308	10.80	0.20	1.6	0.50

This phenomenon is visible also at other energies, see Table II and Fig. 2 [7]. Recently, it was studied and discussed in a number of papers, see [8, 9] and earlier references therein.

The phenomenon was not seen at the Tevatron. CDF had Roman pots only on the antiproton side, *i.e.* it could not align the proton/antiproton measurements by using the track on the opposite side as is normally done. Instead, they used the assumed t-distribution. This technique prevents any measurement of the t-distribution. The D0 experiment [10], on the other

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[1] Preliminary results of this paper were presented in June, 2014 at the Protvino Conference on High-energy Physics [5].
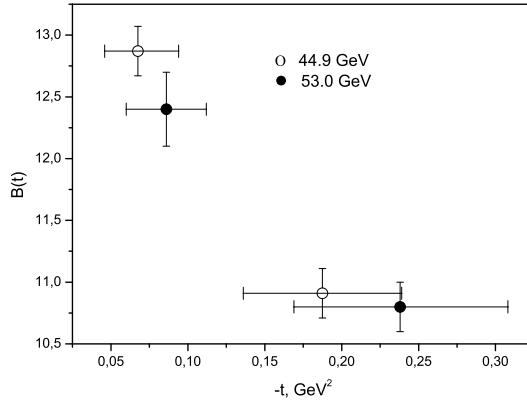
Fig. 1. Local slopes $B(t)$ calculated at ISR energies with two different exponentials (see Table I).

hand, did publish a t-distribution in the interval of interest. Their measurement covers the interval of $0.26 < |t| < 1.2\ \mathrm{GeV}^2$, where a change of the slope at $-0.6\ \mathrm{GeV}^2$ was observed ('rudiments' of the diffraction minimum). Predictions for high-energy, low-t $p\bar{p}$ scattering are possible, however, without prospects to be tested experimentally in the near future.

TABLE II

Average slope values for fits in different bins of $|t|$ for the UA4 data [7].

	$|t|$-range [GeV^2]	B [GeV^{-2}]	Err. [GeV^{-2}]
I	0.03–0.10	15.3	0.3
II	0.03–0.15	15.2	0.2
III	0.15–0.32	14.2	0.4
IV	0.21–0.32	13.6	0.8
V	0.21–0.50	13.4	0.3

Thus, the magnitude of the slope break at different ISR energies, calculated in a simple exponential approximation for nearly the same range of momentum transfer $0.05\ \mathrm{GeV}^2 < |t| < 0.10\ \mathrm{GeV}^2$ and $0.14\ \mathrm{GeV}^2 < |t| < 0.25\ \mathrm{GeV}^2$, varies within the range of $\Delta B(t) \approx (1\text{--}2)\ \mathrm{GeV}^{-2}$. Using the data from Table II on the slope parameter for the SPS energy 546 GeV for antiproton–proton scattering, one can obtain different values for the slope break depending on the choice of bin pairs. For example, the largest value of the slope break $\Delta B = 1.9 \pm 0.6\ \mathrm{GeV}^{-2}$ can be obtained by choosing a couple (adjacent bins) made of the farthest values I, V, and the smallest one for the pairs I and III, $\Delta B = 1.1 \pm 0.7\ \mathrm{GeV}^{-2}$, taking into account two
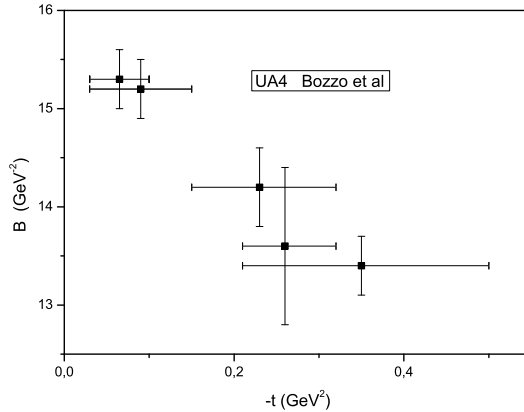
Fig. 2. Local slopes $B(t)$ calculated for the UA4 data [7].

adjacent intervals. A different approach to the choice of the individual t-bins was used in the recent paper [2], where constancy of the slope was stated, at least until $|t| = 0.2$ GeV^2.

Note that the break found at 8 TeV can be obtained even with simpler methods where two single exponential are fitted in non-overlapping t-ranges, the relevant $B(t)$ differing by more than 7σ. The overall behaviour of $B(t)$ as a function of energy is illustrated in [11] and [12].

The 'break' (we recall that in fact it is a smooth curvature approximated by linear exponentials) has a clear physical interpretation: it results from the t-channel branch point at $4m_\pi^2 \approx 0.08$ GeV^2 imposed by unitarity. The 'break' due to the two pion threshold is related to the pionic atmosphere (cloud) of the nucleon [13] (for more details, see the next section).

An immediate conclusion is that in the calculation of $B(t)$ the result depends on the bins in t chosen. Generally speaking, the bins can be chosen arbitrarily: small (containing at least three data points) or large. They may be chosen in a touching sequence or overlap. The latter option (so-called overlapping bins method (OBM)) was studied in details in a number of papers [11, 12, 14–16], whose ideas and results are summarized in Appendix.

Below, we discuss in more details all these results and make predictions for 8 TeV. A preliminary version of this study was presented at the Protvino Conference on High-energy Physics in June, 2014 [5].

To start with, we recalculate the local slope with account for both the statistical and systematic errors. To this end, we will choose the compiled data from [17]. At the LHC, we include only data from the first cone [2, 3] for $|t| < 0.2$ GeV^2.

For the ISR data, we have chosen the Amaldi *et al.* data [18] for center-of-mass energies 23.5, 30.8 and 44.7 GeV. The results are quoted in Table III.

TABLE III

Local slopes $B(t)$ recalculated with different exponentials for the ISR data [18].

$\sqrt{s}$ [GeV]	$|t|$-range [GeV^2]	B [GeV^{-2}]	Err. [GeV^{-2}]	ΔB [GeV^{-2}]	Err. [GeV^{-2}]
23.5	0.05–0.102	11.5	0.7		
	0.138–0.238	10.2	0.4	1.22	1.1
32.0	0.05–0.094	11.6	0.4		
	0.138–0.240	10.9	0.3	0.7	0.7
44.7	0.05–0.096	13.3	0.3		
	0.138–0.238	10.6	0.2	2.7	0.5

The results of the calculations coincide within errors with the data from Table I of Ref. [6].

There is a gap between adjacent bins in the ISR data, seemingly increasing the 'break' of the slope (see Fig. 1). For example, the choice of the bins adopted in Ref. [6] is not unique. For example, by selecting the location of the second bin at $\sqrt{s} = 23.5$, $(0.5 < |t| < 1.0)$ GeV^2, the averaged slope within this bin will be $B = 9.5 \pm 0.1$ GeV^{-2}, implying $\Delta B = (1.9 \pm 0.8)$ GeV^{-2}, which is more reliable than the results of Table III.

On the other hand, it is more natural to calculate the 'break' for neighbouring bins in the vicinity of $|t| \approx 0.1$ GeV^2, close to the 'break' we are scrutinizing. The local slopes calculated for the ISR and SPS data [7, 18] (compiled in [17]) for bin intervals being near the same as in [6] are shown in Table III.

It is interesting to study whether this effect (the 'break' of the local slope) persists up to the LHC energies. To this end, we construct a similar plot for the slopes $B(t)$ (and compare it with Table 6 of [2]) for the differential cross sections measured at $\sqrt{s} = 7$ TeV in the interval $(0.005 < |t| < 0.3)$ GeV^2 (Fig. 3).

For clarity sake, we performed the calculations of local slopes in adjacent bins around $|t| = 0.1$ GeV^2 for nearly the same length as in [6]: the first bin is $(0.05 < |t| < 0.1)$ GeV^2; the second one is $(0.1 < |t| < 0.14)$ GeV^2. As a result, the value of the 'break' ΔB varies within 0.5–1.2 GeV^{-2} for the ISR and TOTEM energies (see Table IV).

This estimate of the 'break' is less reliable, but shows the trend with energy. One concludes from Table IV that the break does not diminish with energy.
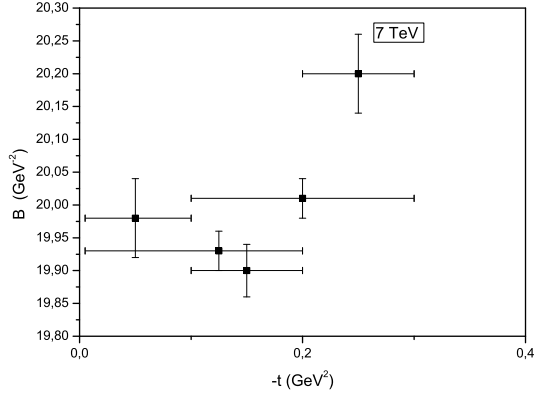
Fig. 3. Local slopes $B(t)$ calculated at 7 TeV up to $|t| = 0.3$ GeV^2, Table III (see also Table 6 in [2]).

<div align="right">TABLE IV</div>

Calculated break ΔB from adjacent bins for the ISR and TOTEM data.

$\sqrt{s}$ [GeV]	t-bin [GeV^2]	B [GeV^{-2}]	Err. [GeV^{-2}]	ΔB [GeV^{-2}]	Err. [GeV^{-2}]
23.5	0.05–0.094	11.5	0.7		
	0.094–0.138	10.2	0.9	1.3	1.6
30.7	0.05–0.094	11.6	0.4		
	0.090–0.138	11.2	0.6	0.4	1.0
44.7	0.05–0.094	13.1	0.3		
	0.094–0.136	12.3	0.5	0.8	0.8
546.	0.05–0.10	15.3	1.2		
	0.10–0.138	13.9	1.5	1.4	2.7
7000	0.046–0.091	19.8	0.7		
	0.091–0.137	19.3	0.9	0.5	1.6
8000	0.05–0.095	19.8	0.2		
	0.095–0.137	18.8	0.4	1.0	0.6
8000	0.05–0.095	19.8	0.2		
	0.095–0.189	19.1	0.3	0.7	0.5

3. Physics of the 'break' phenomenon

The physics of the phenomenon was explained in Ref. [13]. The 'break' is, in fact, a smooth concave over the linear exponential, approximated by two linear exponentials (*cf.* [3]). This structure is due to the lowest two-pion

exchange in the t-channel required by unitarity [19], see Fig. 4. The threshold appears at $t = 4m_\pi^2 \approx 0.08$ GeV^2, which is the mirror (with opposite sign of t) position of the 'break' of the cone. We recall that any analytic function (here, the scattering amplitude) is sensitive to 'mirror reflection' of its singularities (here, the $4m_\pi^2$ branch point in the amplitude).
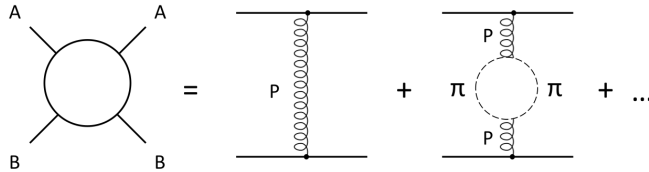

Fig. 4. Feynman diagram for elastic scattering with a t-channel exchange containing a branch point at $t = 4m_\pi^2$.

According to the ideas of duality [19, 20], the singularities enter the amplitude through Regge trajectories. Below, following Ref. [13], we present a model amplitude realizing this principle and reproducing the observed 'break'.

The t-channel threshold shown in Fig. 4 may enter both through leading (Pomeron, Odderon) or non-leading (f, ω) trajectories. While at the LHC, the low-$|t|$ are dominated completely by the Pomeron contribution (whatever it be!) [12], at the ISR energies, secondary Reggeons are not negligible, at least in nearly forward scattering.

A cut Pomeron trajectory including the lowest-lying $2m_\pi$ cut may be approximated by [13, 20]

$$\alpha(t) = 1 + \delta + \alpha' t - \gamma \left(\sqrt{4m_\pi^2 - t} - 2m_\pi \right). \tag{1}$$

The linear term in Eq. (1) is an effective contribution from heavy thresholds in the trajectory. It can be ignored in a limited range of small $|t|$, as shown *e.g.* in Ref. [21], Eq. (1) therein.

In the next section, we extrapolate in s the forward cone from the ISR to the LHC energies. This is not a trivial task since a detailed fit requires the inclusion at 'low' ISR energies the contribution from at least four trajectories, namely that of the Pomeron, eventually the Odderon, and two secondary Reggeons, f and ω. Postponing this discussion to a forthcoming detailed analyses, here we use a single 'effective' trajectory that at the low-energy part mimics all contributions mentioned (at the LHC energies, it is the Pomeron alone, see Ref. [12]).

4. Extrapolating from the ISR to the LHC

To extrapolate the cross section (or just the slope) from 'low' (ISR) energies to those at the LHC, we use a simple single Regge pole amplitude with an 'effective' trajectory that is close to the Pomeron dominating the high-energy region. The intercept of the trajectory $\alpha(0) = 1 + \delta$, following the Donnachie and Landshoff approach [22] to high-energy phenomenology, will be set slightly above 1. The relevant scattering amplitude reads

$$A(s,t) = g\, e^{bt} \tilde{s}^{\alpha(t)}\,, \qquad \tilde{s} = -i\frac{s}{s_0}\,, \tag{2}$$

$$\left.\frac{d\sigma}{dt}\right|_{\mathrm{th}} = \frac{\pi}{s^2}\,|A(s,t)|^2\,. \tag{3}$$

We use a representative Pomeron trajectory, namely that with a two-pion square-root threshold, Eq. (1), required by t-channel unitarity and accounting for the small-t 'break' [13].

The normalized 'experimental' points of $R(t)$ are defined as

$$R(t) = \left((d\sigma/dt)_{\mathrm{exp}} - (d\sigma/dt)_{\mathrm{lin}}\right)/(d\sigma/dt)_{\mathrm{lin}}\,, \tag{4}$$

where

$$\left(\frac{d\sigma}{dt}\right)_{\mathrm{lin}} = a e^{bt}\,. \tag{5}$$

The theoretical values of $R(t)$ are calculated from

$$R(t) = \left((d\sigma/dt)_{\mathrm{th}} - (d\sigma/dt)_{\mathrm{lin}}\right)/(d\sigma/dt)_{\mathrm{lin}}\,. \tag{6}$$

Those for $(d\sigma/dt)_{\mathrm{th}}$ correspond to the solid curve in Figs. 5 and 6 calculated as the best fit of (1)–(3) to the experimental differential cross sections for a given energy with the free parameters a, b, and fixed $\delta = 0.08$ and $\alpha' = 0.23$, $\gamma = 0.10$ and $t_0 = 4m_\pi^2$, where m_π is the pion mass.

For all energies, the value of $R(t)$ clearly demonstrates concavity at $t = -0.1$ GeV^2, which is in qualitative agreement with the 'experimental' one in $R(t)_{\mathrm{exp}}$.

The value $R(t) \neq 0$ around $t = -0.1$ GeV^2 means that the experimental data $(d\sigma/dt)_{\mathrm{exp}}$ are not compatible with a simple exponential.
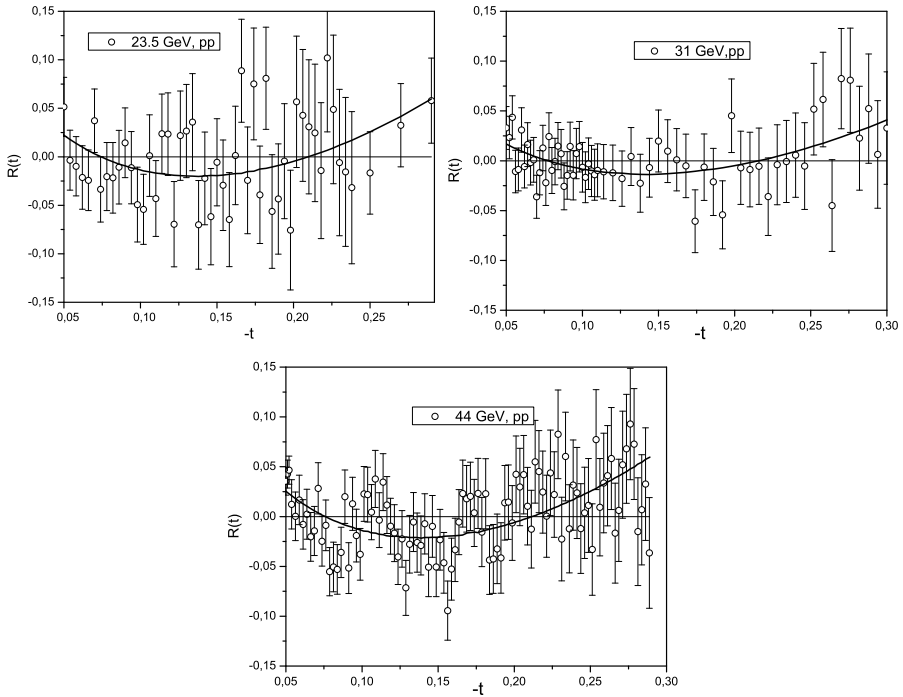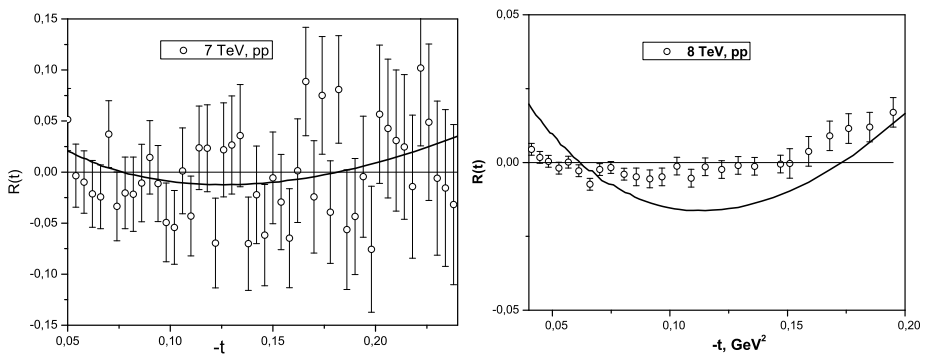
Fig. 5. $R(t)$ calculated at ISR energies.

Fig. 6. $R(t)$ calculated at 7 and 8 TeV.

5. Tiny oscillations?

Besides the 'break' discussed above, small-$|t|$ oscillations on the smooth exponential cone may also be present. They were discussed in a number of papers — theoretical and experimental [15, 24–28]. Since the amplitude of the possible oscillations appears to be close to the error bars, it is still not

clear whether this is an experimental fact or an artefact. In Refs. [14, 15], the low-t data were fitted to a model which, apart from the $t_0 = 4m_\pi^2$ cut, contains also an oscillating term (in the cross section or in the slope $B(t)$)

$$B(t) = 2\left[b + \left(\alpha' - \frac{\gamma}{2\sqrt{t_0 - t}}\right)\ln(s/s_o)\right] + a\cos\left(\omega t + \phi\right). \quad (7)$$

The result is shown in Figs. 7 and 8. The dashed curves are calculated from Eqs. (1)–(3) and they correspond to the smoothed part of (7), see Ref. [15]. Note that, since the derivative of an oscillating function is also oscillating, it makes little difference whether one is fitting the cross sections or the slope.
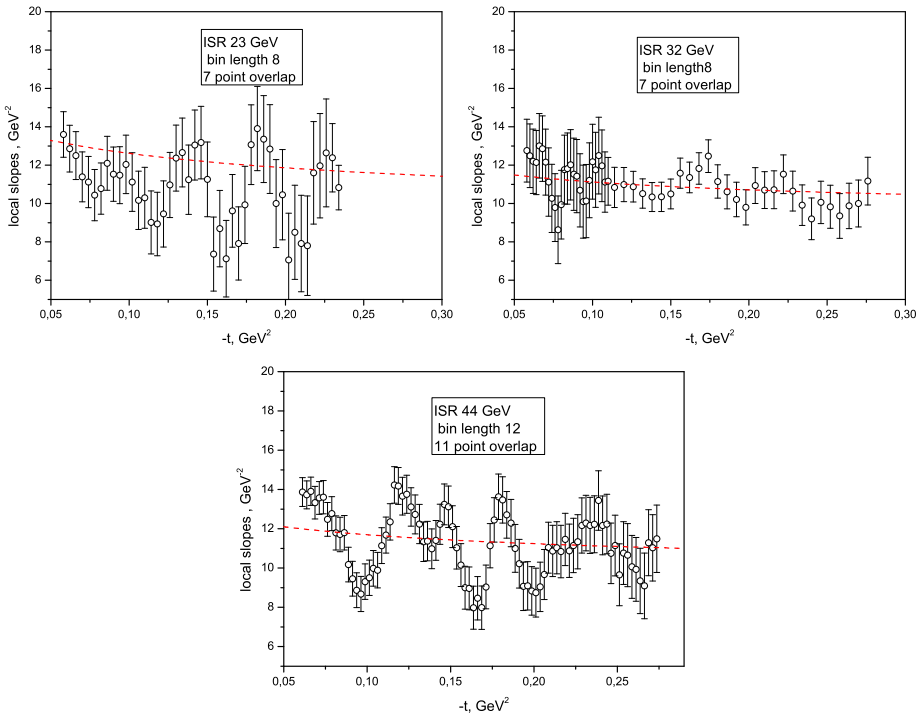

Fig. 7. Local slope $B(t)$ calculated at 23.5 GeV, 32.5 GeV and 45 GeV for overlapping bins.

The overlapping bins method (see Appendix) may be extremely useful in performing this delicate analysis. Since the earlier (theoretical and experimental) results are still inconclusive, new measurements (*e.g.* those by the Denisov group in Protvino [27]) are very important to shed new light on this phenomenon.
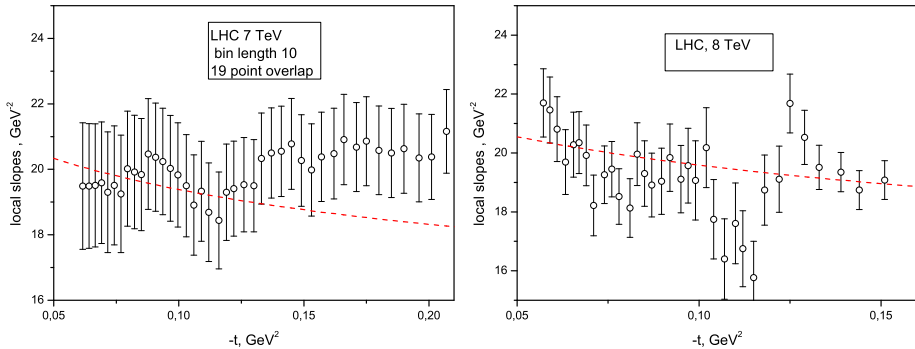
Fig. 8. Local slopes with overlapping bins calculated at 7 and 8 TeV.

The physics behind the possible low-t, small amplitude oscillations may be related to those at large impact parameters. As discussed in Ref. [35], large-distance residual van der Waals forces may be responsible for these oscillations.

6. Conclusions

Figures 1–3 are introductory, showing published results on the 'break' (we reiterate that the 'break' implies a smooth curvature). Subsequent figures show the results of our calculations, including the errors quoted also in the tables. We conclude that the 'break' observed [3] by TOTEM near $t = -0.1$ GeV^2 at 8 GeV is a 'recurrence' of a similar structure seen in 1972 at the ISR.

Note that no 'break' was seen at the Tevatron at 1.8 TeV. Possible reasons for the non-appearance of the 'break' in $\bar{p}p$ may be related to the Odderon contribution masking it. In any case, poor statistics of those data prevents from any definite conclusion concerning the presence of fine structures at the Tevatron.

While the change of the slope $B(t)$ near $-t \approx 0.1$ GeV^2 appears to be a universal and well established phenomenon (although its energy (in)dependence needs better understanding), the status of the tiny oscillations is still ambiguous. It may be that the 'break' near $t = -0.1$ GeV^2 is part of the oscillations [15].

In the present paper, the low-t structured was scrutinized by calculating both the ratio $R(s)$, Eqs. (4)–(6), and the slope $B(t)$. In our opinion both are equally useful and they are complementary.

Appendix

The overlapping bins method (OBM)

The fine structure of the diffraction peak in the differential $\bar{p}p$- and pp-elastic cross section was first observed in [23] at the ISR [6], followed by the UA4/2 experiment [7] by normalizing the differential cross section to the smoothly varying background in the impact parameter representation [25]. In [24], an attempt was made to relate the observed structure near $|t| = 0.1$ GeV^2 to the variation of the opacity in b-space, probably reflecting the density oscillation in matter. The possible existence of oscillations with even smaller periods was discussed by several authors [31].

In Ref. [14], an entirely different method to identify the fine structure in the $\bar{p}p$- and pp-elastic scattering was proposed. The method, unlike that of [32], is based on the use of overlapping bins of local slopes. Small oscillations, over the exponential cone, with a characteristic period were discovered. It is obvious that in order to determine the nature and periods of the oscillations, one first has to improve the reliability of the initial information contained in the experimental data by suppressing the influence of statistical fluctuations. This problem can be settled by means of the well known method of maximum entropy [36], used in many areas of physics. Recently, it has been applied [37] to the hadron scattering data.

The method is based on the use of overlapping bins of local slopes.

To check the expected behaviour of the slope

$$B(s,t) = \frac{d}{dt} \ln \left(\frac{d\sigma(s,t)}{dt} \right) \tag{8}$$

over t, one must operate with its 'experimental' value.

Provided that

$$\left(\frac{d\sigma}{dt} \right)_i = \left| a_i e^{b_i t} \right|^2 \tag{9}$$

has been measured for a given s at N $|t|$-points lying in some interval $[|t|_{\min}, |t|_{\max}]$, we adopt the following procedure. First, we divide this interval into subintervals or elementary 'bins' (with n_b measurements in each of them, assumed for simplicity to be the same for all bins). Once the first bin is chosen, the second bin is obtained from the first one by shifting only one point of measurement (of course, one could shift it by any number of points less or equal to n_b, the shift of one point is the minimal one giving rise to the maximal number of overlapping bins). The third bin is obtained from the second bin by shifting of one data point *etc.* Thus, we define $N - n_b + 1$ overlapping bins for a given s. For each (k^{th}) bin, n_b must be large enough and its width (in $|t|$) small enough to allow fitting $\left(\frac{d\sigma}{dt} \right)$ with the simplest

form directly involving the t-slope b (9). The parameter b represents the value of the t-slope $B\left(\langle t \rangle_k, s\right)$ 'measured' at s and 'weighted average' $\langle t \rangle_k$. This yields the 'experimental' values of $b_k(s, t_k)$ with the corresponding standard errors determined in the fit of (9) to the data. Then, the procedure is to be repeated for all bins and ultimately for other ts at which $\left(\frac{d\sigma}{dt}\right)$ was measured. A regular structure in the local slope of diffraction cone $B(s, t)$ was found by the procedure of the overlapping bins described above and applied to experimental data [14]. However, if one has the bins of ~ 10 points and shifts them at each step by one only, the overlap my be so strong that the information from the neighbours is extremely correlated and one can ascribe the regular tendency of the final plot increase (or decrease) in many neighbouring bins to this correlation.

To resolve these doubts, the local slopes were re-evaluated with the help of the Overlapping Bins Metod (OBM) by the LSQ method with the so-called 'non-independent' ys, [34].

Rather than minimizing the functional

$$s = \sum_1^N \left(\frac{f\left(t_i, \boldsymbol{a}\right) - y_i}{\Delta y_i}\right)^2, \tag{10}$$

the form

$$s = \sum_{ij}^N \left(f\left(t_i, \boldsymbol{a}\right) - y_i\right) w_{ij} \left(f\left(t_j, \boldsymbol{a}\right) - y_j\right) \tag{11}$$

can be used.

In the framework of the OBM calculus

$$\begin{aligned} s &= \sum_{j=1}^N \sum_{i=j}^{j+n-1} \frac{\left(f\left(t_i, a_j\right) - y_i\right)^2}{\Delta y_i^2} \\ &\quad - \sum_{j=1}^N \sum_{i=j+1}^{j+n-2} \frac{\left(f\left(t_i, a_j\right) - y_i\right)\left(f\left(t_i, a_{j+1}\right) - y_i\right)}{\Delta y_i^2}. \end{aligned} \tag{12}$$

Calculations using correlation over Eq. (12) for the same set of points indicate that the errors Δb_i are reduced and oscillations are revealed more distinctly, *i.e.* the relation between the value of errors and the amplitude of oscillation can be improved.

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