INFORMATION PROPERTIES OF CO-PROCESSING MODEL ON COMMUNICATION NETWORKS

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Information properties of co-processing model on communication networks are investigated in this paper. As one crucial factor to determine the processing ability of nodes, the information flow with potential time lag is modeled by co-processing diffusion which couples the continuous time processing and the discrete diffusing dynamics. Exact results on master equation and stationary state are achieved to disclose the formation. Considering the influence of a node to the global dynamical behavior, co-processing centrality is introduced for each node, which determines the relative importance of nodes and exhibits the capability that a node communicates information with its neighbor environment over the network in the diffusion process. Furthermore, a new parameter, co-processing entropy, is proposed to measure the interplay between co-processing centrality and diffusion dynamics. At last, the information function of the co-processing model is investigated to deeply detect the properties of the diffusion process. The experimental results on large-scale complex networks with Poisson distribution confirm our analytical prediction.

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1. Introduction

In recent years, various topological and dynamical properties of networks resulting from real systems have attracted many researchers in diverse fields [1–5]. Rich behaviors in the dynamical processes of the physical systems depending on the topological structure of networks result from the inherent complexity of networks. For example, random walk [5–9] has been widely investigated to understand the essential dynamical properties of physical systems in networks [10–12] and also has many practical applications to real networks such as *e.g.* information searching in the Internet [13–16].

However, congestion often occurs in diffusion process, which is mainly determined by the capability of nodes. Zero-range process (ZRP), as a powerful technique to control the congestion by modulating the capability of nodes, has been investigated deeply. While in the traffic flow [17–21], the car may be jammed at a crossing for a long time before it travels to another crossing. For the information flow in the networks, the information packet may mass at one processor so that it cannot be transferred to the destination in time. The traffic flow or the information packet is jammed at nodes for that there is time consumed on nodes. Thus, the continuous stochastic process with time consumed on nodes corresponds much more to the practical situation in our usual life.

Based on the diffusion process on complex networks, evaluation of the importance of nodes and edges is widely used in analysis of complex networks. To evaluate the importance, various centrality measures, *e.g.*, degree centrality, closeness centrality, and betweenness centrality, have been proposed [22–24]. For example, betweenness centrality BC [25] is introduced as a good approximation for the quantity of information passing through a node in communication networks [26, 27].

Besides, entropy is a measure of the uncertainty about dynamical behaviors of the network. In a network with higher entropy, more information is needed to describe its future behavior, and its effective complexity is higher [28–31]. In the field of complex networks, entropy has been applied to characterize the topological properties, such as the degree distribution [32], the shortest paths between couples of nodes [33], and even more the dynamical processes on complex networks. Recently, the entropy rate of a diffusion process was introduced to characterize a diffusion process [34]. Combining the maximum entropy principle, it is possible to design the optimal diffusion processes. Furthermore, in Ref. [35], a new class of random walk processes was introduced, the maximal entropy random walk (MERW), which induced a surprising effect of localization in the presence of weak disorder.

In this paper, we investigate the information properties of the co-processing model on complex networks. The structure of this paper is as follows. In Section 2, we review the co-processing model on complex networks, which couples the continuous time processing and the discrete diffusing dynamics. In Section 3, the co-processing centrality, which exhibits the capability that a node communicates information with its neighbor environment over the network in this diffusion process, is proposed. Furthermore, to measure the interplay between co-processing centrality and diffusion process, a new parameter, co-processing entropy, is deeply investigated and the experimental results on large-scale complex networks with Poisson distribution are given. In Section 4, we discuss the information function of the co-processing model to deeply investigate the diffusion process of this model. In Section 5, the conclusion is given and the prospect is deeply discussed.

2. Review of co-processing model in communication networks

To investigate the time-lag diffusion process, we consider a co-processing model in a network with nodes set $S = \{1, 2, \dots, N\}$: after staying at the initial node i_0 for time t_0, the particle hops to a neighbor node i_1 at time t_0 with probability $t_{i_0 i_1}$; it will hop to node i_2 with probability $t_{i_1 i_2}$ at time t_1, with the sojourn time $t_1 - t_0$ at node i_1, and so on, introduced in Ref. [36].

Denote $P(t) = (p_{ij}(t))_{N\times N}$ as the probability transition matrix for this process $\{X(t) \in S,\, t \geq 0\}$, where $p_{ij}(t)$ is the probability that the particle reaches node j at time t with initial node i. We just consider that $p_{ij}(t)$ does not rely on the initial time, but the time interval t. Neglecting the time lag at each node, the jumps can be recorded by a discrete random walk with probability transition matrix $T = (t_{ij})_{N\times N}$, where t_{ij} is the probability of the jump from node i to node j.

By the knowledge of stochastic process, the transition matrix $P(t) = (p_{ij}(t))_{N\times N}$ of $\{X(t)\}$ must satisfy the following properties:

(1) $0 \leq p_{ij}(t) \leq 1\,, \qquad p_{ij}(0) = \delta_{ij}\,,$

(2) $p_{ij}(t+s) = \sum_k p_{ik}(t) p_{kj}(s)\,,$

(3) $\lim_{t\to 0+} p_{ij}(t) = p_{ij}(0) = \delta_{ij}\,.$

For the properties above, we can derive that

$$\lim_{t\to 0+} \frac{p_{ij}(t) - \delta_{ij}}{t} \doteq r_{ij}\,,$$

in which the limit r_{ij} does exist and $r_{ij} < \infty$ for all $i, j \in S$.

For

$$\sum_{i\neq j} \frac{p_{ij}(t)}{t} = \frac{1 - p_{ii}(t)}{t}\,,$$

then

$$\sum_{i\neq j} r_{ij} = -r_{ii} \doteq r_i\,.$$

$\{r_{ij},\ i \neq j\}$ reflects the rate of the transition probability from node i to j, and $R = (r_{ij})_{N\times N}$ is the transition rate matrix of the process $\{X(t)\}$.

By property (2)

$$p_{ij}(t+\triangle t) = \sum_{k\in S} p_{ik}(t)p_{kj}(\triangle t)\,,$$

and then, the *master equation* for this time-lag process $\{X(t)\}$ can be derived

$$\frac{dp_{ij}(t)}{dt} = \sum_{k\in S} p_{ik}(t)r_{kj}\,. \tag{1}$$

In practice, it is difficult to determine the transition matrix $P(t) = (p_{ij}(t))_{N\times N}$. However, the rate matrix $R = (r_{ij})_{N\times N}$ for this process consists of the differential coefficient of $\{p_{ij}(t)\}$ at $t = 0$, and it is easy to measure $\{p_{ij}(t)\}$ nearby $t = 0$. Usually, we get $R = (r_{ij})_{N\times N}$ at first, and then deduce the probability transition matrix $P(t)$ according to equation (1).

In the following, an interpretation of r_i will be given to have a well understanding of the transition rate matrix R. Denote τ as the time the particle firstly departs from the initial node i, then

$$\begin{aligned}
P\{\tau > t|X(0)=i\} &= P\{X(u)=i, 0<u<t|X(0)=i\}\\
&= \lim_{n\to\infty} P\{X(kt/2^n)=i, k=1,2,\cdots,2^n|X(0)=i\}\\
&= \lim_{n\to\infty} [p_{ii}(t/2^n)]^{2^n}\\
&= \lim_{n\to\infty} \exp\left(\frac{\ln p_{ii}(t/2^n)}{t/2^n}\,\frac{t}{2^n}\,2^n\right)\\
&= \exp(-r_i t)\,.
\end{aligned}$$

This formula illustrates that the sojourn time at node i follows the exponential distribution with parameter r_i, which determines the transition rate that the particle departs from node i.

Supposing that j and k are neighbors of node i, the relation between t_{ij} and t_{ik} is

$$\frac{t_{ij}}{t_{ik}} = \lim_{t\to 0+}\frac{p_{ij}(t)}{p_{ik}(t)} = \frac{r_{ij}}{r_{ik}}$$

and $r_i = -r_{ii} = \sum_{i\neq j} r_{ij}$, then

$$t_{ij} = \frac{r_{ij}}{r_i}\,.$$

At every jump, the particle hops to node j from node i with probability $t_{ij} = \frac{r_{ij}}{r_i}$.

Therefore, this process $X(t)$ is that the particle hops to node j from node i with probability $t_{ij} = r_{ij}/r_i$ in discrete time series, and the time it stays at the node i before hopping to node j follows the exponential distribution with parameter r_i.

2.1. The stationary distribution and mean first passage time in the process

The stationary distribution, which reflects the importance for nodes, is explored in the following. For the connected network, the stationary distribution $\{\mu_j = \lim_{t\to\infty} p_{ij}(t), \forall\, j \in \boldsymbol{S}\}$ uniquely exists (not relying on the initial state i [26]) and satisfies

(1) $\mu P = \mu\,, \qquad \forall\, t \geq 0\,,$

(2) $\sum_i \mu_i = 1\,.$

For

$$\lim_{t\to 0+} \frac{p_{ij}(t) - \delta_{ij}}{t} = r_{ij}\,,$$

μ satisfies

$$\mu R = 0$$

with the transition rate matrix R, and the stationary probability can be derived.

The probability in the stationary distribution of the co-processing model equals the number of times that the particle passes the node multiplied by the mean sojourn time at the node. Then, the relation of the stationary distribution between the process $\{\mu_i\}_{i=1}^N$ and the transition matrix T [21] is

$$\mu_i \propto P_i^\infty \times \frac{1}{r_i} = \frac{P_i^\infty}{r_i}\,, \tag{2}$$

which reflects that the probability in the stationary state relies not only on the times that the particle arrives at the node, but also the sojourn time the particle stays at this node. This can be used against the attack of a hacker, that is, we could adjust the sojourn time at some node to modify the stationary distribution against the hacker capturing the secrete information.

To reveal the impact of the time lag on the transition efficiency of the process, we investigate the relationship between the mean first passage time and the stationary distribution.

Denote σ_{ij} as the first passage time from node i to node j, that is, the first time the particle arrives at node j after departing from node i, and denote $\langle\sigma_{ij}\rangle$ as the mean first passage time (MFPT) from node i to node j.

For $i=j$, $1/\langle\sigma_{ii}\rangle$ is the mean time of the particle returning to node i itself in unit time. For each arrival, the average time lag at node i is $1/r_i$, thus there exists

$$\mu_i = \frac{1}{r_i\langle\sigma_{ii}\rangle}\,,$$

that is,

$$\langle\sigma_{ii}\rangle = \frac{1}{r_i\mu_i}\,.$$

For $i\neq j$, the MFPT from i to j is the sum of the mean time lag at node i and the mean MFPT from the neighbors of i to j, so

$$\langle\sigma_{ij}\rangle = \frac{1}{r_i} + \sum_{k\neq i,k\neq j} t_{ik}\langle\sigma_{kj}\rangle\,, \qquad i\neq j\,. \tag{3}$$

With the expression of $\langle\sigma_{ii}\rangle$, for each pair (i,j), $\langle\sigma_{ij}\rangle$ can be calculated accurately.

As an application in information security, a co-processing model on a random graph, the degree distribution of which is $p(k)=e^{-c}\frac{c^k}{k!}$ with average degree c can be introduced as follows: the particle at node i hops to its neighbors with equal probability $\frac{1}{k_i}$ and the time it stays at node i follows the exponential distribution with parameter $r_i = k_i^{\alpha}$, in which k_i is the degree of node i and α is a parameter reflecting the handling ability of the nodes. If the graph is unconnected, we consider the random walk on each connected component respectively, and then make a normalization over the whole graph. The discrete diffusion process corresponds to the classical unbiased random walk [26]. For that

$$t_{ij} = \frac{A_{ij}}{k_i} = \frac{r_{ij}}{r_i} = \frac{r_{ij}}{k_i^{\alpha}}\,,$$

then

$$r_{ij} = A_{ij}k_i^{\alpha-1}\,.$$

Thus, the transition rate matrix of this time-lag process is

$$R = \begin{pmatrix} -k_1^{\alpha} & A_{12}k_1^{\alpha-1} & \cdots & A_{1N}k_1^{\alpha-1} \\ A_{21}k_2^{\alpha-1} & -k_2^{\alpha} & \cdots & A_{2N}k_2^{\alpha-1} \\ \cdots & \cdots & \cdots & \cdots \\ A_{N1}k_N^{\alpha-1} & A_{N2}k_N^{\alpha-1} & \cdots & -k_N^{\alpha} \end{pmatrix}.$$

For $\mu R = 0$, thus, the stationary distribution μ and the MFPT from node i to itself $\langle\sigma_{ii}\rangle$ are separately

$$\mu \;=\; \left(\frac{k_1^{1-\alpha}}{\sum\limits_i k_i^{1-\alpha}}, \frac{k_2^{1-\alpha}}{\sum\limits_i k_i^{1-\alpha}}, \cdots, \frac{k_N^{1-\alpha}}{\sum\limits_i k_i^{1-\alpha}}\right)\,,$$

$$\langle \sigma_{ii} \rangle \;=\; \frac{1}{\mu_i r_i} = \frac{\sum\limits_l k_l^{1-\alpha}}{k_i}\,.$$

For $i \neq j$, by equation (3)

$$\langle \sigma_{ij} \rangle = \frac{1}{k_i^\alpha} + \sum_{k\neq i, k\neq j} \frac{A_{ik}}{k_i} \langle \sigma_{kj} \rangle\,.$$

For $\alpha > 1$, nodes with larger degree have weaker importance, and *vice versa*, which is reflected by the stationary distribution. Against the attack of hacker, we can adjust the handling ability of the node locally to strengthen the robustness and optimize the function of the network.

3. The co-processing centrality and co-processing entropy

The time-lag dynamical process can be expressed by co-processing model, we can investigate the information properties of the co-processing model to study the properties of the time-lag dynamical process.

3.1. Co-processing centrality

In the co-processing model, μ_i reflects the probability that we can find the particle at the steady state. For node i with degree k, that is, node i has k neighbors, μ_i^k reflects the contribution of node i to its neighbors, thus $w_i = \sum_{k=1}^{\infty} p(k)\mu_i^k$ is the mean contribution of node i for different degrees to attract the random walker at the stationary state. Therefore, we define the *co-processing centrality of vertex* i as follows

$$h_i = \frac{w_i}{\sum\limits_l w_l} = \frac{\sum\limits_{k=1}^{\infty} p(k)\mu_i^k}{\sum\limits_N \sum\limits_{k=1}^{\infty} p(k)\mu_i^k}\,,$$

which is the normalized contribution of node i to attract the random walker. As the application in Section 2, considering the co-processing model on random networks with different α, the co-processing centrality of node i is

$$h_i = \frac{e^{c\left(\frac{k_i^{1-\alpha}}{\sum_l k_l^{1-\alpha}} - 1\right)} - e^{-c}}{\sum\limits_{i=1}^{N} e^{c\left(\frac{k_i^{1-\alpha}}{\sum_l k_l^{1-\alpha}} - 1\right)} - Ne^{-c}}\,. \tag{4}$$

For $\alpha = -2, 0, 2$, the co-processing centrality h for the nodes with different degrees is shown in figure 1.

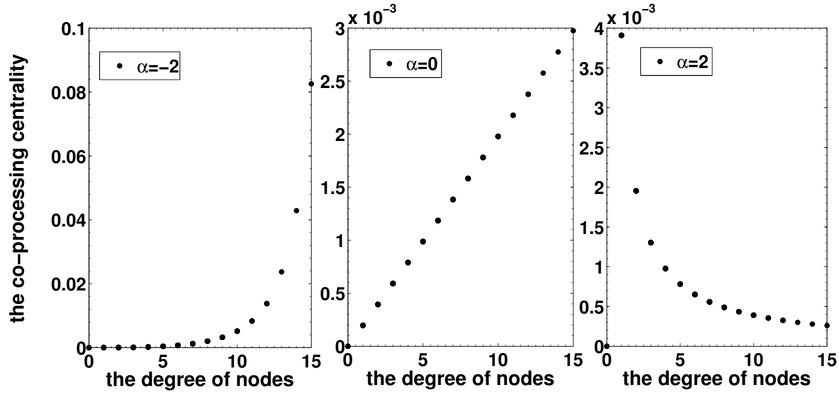

Fig. 1. The co-processing centrality h changes with the degree of node k when $\alpha = -2, 0, 2$.

As shown in figure 1, when $\alpha < 0$, nodes with higher degree have weaker handling ability, which results that the sojourn time at such nodes is longer. Besides, the random walker tends to stay at nodes with higher degree, and then the co-processing centrality of higher nodes is relatively larger than that of the nodes with lower degree. When $\alpha = 0$, there is no difference between the handling ability of nodes, but as nodes with higher degree have stronger connectedness, the co-processing centrality is decided directly by the degree. However, the higher the degree, the more important the node, *i.e.*, the co-processing centrality is decided by the degree directly. In contrast to the case of $\alpha < 0$, when $\alpha > 0$, nodes with higher degree have stronger handling ability, the random walker tends to stay at nodes with low degree, which perform more importance than those with higher degree.

Generally, different cases of α result in different handling ability of nodes. The stationary distribution depends not only on the degree, but also on the handling ability of nodes. Thus, α can be modulated to distinguish the different important nodes in the co-processing model.

3.2. Co-processing entropy

Entropy is a measure of randomness and confusion. To measure the interplay between co-processing centrality and diffusion dynamics, the information properties can be accounted by *co-processing entropy* which is defined as follows

$$\mathrm{CE} = -\sum_{i=1}^{N} h_i \ln(h_i)\,, \tag{5}$$

where h_i is the co-processing centrality of node i. For h_i reflects the importance of node i attracting the random walker, CE measures the confusion state of the walker at the stationary state.

For random graph and different α,

$$h_i = \frac{e^{c\left(\frac{k_i^{1-\alpha}}{\sum_l k_l^{1-\alpha}}-1\right)} - e^{-c}}{\sum_{i=1}^{N} e^{c\left(\frac{k_i^{1-\alpha}}{\sum_l k_l^{1-\alpha}}-1\right)} - Ne^{-c}}. \tag{6}$$

Applying equation (6) into equation (5), experimental results of the co-processing entropy for $\alpha = 0, 1, 2$ and the average trend for CE along with average degree c in detail are shown in figure 2.

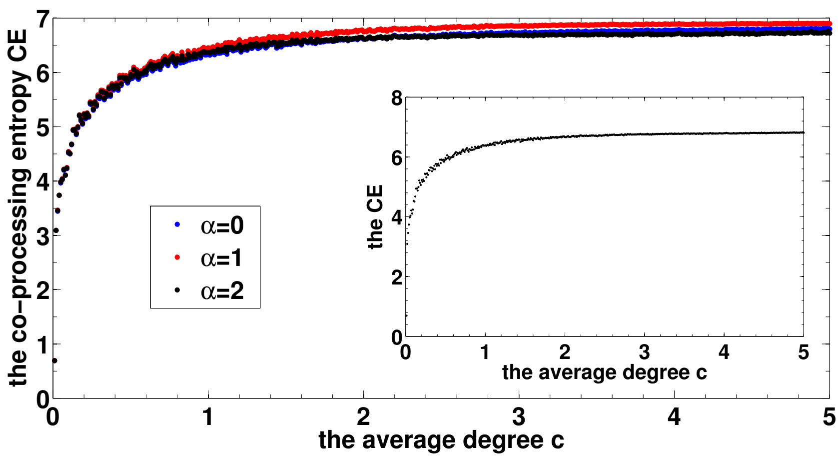

Fig. 2. The co-processing entropy CE as a function of the average degree c of the random graph with $N = 10^3$ and $\alpha = 0, 1, 2$ is in the main graph, and the subgraph is the average co-processing entropy as a function of c.

As the average degree c increases, the random walker can arrive at more and more nodes, and the final state is more and more jumbled, which induce the increase of the co-processing entropy CE. As the scale of the giant component increases slowly when $c > 1$ [37], the co-processing entropy CE performs little increase. According to the Maximum Entropy Principle [38], to investigate dynamical process on the network globally, we can add edges into the network until the giant cluster appears.

To show the detailed difference of the CE among different parameter α, numerical verifications are carried out at $c = 3, 4, 5$ and the average trend for CE along with the parameter α is described, which is shown in figure 3. When α increases, nodes with higher degree have stronger handling ability. The nodes with large co-processing centrality concentrate on fewer low-degree nodes as α increases, and the walker is more inclined to such more dynamical important nodes, which cause decrease of the level of the confusion and the co-processing entropy CE decreases, too.

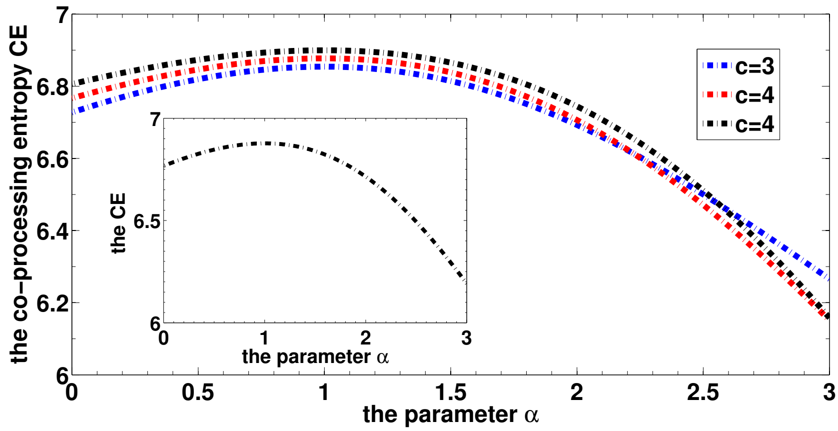

Fig. 3. The co-processing entropy CE as a function of the parameter α of random graph with $N = 10^3$ and $c = 3, 4, 5$ is in the main graph, and the subgraph is the average co-processing entropy as a function of α.

To measure the co-processing centrality on the complex networks, we consider the co-processing model on networks with 1000 nodes and the average degree $c = 5$, and remove either *(i)* the most co-processing important nodes, *(ii)* the nodes with highest degree k_i, *(iii)* random nodes. After removing m nodes, we recalculate the co-processing entropy CE(m). In figure 4, CE(m)/CE(0) is shown as a function of m. When $\alpha > 1$, the nodes

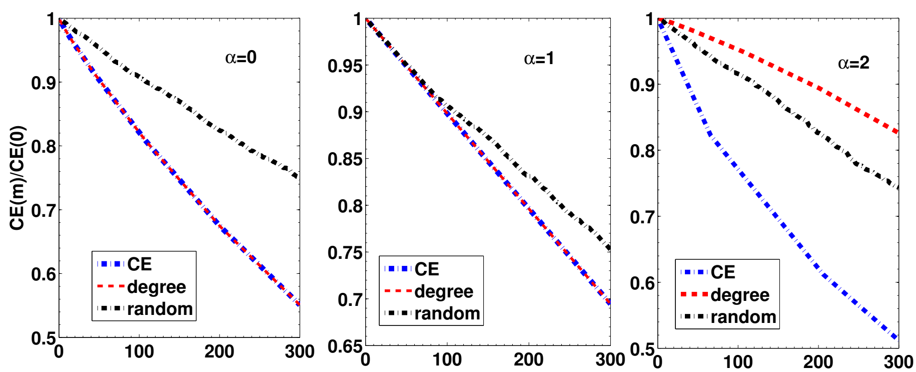

Fig. 4. For different parameter $\alpha = 0, 1, 2$ of random walk, the co-processing entropy changes with the number of vertices taken off, for different cases: by the co-processing centrality (CE), degree, and random.

with high degree have stronger handling ability and contribute more in the co-processing model. On the contrary, the nodes with low degree contribute more when $\alpha < 1$, and the degree centrality is just a special situation of the co-processing centrality when $\alpha < 1$, which is reflected in figure 6. All these numerical figures show that the co-processing centrality has great influence on the dynamical process on the complex networks.

4. Information function of co-processing model

Since the time-lag dynamical process can be expressed by co-processing model, we define the *information function* of the diffusion process as

$$H = \sum_{i=1}^{N} \sum_{k=1}^{\infty} p(k) \mu_i^k = \sum_{i=1}^{N} w_i \,, \tag{7}$$

where $w_i = \sum_{k=1}^{\infty} p(k)\mu_i^k$. The information function H is the co-information measure of the co-processing model on the complex networks.

As an application, we analyze the co-processing model on a random graph, the degree distribution of which is $p(k) = e^{-c}\frac{c^k}{k!}$ with average degree c.

For each α,

$$\mu_i = \frac{k_i^{1-\alpha}}{\sum\limits_l k_l^{1-\alpha}} \,,$$

the average contribution of node i is

$$w_i = \sum_{k=1}^{N} e^{-c} \frac{c^k}{k!} \left(\frac{k_i^{1-\alpha}}{\sum\limits_l k_l^{1-\alpha}} \right)^k = e^{c\left(\frac{k_i^{1-\alpha}}{\sum_l k_l^{1-\alpha}} - 1 \right)} - e^{-c} \,, \tag{8}$$

and the relative information function is

$$H = \sum_{i=1}^{N} e^{c\left(\frac{k_i^{1-\alpha}}{\sum_l k_l^{1-\alpha}} - 1 \right)} - N e^{-c} \,. \tag{9}$$

For $\alpha = 0$,

$$H = \sum_{i=1}^{N} e^{c\left(\frac{k_i}{\sum_l k_l} - 1 \right)} - N e^{-c} \,. \tag{10}$$

To get the average degree c at which the information function reaches the maximum value, let

$$\frac{\partial H}{\partial c} = \sum_{i=1}^{N} e^{c\left(\frac{k_i}{\sum_l k_l}-1\right)} \left(\frac{k_i}{\sum_l k_l} - 1\right) + Ne^c = 0\,. \tag{11}$$

We apply the mean field approximation to equation (11), then

$$\sum_k p(k) e^{c\left(\frac{k}{\sum_l k_l}-1\right)} \left(\frac{k}{\sum_l k_l} - 1\right) + e^c = \sum_k p(k) e^{c\left(\frac{k}{Nc}-1\right)} \left(\frac{k}{Nc} - 1\right) + e^c = 0\,,$$

where $p(k)$ is the degree distribution. Thus,

$$c = \ln\left(1 + \frac{1}{N-1}\right)^N\,. \tag{12}$$

From equation (12), we obtain that the information function achieves the maximum value at $c = 1$ when $N \to \infty$.

For random graphs with $N = 10^3$ but different average degrees, we simulate the information function H for different $\alpha = 0, 1, 2$ and show the corresponding results in figure 5 which agree with our analysis perfectly.

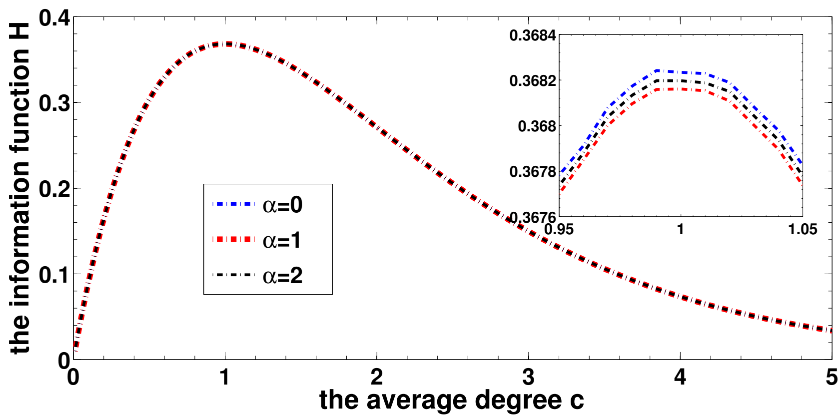

Fig. 5. The information function H changes with the average degree c of the random graph with $N = 10^3$ and $\alpha = 0, 1, 2$.

It reveals that the giant component appears when the average degree c gets over 1 [37]. Once $c < 1$, the largest component increases quickly along with the increase of the average degree c, and the amount of information received by the random walker also increases. As $c > 1$, the increasing speed of the giant component size slows down [37], and the amount of information gotten by the random walker continues increasing. However, the increasing rate is lower than that of the time consumed, which induces that the average efficient information H decreases along with the increase of c. In the inset in figure 5, we can see that the information function decreases along with the increase of the parameter α for $\alpha < 1$ and increases with the increase of α for $\alpha > 1$. We verify it as follows.

When $\alpha < 0$, nodes with lower degree have stronger handling ability, thus the random walker stays at these nodes with shorter time, besides, nodes with larger degree have stronger connectedness, which makes the total amount of information small. While $0 < \alpha < 1$, nodes with higher degree have better connectedness, which is balanced by out their stronger handling ability, thus, the information of the diffusion decreases with the increase of α. When the parameter $\alpha > 1$ increases, nodes with higher degree have stronger handling ability, thus the random walker stays at these nodes with shorter time, in addition with their stronger connectedness, which leads to the increase of value of information along with the increase of parameter $\alpha > 1$. The information function H on the random graph for different adjustment parameter α is shown in figure 6, which fits our analysis very well.

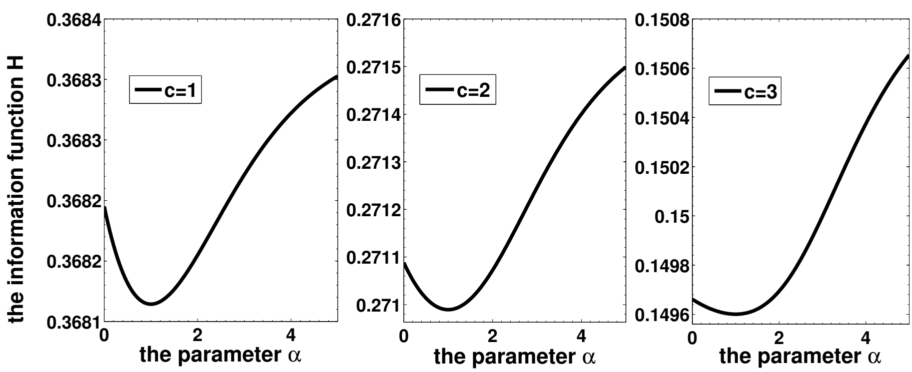

Fig. 6. The information function H changes with the parameter α of the random graph with $N = 10^3$ and $c = 1, 2, 3$.

5. Conclusion

In summary, we have investigated in detail the information properties of the co-processing model on complex networks, in which the information flow with potential time lag couples the continuous time processing and the discrete diffusing dynamics. Furthermore, the information function of process is discussed to explore the coupled properties. For various centrality measures which have been discussed mainly based on diffusion process without time lag, the co-processing centrality of nodes, which exhibits the capability that a node communicates information with its neighbor environment over the network in the co-processing model, is introduced and employed to evaluate the average contribution of node during the process. Furthermore, a new parameter, co-processing entropy, is proposed to measure the interplay between co-processing centrality and diffusion dynamics, which can measure the confusion at the stationary state. Experimental results on large-scale complex networks confirm our analytical prediction.

In this paper, the node handling ability parameter α is considered as a constant all over the process, but in the diffusion process of realistic self-adaptive networks, *e.g.* traffic network and World Wide Web, it has to regulate itself in the diffusion process to optimize the transmission capability over the network. That is, when there are some nodes overloaded, we should modulate α to mitigate the transmission loads to others. In the future work, dynamical self-adaptive parameter $\alpha(t)$ will be considered to realize the network optimization.

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