STUDY OF ANOMALOUS FRACTAL DIMENSIONS AND SCALING EXPONENT IN GINZBURG–LANDAU PHASE TRANSITION IN 14.5 $A\,\mathrm{GeV}/c$ ^{28}Si–AgBr INTERACTIONS

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In this paper, multiplicity distributions of charged particles produced in 14.5 $A\,\mathrm{GeV}/c$ ^{28}Si–AgBr interactions and the events generated by Monte Carlo models, FRITIOF and HIJING, are analysed using Scaled Factorial Moments (SFMs) approach in terms of the Ginzburg–Landau formalism. The reported power-law behaviour of the higher-order and second-order scaled factorial moments is searched for in the experimental and simulated data. The analysis of anomalous fractal dimensions does not favour occurrence of any exotic phenomenon. Further, the values of the slopes, β_q, are used to determine a universal scaling exponent, ν, for both the experimental and simulated data in terms of Ginzburg–Landau approach in order to investigate the nature of phase transition from the de-confined quark–gluon medium to the confined hadronic state. The analysis indicates that the values of ν for the experimental and FRITIOF data compare reasonably well with its critical value. Nevertheless, the value of ν for the HIJING data is in marked disagreement. This may be attributed to the fact that local fluctuations are not addressed effectively by HIJING model; however, it successfully explains the phenomenon of flow. Finally, some evidences regarding second-order phase transition are found.

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1. Introduction

To understand the dynamics of multiparticle production in relativistic nuclear collisions, a useful and attractive probe is intermittency [1, 2], particularly for disentangling information regarding dynamical fluctuations. In particle physics, intermittency refers to power-law behaviour of the scaled factorial moments with decreasing rapidity bin size; this also helps extract dynamical fluctuations after eliminating statistical contribution. This behaviour [1, 2] suggests that mechanism of multiparticle production in relativistic nuclear collisions is self-similar. The possibility of occurrence of intermittent spectra of particles produced in high energy nuclear collisions is envisaged [1] to be a vital tool to explain the appearance of spikes in rapidity distributions in high energy physics experiments [3–5].

Existence of dynamical fluctuations in high energy nuclear collisions may be investigated in terms of intermittency. Study of this aspect of multiparticle production has recently provided great impetus to understand the dynamics of multiparticle production in such collisions. The idea which motivated Bialas and Peschanski [1, 2] to study intermittency, originated from the observation of rapidity fluctuations in an unusually high multiplicity cosmic ray event, commonly known as JACEE [3] event. Nevertheless, success in intermittency analysis in disentangling dynamical fluctuations in the JACEE event has influenced many other workers to study it more critically with the accelerator data involving several projectiles, targets and incident energies [4, 6, 7]. Importance of studying fluctuations stems from the fact that these fluctuations may manifest during QGP formation. Hence, occurrence of dynamical fluctuations in relativistic nuclear collisions may be considered as an important signature of QGP production [8, 9].

Currently, one of the main objectives for undertaking high-energy heavy-ion experiments is to search for possible signals of QGP formation in laboratory. In these collisions, QGP is envisaged to be formed and the system is predicted to cool, while expanding and undergo a phase transition from the deconfined QGP medium to confined hadrons. Since only the final state particles in the collisions are observable experimentally, one can, therefore, search for the signatures of the phase transition by analysing characteristics of only these particles; order of the phase transition, whether first or second order, also needs to be investigated critically.

Lattice Quantum Chromodynamics calculations predict that if nuclear matter is heated above a critical temperature $T_{\mathrm{c}} \simeq 170$ MeV, quarks and gluons are liberated and a hot, dense and deconfined state of matter, quark–gluon plasma (QGP), is formed [10–14]. The results of the study of Pb–Pb collisions at LHC energies with the ATLAS detector hint towards formation of QGP [15]. However, there are reports that QGP phase transition does not take place in certain types of interactions [7, 16, 17]. It may be noted that analysis in terms of intermittency can provide useful information regarding formation of QGP and the order of the phase transition. Until now, the issue relating to possible signals of QGP formation is still at discussion stage [18]. Debate on the possible signatures of phase transitions is still going on; the answer to the fundamental question, whether the phase transition is of the first or second order, is being sought.

Large multiplicity fluctuations associated with the hadronization process during the phase transitions occur; fluctuations with different features are visualized to occur in different types of phase transitions. A study of multiplicity fluctuations in the final states might enable one to find out some new observables linked to the characteristics of different types of phase transitions. In statistical physics, the issue of phase transitions can be addressed in terms of the Ginzburg–Landau model [19], which is used to explain multiparticle production [20–23] phenomenon in ultrarelativistic heavy-ion collisions. The values of the parameters of the model are determined for both the second-order [24, 25] and first-order [26, 27] phase transitions; values of scaled factorial moments and scaling exponent, ν, are also calculated. It has been shown [24] that in the Ginzburg–Landau model, the anomalous fractal dimensions are not constant but satisfy the relation, $\frac{d_q}{d_2} = (q-1)^{\nu-1}$, where ν is the scaling exponent, which has a critical value of 1.304. It may be noted that ν is a universal quantity, strikingly the same for all the systems describable by the Ginzburg–Landau model and independent of the parameters of the model. It is interesting to mention that when anomalous fractal dimensions, d_q, become constant, the inner structure of the fluctuations is represented by monofractal patterns, which may be regarded a possible signal of QGP formation corresponding to the second-order phase transition. On the other hand, dependence of d_q on the order of the moments, q, would indicate the absence of such a phase transition.

As stated earlier, the value of ν is used to investigate the presence of the second-order phase transition in the hadronization process in high energy nuclear interactions. If for an experimental data, the value of ν turns out to be less than its critical value, occurrence of a second-order quark–hadron phase transition is a clear possibility [24]; value of ν higher than its critical value would indicate the absence of such a phase transition. Experimental values [6, 7, 14, 24, 27–32] of the scaling exponents are reported to be more than the critical value. Thus, no clear evidences supporting existence of the second-order phase transition are found in these experiments. Several workers [16, 17, 33–38] have investigated features of intermittency and evidence of phase transition in terms of the Levy index using F_q, G_q and Takagi moments. Therefore, it was considered worthwhile to investigate multiplicity fluctuations as a phenomenological manifestation of quark–hadron phase transition in the framework of the Ginzburg–Landau model [19]. We would also like to point out that till date very few attempts have been made to calculate the value of the scaling exponent, ν, using scaled factorial moments for HIJING generated events for examining the nature of transition from the deconfined quark–gluon plasma to the confined hadron state in high multiplicity nuclear collisions at low energies.

We have studied multiplicity fluctuations in 14.5 $A\,\mathrm{GeV}/c$ ^{28}Si–AgBr collisions in terms of scaled factorial moments using the Ginzburg–Landau formalism. We have also investigated a relationship between higher-order and second-order scaled factorial moments for the experimental and Monte Carlo models, FRITIOF and HIJING, generated data. The dependence of power-law slopes on the order of the moments is looked into and the value of scaling exponent, ν, is calculated to investigate mechanism of hadronization. It may be of interest to note that we have investigated the differences in the values of scaling exponent, ν, calculated for the experimental and HIJING generated events. The main motivation behind using HIJING [39, 40] Monte Carlo model is to compare the experimental results with those obtained using FRITIOF model because of the realization that this is the most popular Monte Carlo program to study multiparticle production in high energy pp, pA and AA collisions. It addresses issues relating to soft and hard interactions, nuclear modification of structure functions, jet quenching and true geometry of nuclear collisions. It is worth mentioning that this model was developed and designed for understanding and explaining RHIC and LHC physics with special emphasis on the role of minijets in pp, pA and AA collisions at collider energies.

The value of ν for HIJING data comes out to be 1.824 ± 0.233, which is markedly different from its critical value of 1.304. However, the values of ν for the experimental and FRITIOF data match fairly well with the critical value. It may be noted that the value of ν for the HIJING data is not in agreement with its predicted value as well as the value predicted by AMPT (A Multi-Phase Transport) model. It may be attributed to the fact that local fluctuations are not very well addressed by HIJING model as it mainly deals with physics issues relating to flow.

2. A brief description of the data sample

Data collected from ILFORD G5 emulsion exposed to the beam of 14.5 $A\,\mathrm{GeV}/c$ ^{28}Si nuclei from AGS, BNL is used. The events were searched by the method of line scanning and the space angles of the produced particles, θ_{s}, were measured using the coordinate method. The other relevant details regarding scanning procedure, criteria for selecting events, method of measurements, *etc.*, may be found in the book by Powell *et al.* [41] and our earlier publications [42–44]. A random sample comprising 605 interactions having $n_h \geq 0$, where n_h represents the number of charged particles produced in an interaction with relative velocities $\beta \leq 0.7$, are analysed. Furthermore, the experimental results are compared with the corresponding results obtained for the events with the same description as the experimental ones generated by FRITIOF and HIJING models.

3. Method of analysis

3.1. The scaled factorial moments

For explaining occurrence of intermittent behaviour in relativistic nuclear interactions, Bialas and Peschanski [1, 2] have proposed a method of scaled factorial moments [1, 2]. For this, a limited pseudorapidity range $\Delta\eta$ is divided into M bins of equal width, $\delta\eta = \Delta\eta/M$. Then, for events of varying multiplicities, the $q^{\rm th}$ order moment is calculated [1, 2] using

$$\langle F_q \rangle = \frac{M^{q-1}}{N_{\rm evt}} \sum_{N_{\rm evt}} \sum_{m=1}^{M} \frac{n_m(n_{m-1})\ldots(n_m - q + 1)}{\langle N \rangle^q}\,, \tag{1}$$

where n_m is the number of particles in $m^{\rm th}$ bin for a single event, $\langle N \rangle$ is the average particle multiplicity in the pseudorapidity range $\Delta\eta$ and $N_{\rm evt}$ denotes the total number of events considered.

It has been shown [1, 2] that for a smooth rapidity distribution, not exhibiting any fluctuations other than the statistical ones, $\langle F_q(\delta\eta) \rangle$ is essentially independent of the resolution $\delta\eta$ in the limit $\delta\eta \to 0$. However, if the dynamical fluctuations are self-similar in nature in the limit of small bin size, the scaled factorial moments are visualized to follow a power-law behaviour of the type

$$\langle F_q \rangle = \left[\frac{\Delta\eta}{\delta\eta}\right]^{\phi_q} . \tag{2}$$

Alternatively, Eq. (2) can be expressed as

$$\ln\langle F_q \rangle = k - \phi_q \ \ln \delta\eta\,, \tag{3}$$

where k is proportionality constant. Such a power-law behaviour of the scaled factorial moments, as indicated by Eq. (2), is referred to as intermittency [1] or indication of a self-similar fractal structure of multiparticle production, which predicts a linear rise of $\ln\langle F_q \rangle$ as a function of $-\ln \delta\eta$. It may be of interest to note that ϕ_q appearing in Eqs. (2) and (3) measure the strength of intermittency and is referred to as intermittency exponent. The values of the intermittency exponents can be obtained by performing best fits to the data using Eq. (3). Intermittency is envisaged [45, 46] to be a reflection of the phase transition which can be studied through anomalous fractal dimensions, d_q [47, 48]

$$d_q = \frac{\phi_q}{q-1} \,. \tag{4}$$

The intermittency indices, ϕ_q, of the power law can also be used to determine the ratio of higher-order anomalous fractal dimensions and second-order anomalous fractal dimension [24, 49]. Dependence of the ratio $\frac{d_q}{d_2}$ on the order of the moment q can provide an insight into the process leading to intermittent behaviour.

3.2. Scaling exponent

The Ginzburg–Landau model [19] has been extensively used to explain scaling of scaled factorial moments in both second-order [24, 25] and first-order [26, 27] phase transitions. A distinct power-law behaviour of the moments $\langle F_q \rangle$ of the order of q is not envisaged [24] in the Ginzburg–Landau theory. It has been pointed out by Ochs and Wosiek [50] that one-dimensional scaled factorial moments follow a modified power-law of the type

$$\langle F_q(\delta\eta)\rangle \;\propto\; [g(\delta\eta)]^{\phi_q} \;, \tag{5}$$

where bin-width $\delta\eta$ in the full range with a non-linear function, or equivalently pairs of moments are related [50] as

$$\ln\langle F_q\rangle = \beta_q \;\ln\langle F_2\rangle + G_q \,, \tag{6}$$

where $\beta_q = \frac{\phi_q}{\phi_2}$ and G_q is a constant. The values of the slopes β_q can be determined by plotting $\ln\langle F_q\rangle$ against $\ln\langle F_2\rangle$.

It was shown recently that self-similar behaviour in multiplicity fluctuations exists in the Ginzburg–Landau description of the second-order phase transition [24, 25]. It may be noted that Eq. (6) represents the variation of $\ln\langle F_q\rangle$ with $\ln\langle F_2\rangle$ irrespective of their separate dependence on $\delta\eta$. Thus, slopes β_q appearing in Eq. (6) summarize scale-invariant property on global scale. Hwa and Nazirov [24, 25] have also shown that β_q can be used to probe the nature of intermittent systems. They have discussed intermittency phenomenon of hadrons arising from quark–gluon plasma in terms of Ginzburg–Landau description of the second-order phase transition and have obtained the following relationship

$$\beta_q \;= (q-1)^{\nu} \,. \tag{7}$$

The scaling behaviour represented by Eq. (7) is universal and scaling exponent, ν, is independent of the details of the phase transition. The slopes β_q and ν are independent of the phase space bin size as well as dimensions of the phase space [24, 25, 28, 51]. The scaling exponent can be used to investigate the presence of second-order phase transition in the hadronization process in high energy nuclear interactions.

Hwa and Yang [52] have reviewed many papers on local behaviours of multiplicity fluctuations of the systems undergoing a second-order phase transition describable by the Ginzburg–Landau theory. These are then connected to the distribution of clusters produced that can be used to simulate initial configurations before start of hadronization. It may be stressed that one parameter is required to describe clustering. Later, for addressing the non-critical case, this parameter needs to be adjusted so as to obtain random distribution. It has been reported that normalized FMs(F_q) can be used as a quantitative measure of the local fluctuations [1, 2] and they should follow a power-law of the type: $F_q(\delta) \propto \delta^{-\phi_q}$ over a range of small δ, referred to as intermittency, which has been observed in many colliding systems [53]. An important feature of this method is that it does not only detect large non-statistical fluctuations but also filters out statistical ones. It is worth mentioning that if a multiplicity distribution is written as a convolution of Poisson and dynamical distributions and the underlying dynamics is trivial, then F_q must be unity for all q values [52]. Any deviation from unity may be then a measure of non-trivial dynamical fluctuations, and $F_q(\delta) \propto \delta^{-\phi_q}$ representing power-law behaviour would suggest a dynamics that is not characterized by a particular scale [1, 2]. The footprint of a phase transition resulting in fluctuations of all scales may be possibly reflected in the measurement of intermittency [54].

In order to quantify phase transition in terms of F_q, a study of second-order phase transition in the framework of Ginzburg–Landau theory was carried out [24, 25], in which the order parameter is identified as the multiplicity density. Equation (2) represents the power-law behaviour and both β_q and ν appearing in Eq. (7) are essentially independent of the nature of the GL parameters. Such a behaviour was experimentally observed in the study of photon number fluctuations of a single-mode laser at the threshold of lasing [52, 55], confirming that it is a phase transition problem describable by GL theory [56]. On the theoretical side, it has also been found that using 2-dimensional Ising model to simulate quark–hadron phase transition and the resulting scaling behaviour of F_q is in agreement with Eqs. (6) and (7) [57]. It does not, however, imply that an analysis of F_q using the data on Pb–Pb collisions at the LHC energies can verify or falsify a connection between hadronization and second-order phase transition because of the complications that are inherently present in such systems but are absent in the optical systems. Details of calculations of the parameters of GL theory and critical exponent, ν, can also be found in Refs. [52, 58].

As already stated, anomalous fractal dimensions, d_q, and intermittency indices, ϕ_q, satisfy Eq. (4). In terms of β_q, the ratio of the anomalous fractal dimensions of higher-order($q \geq 3$) and second-order anomalous fractal dimension can be written [24] as

$$\frac{d_q}{d_2} = \frac{\beta_q}{(q-1)} = (q-1)^{\nu-1}\,, \tag{8}$$

where

$$\beta_q = \frac{\phi_q}{\phi_2}\,. \tag{9}$$

Alternatively, Eq. (8) can be expressed as

$$\beta_q = (q-1)\frac{d_q}{d_2}\,. \tag{10}$$

It is quite important to note that the scaling behaviour characterized by Eq. (8) is universal and the scaling exponent, ν, is a universal constant. If the measured value of ν is close to the critical value, then the system characteristics can be explained in terms of Ginzburg–Landau formalism and phase transition of the second order may take place.

3.3. Phase transition and Ginzburg–Landau model

As stated earlier, investigation of phase transition is quite common in statistical physics described by the Ginzburg–Landau model. This analogy may be extended to the study of multiparticle production in high energy nuclear collisions. To explain the phenomenon, a clear understanding of fluctuations and their characteristics linked with quark–gluon plasma and first-/second-order phase transition is essentially required [24–27]. Systematic theoretical studies [24, 30] of fluctuations have been carried out to explain the dynamics of phase transition from QGP to hadronic matter. Attempts have also been made during the last several years to investigate theoretically the occurrence of phase transition under extreme conditions of energy density and/or temperature. Several theoretical models have been proposed to explain the occurrence of this novel phase transition; the Ginzburg–Landau model successfully explains the features of this phase transition, which can be tested by comparing the predictions of this model with the corresponding experimental results [59, 60].

As already stated, the Ginzburg–Landau model has been extensively used to describe the scaling behaviour of the scaled factorial moments in the first- [26, 27] and second-order [24, 25] phase transitions. The multiplicity difference correlators [61] and multiplicity distributions in the entire phase space [27, 62, 63] are also nicely explained by this model. According to the Ginzburg–Landau model, scaled factorial moments are defined [25] as

$$F_q(\delta) = \frac{f_q}{f_1^q}\,, \tag{11}$$

where

$$f_q = \frac{1}{z}\int D\phi \left(\int_\delta dz|\phi|^2\right)^q \exp(-F|\phi|) \tag{12}$$

and

$$z = \int D\phi \exp(-F|\phi|)\,. \tag{13}$$

Free energy function, F, is defined as

$$F(\phi) = \int dz \left[a\left|\phi^2\right| + b\left|\phi^4\right| + c\left|\Delta\phi\right|^2\right]\,, \tag{14}$$

where a in the above expression, if turns out to be proportional to $(T - T_{\rm c})$, would represent deviation from the critical point; T and $T_{\rm c}$ represent respectively the temperature and critical temperature. The constants b and c in Eq. (14) have positive values and $|\phi|^2$ defines multiplicity density of the system.

4. Results and discussion

For studying intermittent behaviour in multiparticle production in 14.5 A GeV/c ^{28}Si–AgBr collisions, $\langle F_q\rangle$ are calculated using Eq. (1) for $q = 2$–6. Figure 1 exhibits the variations of $\ln\langle F_q\rangle$ with $-\ln\delta\eta$ for the experimental, FRITIOF and HIJING simulated data; the figure clearly indicates a power-law rise of the scaled factorial moments with decreasing bin size, $\delta\eta$, indicating thereby the presence of intermittency. The solid lines in the figure are obtained by carrying out least squares fits to the data. Intermittency indices, ϕ_q, which are related to the strength of intermittency effect are extracted from the observed linear dependence of $\ln\langle F_q\rangle$ on $-\ln\delta\eta$. It may be of interest to note that ϕ_q are the slopes of the fits to the $\ln\langle F_q\rangle$ and $-\ln\delta\eta$ plots.

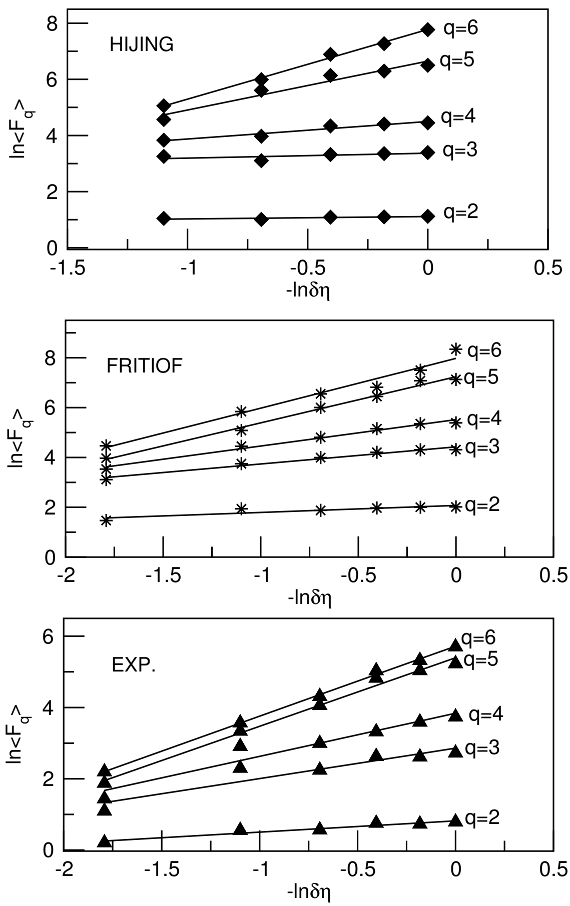

Fig. 1. Variations of $\ln\langle F_q\rangle$ with $-\ln\delta\eta$ for the experimental and simulated data on 14.5 A GeV/c ^{28}Si–AgBr interactions.

Figure 2 displays the variations of ϕ_q with q for the experimental, FRITIOF and HIJING simulated data sets. Statistical errors are also indicated in the plots. Linear increases in the values of ϕ_q with q and decreasing bin size, $\delta\eta$, for all the data sets are discernible. However, the linear increase of ϕ_q with q is very distinct in the case of the experimental data as compared to those for the FRITIOF and HIJING data. Moreover, it is seen in the figure that the values of ϕ_q are relatively higher for the experimental data as compared to those for the FRITIOF and HIJING generated data. It may be stated that intermittency effects are stronger in the case of events with lower multiplicities [16]. A convolution of several independent sources, contributing to particle production in higher multiplicity events, has a tendency to smear out the fluctuations, thus reducing the intermittency effects [47]. However, the trends of the variations of ϕ_q with q are observed to be essentially similar for all the three types of the data sets.

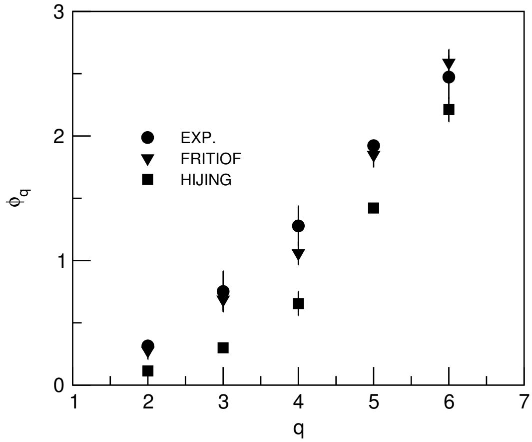

Fig. 2. Variations of ϕ_q with q for the experimental and simulated data on 14.5 $A\,\mathrm{GeV}/c$ ^{28}Si–AgBr interactions.

The intermittency patterns do not only suggest dynamical nature of the fluctuations but also reveal inner fractal structure of the fluctuations, reflected by the values of anomalous fractal dimensions [64], d_q. Table I gives the values of d_q, β_q and ν obtained using Eqs. (4), (7) and (10) for different order of the moments, q, for the experimental as well as simulated data. It can be seen from the table that the values of d_q increase with the order of the moments for all the data sets. Increase in the values of d_q with q reveals the presence of multifractality accompanied by non-thermal phase transition or self-similar cascade mechanism. Nevertheless, the behaviour of anomalous fractal dimensions, d_q, does not favour occurrence of any exotic phenomenon. Furthermore, it is noted from the table that the values of d_q decrease with increasing multiplicity for the FRITIOF and HIJING simulated data for each order of the moments.

A power-law behaviour between $\langle F_q\rangle$ and $\langle F_2\rangle$ is envisaged in the Ginzburg–Landau description of the scaled factorial moments, expressible by Eq. (6). Figure 3 exhibits the variations of $\ln\langle F_q\rangle$ with $\ln\langle F_2\rangle$ for the three types of data on 14.5 $A\,\mathrm{GeV}/c$ ^{28}Si–AgBr interactions. Linear rises in $\ln\langle F_q\rangle$ when plotted against $\ln\langle F_2\rangle$, are discernible for all the three data sets. Further, it can be seen from the figure that $\ln\langle F_q\rangle$ are linearly related to $\ln\langle F_2\rangle$, which confirms the validity of Eq. (6). Almost similar patterns are observed for the FRITIOF and HIJING simulated data. The solid lines in the figure are obtained by carrying out least squares fits to the data.

TABLE I

Values of d_q, β_q and ν for different order of the moments, q, for the experimental and simulated data on 14.5 A GeV/c ^{28}Si–AgBr collisions.

Data type	q	d_q	β_q	ν
Expt.	2	0.315 ±0.042	1.000	1.287 ±0.017
	3	0.376 ±0.082	2.388 ±0.410	
	4	0.426 ±0.053	4.057 ±0.348	
	5	0.481 ±0.010	6.108 ±0.235	
	6	0.495 ±0.034	7.857 ±0.317	
FRITIOF	2	0.277 ±0.071	1.000	1.371 ±0.083
	3	0.344 ±0.045	2.484 ±0.481	
	4	0.353 ±0.031	3.823 ±0.438	
	5	0.462 ±0.025	6.671 ±0.518	
	6	0.517 ±0.021	9.332 ±0.554	
HIJING	2	0.114 ±0.025	1.000	1.824 ±0.233
	3	0.141 ±0.002	2.474 ±0.289	
	4	0.218 ±0.032	5.737 ±0.691	
	5	0.356 ±0.007	12.491 ±0.746	
	6	0.442 ±0.019	19.386 ±0.914	

It may be mentioned that the slope parameters, β_q, in the framework of Ginzburg–Landau model are predicted [19] to satisfy Eq. (7). The values of β_q are calculated using Eq. (10) and are given in Table I. It may be stressed that obviously the value of β_2 shall always be unity. Figure 4 exhibits the variations of β_q with q for the three data sets. It may be of interest to note that the values of β_q increase with q, for all the three types of the data sets. The errors shown in the plots are the standard statistical errors. The values

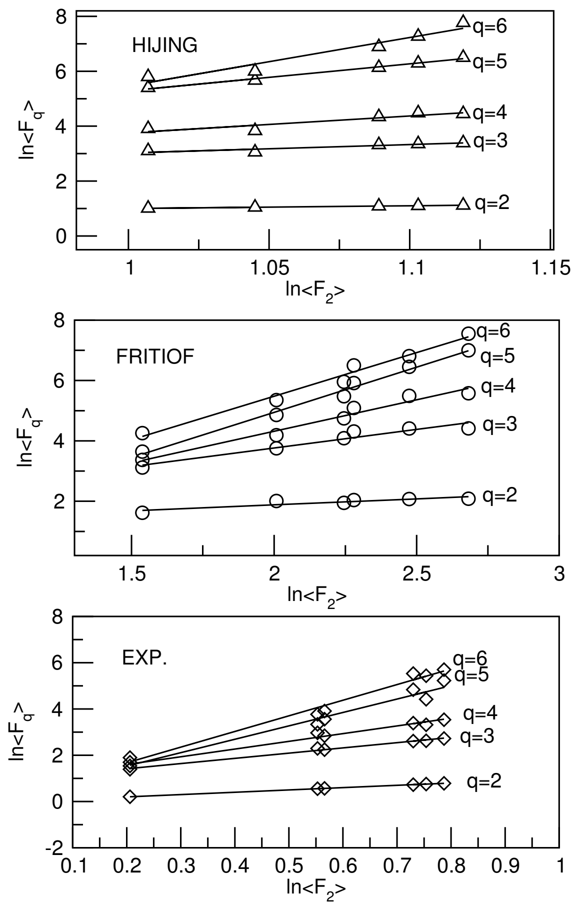

Fig. 3. Dependance of $\ln\langle F_q\rangle$ on $\ln\langle F_2\rangle$ for different order of the moments q, for the experimental and simulated data on 14.5 $A\,\mathrm{GeV}/c$ ^{28}Si–AgBr interactions. Solid lines represent the best linear fits to the data.

of β_q, corresponding to the critical value of the scaling exponent, are also shown by dashed lines in Fig. 4. The values of β_q for the experimental and simulated data are fitted by the scaling function, Eq. (7), as shown by the solid lines to determine the value of the scaling exponent, ν.

It is worth mentioning that the values of ν obtained from Eq. (7) for the experimental as well as FRITIOF data are lower than its critical value. However, for HIJING data ν is significantly higher than its critical value as well as those predicted by AMPT model. It may be due to the fact that local fluctuations are not very well addressed by HIJING model. However, it successfully describes the phenomenon of flow. The values of ν for all the three data sets are listed in Table I. Incidentally, the value of ν, for

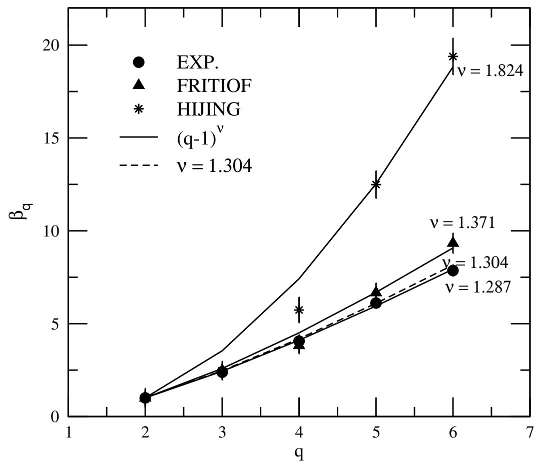

Fig. 4. Variations of β_q with q for the experimental and simulated data for 14.5 $A\,$GeV$/c$ ^{28}Si–AgBr interactions along with the fitted curves.

14.5 $A\,$GeV$/c$ ^{28}Si–AgBr interactions is 1.287 ± 0.017, which is quite closer to its universal value. Since β_q describe the scaling behaviour of F_q with respect to F_2, a larger ν may be a consequence of a smaller value of intermittency index, ϕ_2. Thus, one may say that there is a possibility of occurrence of phase transition of the second order in the interactions considered, which is in accord with the predictions of Ginzburg–Landau theory.

5. Conclusions

As stated earlier, a power-law behaviour between the higher-order and second-order scaled factorial moments is envisaged in the Ginzburg–Landau description, which supports self-similar cascade mechanism in multiparticle production in relativistic hadronic and nuclear collisions. It may be noted that the values of intermittency indices, ϕ_q, are relatively higher for the experimental data in comparison to those for the FRITIOF and HIJING generated events, which thereby reveals that intermittency effects are stronger in the events having lower multiplicities. On the other hand, increase in the values of d_q with q does not favour any exotic phenomenon.

The values of the slopes, β_q, are used to investigate a universal scaling behaviour and to determine the value of scaling exponent. The value of scaling exponent, ν, obtained for the experimental and FRITIOF data beautifully matches its critical value of 1.304. However, for HIJING data, ν is significantly higher than its critical value as well as those predicted by AMPT (A Multi-Phase Transport) model. It may be due to the fact that local fluctuations are not very well addressed by HIJING model. However, it successfully describes the phenomenon of flow. This implies that the Ginzburg–Landau theory is inapplicable to HIJING data and more appropriately it can be inferred that there is no quark–gluon phase transition at low energies. Moreover, if simulation could be performed using HIJING model at RHIC and LHC energies, it may provide conclusive signature regarding the nature of phase transition from deconfined quark–gluon state to the confined hadron state. However, some evidences of phase transition are indicated.

Finally, it may be remarked that a new frontier opened up by the high multiplicity events at LHC energies provide a fertile ground for examining the nature of phase transition, which was certainly not possible at lower energies in the past.

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