GENERALIZED QHD IN BARYONIC MATTER

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We provide here a theoretical frame for Quantum Hadro-Dynamics able to provide a well-behaved approximation scheme that preserves sum rules and general theorems. This scheme is extended to strange particles. A mean field evaluation follows, including strangeness in the ground state, so to reproduce the hypernuclei properties. The theoretical frame is constructed in such a way that the calculation at the Next-To-Leading-Order becomes only a numerical problem.

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1. Introduction

The present paper is devoted to the study of strange matter in the frame of the Quantum Hadro-Dynamics (QHD). Our aim is on the one hand to get some results for the thermodynamical properties of the strange matter at the mean field (MF) level, but also, on the other hand, to provide a well-behaved approximation scheme that preserves Ward identities and general theorems of the theory and embodies the MF as its lowest level.

The theoretical frame we choose is the Boson Loop Expansion (BLE), also known as modified loop expansion. We shall see that BLE has the advantage of remaining finite against a wild increase of the coupling constants.

At the beginning, the BLE was introduced in order to evaluate the nuclear response to an e.m. probe, the formal apparatus being exposed in [1] and the most relevant outcomes in [2]. The same scheme (with the name of "modified loop expansion") was also introduced, in the frame of QHD, in [3]. Later, the BLE has been used in the Λ–hypernuclei decay [4] and then embedded in the QHD frame in [5], where a particular care has been devoted to the renormalization of the theory.

The theoretical frame (BLE) will be shortly resumed in Sec. 3, but in a nutshell it amounts to:

1. describe the observables in terms of Feynman diagrams;
2. shrink to a point each fermionic loop;
3. collect together all the (so modified) diagrams having the same number of (bosonic) loops (in a potential theory a potential line is equivalent to a bosonic one).

In this paper, we shall deal with the 0^{th} (MF) order, extended to embody strange baryons. We adapt the parameters in order to satisfactorily describe the static properties of the symmetric nuclear matter. Then, by imposing the SU(3) symmetry, we generalize the dynamics to include strangeness. We thus may study systems with a sizeable strangeness component and compare (in principle) the outcomes with the binding energies of the known hypernuclei. In this sense, the theory is predictive.

As a matter of fact, however, the dynamics may be improved by going to the Next-to-Leading Order (NTLO), a job that will be pursued in one or more forthcoming papers.

2. The model

2.1. The Hilbert space (baryons and mesons)

We consider a baryons system interacting via (pseudo)scalar and (pseudo) vector mesons.

The baryons are classified according to the SU(3) families, and at MF level only the octet matters. The decuplet enters in the game only at the NTLO (and there its contribution is relevant).

We denote with B the class of baryons of the octet: $B = N, \Lambda, \Xi, \Sigma$, collected according to the SU(2) representations.

Mesons are classified as scalar (denoted with $\varphi_{\text{s}+}$), pseudoscalar ($\varphi_{\text{s}-}$) vector ($A^{\mu}_{\text{v}+}$) and pseudovector ($A^{\mu}_{\text{v}-}$).

In describing a "realistic" dynamics many mesons are required, but at the MF only few of them are active (pseudoscalar and strange mesons are ruled out). The ones accounted for in this paper are listed in Table I together with their masses and properties.

TABLE I

List of the mesons considered in this paper (the second column contains a possible alternative name of the mesons

		M	J^P	I	S
σ		550	0^+	0	0
ω		783	1^-	0	0
δ	$a_0(980)$	980	0^+	1	0
ρ		770	1^-	1	0

Their choice stems from phenomenological reasons [6]. First of all, we remind that the original version of QHD (henceforth referred to as Ur-QHD) required only the σ and ω mesons. The spirit was to reproduce the static properties of the nuclear matter within a simple but fully covariant model using meson masses and coupling constants as free parameters. In this sense, the σ and ω were fictitious particles having no immediate contact with the physical world.

A detailed study of the N–N scattering in the frame of the one boson exchange potential has been carried out by the Bonn group, and Ref. [7] nicely showed that the σ-meson exchange, largely responsible for the N–N attraction in the original version of the Bonn potential [8], could be safely replaced by the so-called box diagrams, namely a two-pion exchange with the simultaneous excitation of one or two nucleons to Δ-isobars.

Nowadays, the existence of a broad resonance with the same quantum numbers of the σ has been firmly established [the $f(500)$] and it could be tempting to identify it with the QHD σ-meson. However, the Bonn group analysis shows that introduction of the box diagrams is quite sufficient to reproduce N–N phase shifts. This means the contribution of the $f(500)$ only leads to marginal corrections and the QHD σ just amounts to parametrizing the box diagrams.

The inclusion of the ω is then consequent, since in the Dirac phenomenology [9] an optical potential like $\Sigma_{\mathrm{S}} + \Sigma_{\mathrm{V}}\gamma_0$ with a large cancellation between the two terms describes quite well the elastic proton scattering. For this reason, we try to group as far as possible our mesons in doublets with a (pseudo)scalar and a (pseudo)vector meson.

We neglect the lowest meson octect (π, K, η, η') because it is not allowed by the MF dynamics. It, however, matters at the NTLO, where the two pions exchange comes into play. Adding the Δ resonance and other technicalities, we recover, in fact, the whole 2-pion exchange that is almost sufficient to give the nuclear binding: a striking example is given in Ref. [10].

Next, other mesons have been utilized previously in literature [6, 11–14] to get a more accurate description of the nuclear matter ground state, accounting for pressure and compressibility as well as for the stiffness of the equation of state (δ, ρ, σ^*, ϕ).

Thus, we should include the scalar octet [15] composed by $a_0(980)$ (also called δ), $f_0(980)$ and $K_0^*(800)$. Again, strange mesons matter only at the NTLO. The mixing between $f_0(980)$ and $f_0(500)$ is neglected. Thus, presently only the δ-exchange matters. Its coupling is left free, while the other couplings are fixed by the SU(3) symmetry.

Finally, we include the vector partners of the above octet, built up by (tentatively) $\rho(770)$, $K^*(892)$ and $\phi(1020)$. Again, only the ρ is active.

2.2. The Lagrangian

The dynamics described above can be written in a compact form by means of the Lagrangian

$$\mathcal{L} = \mathcal{L}_B^0 + \mathcal{L}_{\rm s}^0 + \mathcal{L}_{\rm v}^0 + \mathcal{L}_I - U_\sigma(\sigma) - U_\omega(\omega) + \mathcal{L}_f\,. \tag{1}$$

$\mathcal{L}_B^0$ is the fermion Lagrangian

$$\mathcal{L}_B^{(1/2)0} = \sum_B \bar{B}(i\partial\!\!\!/ - M_B)B\,, \tag{2}$$

the free boson Lagrangian reads

$$\mathcal{L}_{\rm s}^0 = \sum_{\rm s} \tfrac{1}{2}\left[\left(\partial_\mu \varphi_{\rm s}^i\right)\left(\partial^\mu \varphi_{\rm s}^i\right) - \mu_{\rm s}^2\left(\varphi_{\rm s}^i\right)\right] \tag{3}$$

for scalar or pseudoscalar bosons and

$$\mathcal{L}_{\rm v}^0 = \sum_{\rm v}\left(-\tfrac{1}{4}F_{{\rm v}\mu\nu}^i F_{\rm v}^{\mu\nu,i} + \tfrac{1}{2}m_{\rm v}^2 A_{{\rm v}\mu}^i A_{\rm v}^{\mu,i}\right) \tag{4}$$

with

$$F_{\rm v}^{\mu\nu,i} = \partial^\mu A_{\rm v}^{\nu,i} - \partial^\nu A_{\rm v}^{\mu,i}\,, \tag{5}$$

for the (pseudo)vector ones.

Defining

$$\Gamma = \begin{cases} I & \text{for scalar or vector mesons}\,, \\ \gamma_5 & \text{for pseudoscalar or pseudovector mesons :} \end{cases} \tag{6}$$

the interaction term takes the form

$$\mathcal{L}_I = \sum_{{\rm s}BB'} g_{{\rm s}BB'}\varphi_{\rm s}^i \bar{B}\Gamma T_{BB'}^i B' + \sum_{{\rm v}BB'} g_{{\rm v}BB'} A_{{\rm v}\mu}^i \bar{B}\Gamma\gamma^\mu T_{BB'}^i B'\,, \tag{7}$$

where the $T_{BB'}^i$s are some suitable isospin matrices fixed by the Wigner–Eckart theorem. Equation (14) tells us their explicit form.

In the MF approximation, only the diagonal part ($B = B'$) of the $T_{BB'}^i$s (and hence only the $T_3 = 0$ component of the mesonic field) matters. Take now the representation of SU(2) carrying isospin T and let $\mathcal{T}_i^T$ be the generators. We get $T_{BB} \propto \mathcal{T}_3^{T=I(B)}$. Then, we put

$$J^T(B) = \begin{cases} 1 \ \ \forall B & \text{if } T = 0\,, \\ \left(\mathcal{T}_3^{I(B)}\right)_{I_3(B),I_3(B)} & \text{if } T > 0 \end{cases} \tag{8}$$

and redefine the coupling constants as

$$\tilde{g}_{uB} = g_{uBB} J^T(B)\,. \tag{9}$$

Explicitly, $2J^T(B)$ takes the value 0 for an isosinglet (Λ and Ω), $= +1, -1$ for an isodoublet (N,Ξ,Ξ^*), $+2, 0, -2$ for an isotriplet (Σ, Σ^*) [and (not directly relevant for us) $+3, +1, -1, -3$ for $T = 3/2$ (the Δ)].

It is customary in the literature to add to the interaction two self-interaction terms for the σ and the ω in the form of

$$U_\sigma(\sigma) = \frac{1}{3} b M_N (g_{\sigma NN}\sigma)^3 + \frac{1}{4} c (g_{\sigma NN}\sigma)^4\,, \tag{10}$$

$$U_\omega(\omega) = \frac{\mathrm{d}}{4!}\left(g^2_{\omega N}\omega_\mu\omega^\mu\right)^2\,. \tag{11}$$

These terms have a phenomenological origin (they help in controlling the stiffness of the nuclear matter), but, in principle, they should exist *for all* mesons. We consider only σ and ω in order to avoid an uncontrolled increase of free parameters.

The last remark concerns renormalization. No infinities arise at the MF, but in general, as it is, the theory is not renormalizable. However, this disease may be overcome by introducing for each vector meson an associated Stueckelberg ghost, whose effect is that of canceling the $q^\mu q^\nu/m^2$ term in each vector meson propagator [16].

2.3. The SU(3) limit

We classify, as far as possible, the mesons according to the SU(3) representations.

To begin with, we look at the σ meson, whose nature is largely fictitious in the QHD scheme. We do not attribute presently to the σ the role of chiral partner of the π and consider it as a singlet in SU(3), so that each B–B–σ vertex has the same coupling. The same holds for the ω.

Next, we consider a boson octet. To exemplify, we call the mesons π, K and η, with the η' that can be assumed to be a singlet (neglecting his mixing with the η).

The SU(3)-invariant interaction Hamiltonian is known from the early days of the "eightfold way" [17]. The generalized Wigner–Eckart theorem (see, *e.g.*, [18]) tells us that a matrix element $\langle B'|M|B\rangle$, where $|B\rangle$ and $|B'\rangle$ are members of a baryon octet and M belongs to a boson octet can be expressed in terms of only two reduced matrix elements (while in SU(2) only one is required).

We follow de Swart [19] and define the isospin multiplets

$$N=\begin{pmatrix}p\\n\end{pmatrix}\,,\qquad \Xi=\begin{pmatrix}\Xi^0\\\Xi^-\end{pmatrix}\,,\qquad K=\begin{pmatrix}K^+\\K^0\end{pmatrix}\,,\qquad K_c=\begin{pmatrix}\bar{K}^0\\-K^-\end{pmatrix} \quad (12)$$

(isodoublets) and the isovectors $\vec{\pi}$, $\vec{\Sigma}$, $\vec{\Sigma}^\dagger$ and $\vec{\tau}$, the last embodying the Pauli matrices. We also define (with a somehow incoherent notation) the quantities

$$N^\dagger=(\bar{p}\gamma_5,\bar{n}\gamma_5)\,,\qquad \Xi^\dagger=\left(\bar{\Xi}^0\gamma_5,\bar{\Xi}^-\gamma_5\right)\,,\qquad \vec{\Sigma}^\dagger=\left(\bar{\Sigma}^+\gamma_5,\bar{\Sigma}^0\gamma_5,\bar{\Sigma}^-\gamma_5\right)\,. \quad (13)$$

The matrix element of the Hamiltonian interaction can be expressed in terms of only two parameters, g and α

$$\begin{aligned}H_I/g \;=\;& N^\dagger(\vec{\tau}\cdot\vec{\pi})\gamma_5 N-(1-2\alpha)\Xi^\dagger(\vec{\tau}\cdot\vec{\pi})\gamma_5\Xi\\ &+\frac{2}{\sqrt{3}}(1-\alpha)\left(\Lambda^\dagger\gamma_5\vec{\Sigma}+\vec{\Sigma}^\dagger\gamma_5\Lambda\right)\cdot\vec{\pi}-2i\alpha\left(\vec{\Sigma}^\dagger\times\gamma_5\vec{\Sigma}\cdot\vec{\pi}\right)\\ &+\frac{1}{\sqrt{3}}(4\alpha-1)N^\dagger\gamma_5 N\eta-\frac{1}{\sqrt{3}}(2\alpha+1)\Xi^\dagger\gamma_5\Xi\eta\\ &+\frac{2}{\sqrt{3}}(1-\alpha)\Lambda^\dagger\gamma_5\Lambda\eta-\frac{2}{\sqrt{3}}(1-\alpha)\left(\vec{\Sigma}^\dagger\cdot\vec{\Sigma}\right)\eta\\ &-\frac{1}{\sqrt{3}}(2\alpha+1)\left(N^\dagger K\right)\gamma_5\Lambda-\frac{1}{\sqrt{3}}(2\alpha+1)\Lambda^\dagger\gamma_5\left(K^\dagger N\right)\\ &+(1-2\alpha)\left\{\vec{\Sigma}^\dagger\gamma_5\cdot\left(K^\dagger\vec{\tau}N\right)+\left(N^\dagger\vec{\tau}K\right)\cdot\gamma_5\vec{\Sigma}\right\}\\ &-\left\{\vec{\Sigma}^\dagger\gamma_5\cdot\left(K_c^\dagger\vec{\tau}\Xi\right)+\left(\Xi^\dagger\vec{\tau}K_c\right)\cdot\gamma_5\vec{\Sigma}\right\}\,. \end{aligned} \quad (14)$$

The pseudoscalar octet becomes relevant only at the NTLO. The second octet we are interested in is regained with the substitutions

$$\pi\to\delta\,,\qquad K\to K_0^*(800)\,,\qquad \eta\to f_0(980)\,,\qquad \gamma_5\to 1\,.$$

Finally, the third octet comes from

$$\pi\to\rho\,,\qquad K\to K^*(892)\,,\qquad \eta\to\phi\,,\qquad \gamma_5\to\gamma_\mu\,.$$

Needless to say, the above choice hides a huge amount of physics concerning the nature and structure of these mesons. For example, the second octet could have a large tetraquark component.

2.4. The ground state

Finally, we must specify the ground state. We consider a system with N particles in a volume V and we define the total density ρ as

$$\frac{N}{V} = \rho = \frac{k_{\mathrm{F}}^3}{3\pi^2} \tag{15}$$

that introduces a momentum k_{F}, whose meaning is truly that of a Fermi momentum only when one kind of particles is involved.

If the ground state contains many different baryons, we define their partial densities and the corresponding Fermi momenta as

$$\rho_B = \frac{N_B}{V} = \frac{k_{\mathrm{F}_B}}{3\pi^2} \,. \tag{16}$$

Note: in the above, B denotes a single baryon and *not* a single multiplet.

To exemplify, consider the case of symmetric nuclear matter (SNM)

$$N = N_p + N_n = 2N_N$$

(the suffixes p and n denote protons and neutrons, N stays for nucleon). Since this case is particularly relevant, we introduce a specific notation for it, calling p_{F} the usual Fermi momentum of the SNM, *i.e.*, $p_{\mathrm{F}} = k_{\mathrm{F}_N}$, so that

$$\rho_N = \frac{N_N}{V} = \frac{k_{\mathrm{F}_N}^3}{3\pi^2} = \frac{2p_{\mathrm{F}}^3}{3\pi^2} \Longrightarrow k_{\mathrm{F}} = \sqrt[3]{2}\, p_{\mathrm{F}} \,. \tag{17}$$

The ground state $|\Phi_0\rangle$ may contain up to 8 Fermi spheres. Defining $|\Phi_0^B\rangle$ as the one pertaining to the baryon B with Fermi momentum k_{F_B}, we write

$$|\Phi_0\rangle = \bigotimes_B \left|\Phi_0^B\right\rangle \,. \tag{18}$$

Note that, in general, the ground state is *not* isospin invariant (think of a neutron star).

3. The theoretical frame

3.1. The partition function

The theoretical frame we adopt is the Boson Loop Expansion.

Let $\boldsymbol{\Psi}$ be a column vector collecting the baryons of the octet and $\boldsymbol{\Phi}$ the set of bosons. The partition function $Z(\beta)$ can be represented as a path integral by

$$Z(\beta) = \mathrm{Tr}\, e^{-\hat{H}} = \int D\left[\bar{\boldsymbol{\Psi}}, \boldsymbol{\Psi}, \boldsymbol{\Phi}\right] e^{-\int_0^\beta \mathrm{d}\tau \int \mathrm{d}^3\boldsymbol{x}\mathcal{L}_{\mathrm{E}}(\boldsymbol{x},\tau)} \,. \tag{19}$$

Observe that

1. $\mathcal{L}_{\mathrm{E}}$ is the Lagrangian in the Euclidean world: if $\mathcal{H}$ is the Hamiltonian density, then $\mathcal{L}_{\mathrm{E}}(x) = -\mathcal{H}(x)$ provided the dependence of $\mathcal{L}$ upon time derivatives is at most quadratic.

2. Calculations must be performed, in principle, at finite temperature, and hence using suitable boundary conditions for the paths at $\tau = 0$ and $\tau = \beta$. Since, however, we shall eventually work in the limit $\beta \to \infty$ these complications are uneffective and ultimately negligible.

Equation (19) represents the key issue of the formalism, its link with the observables being provided by the next subsection.

3.2. Observables

Once Z is known, the ground state properties follow according to well-known relations.

Binding energy

$$E_0 = \left\langle \hat{H} \right\rangle = \lim_{\beta\to\infty} \left(-\frac{\log Z(\beta)}{\beta} \right) . \tag{20}$$

The binding energy per particle is

$$\frac{\mathrm{BE}}{N} = \frac{E_0}{N} = \frac{\epsilon}{\rho} , \tag{21}$$

where ϵ is the energy density.

Chemical potential: For each baryon B, we have

$$\mu_B = \frac{\partial E}{\partial N_B} = \frac{\partial}{\partial (V\rho_B)} V\epsilon = \frac{\partial \epsilon}{\partial \rho_B} . \tag{22}$$

Pressure:

$$p = -\epsilon - V\frac{\partial \epsilon}{\partial V} = -\epsilon + \sum_B \mu_B \rho_B . \tag{23}$$

Compressibility: According to [20], the compressibility is defined as

$$\chi = -\frac{1}{V}\frac{\partial V}{\partial p} = -\frac{1}{\rho}\left(\frac{\partial p}{\partial \rho}\right)^{-1} . \tag{24}$$

At a variance of [20], we define the compression modulus $\mathcal{K}$ as the inverse

$$\mathcal{K} = \frac{9}{\rho\chi} = -9\frac{\partial p}{\partial \rho} = -\frac{9}{\rho}\sum_{BB'} \frac{\partial^2 \epsilon}{\partial \rho_B \partial \rho_{B'}} \rho_B \rho_{B'} . \tag{25}$$

Our definition extends $\mathcal{K}_\infty$ of [20], namely

$$\mathcal{K}_\infty = -9\rho_0^2 \left[\frac{\partial^2}{\partial\rho^2}\frac{\mathrm{BE}}{N}\right]_{\rho=\rho_0} \tag{26}$$

ρ_0 being the equilibrium density. There is no contrast between the two definitions, since the latter holds only at the equilibrium ($p = 0$) where they exactly coincide.

3.3. The BLE action

The theoretical framework of the BLE has been described *e.g.*, in [2, 21] for the non-relativistic case and in [5] for its application to Ur-QHD. Here, we only need to generalize it to many kinds of fermions what already has been derived in [5]. The extension being trivial, we only quote the final result.

The relevant steps are the following:

1. Evaluate the fermionic integrals in (19): what remains at the exponent is the effective bosonic action.
2. Evaluate the minimum of the effective action. The resulting value of $\boldsymbol{\Phi}$, denoted by $\bar{\boldsymbol{\Phi}}$ is non-trivial, since at least $\langle\sigma\rangle \neq 0$.
3. Get rid of $\bar{\boldsymbol{\Phi}}$ by means of a shift of the integration variable $\boldsymbol{\Phi}$.

Following [5], the effective bosonic action takes the form of

$$A^B[\boldsymbol{\Phi}](\beta) = A^B(\beta)_{\mathrm{mf}} + A^B[\boldsymbol{\Phi}](\beta)_{1-\mathrm{loop}} + A^B[\boldsymbol{\Phi}](\beta)_{\mathrm{int}}\,. \tag{27}$$

The first term, namely

$$A^B(\beta)_{\mathrm{mf}} = \mathrm{Tr}\log\left[S_{\mathrm{H}}^{-1}\right] + \frac{1}{2}\int \bar{\boldsymbol{\Phi}} D^{-1}\bar{\boldsymbol{\Phi}} \tag{28}$$

describes the partition function of an assembly of non-interacting fermions. Here, D is a diagonal matrix built with the propagators on the various mesons, while S_{H} is also a diagonal matrix constructed with the Hatree-dressed fermions propagators, defined by

$$D^{-1}\bar{\boldsymbol{\Phi}} = -\frac{\boldsymbol{S}G}{1+\boldsymbol{S}G\bar{\boldsymbol{\Phi}}} = -\boldsymbol{S}_{\mathrm{H}}G \tag{29}$$

(that also defines the mean field $\bar{\boldsymbol{\Phi}}$).

The second term contains the quadratic part (with respect to the meson field) of the action and reads

$$\begin{aligned} A^B[\boldsymbol{\Phi}](\beta)_{1-\text{loop}} &= \tfrac{1}{2}\int \boldsymbol{\Phi} D^{-1}_{\text{RPA}}\boldsymbol{\Phi} - \tfrac{1}{2}\,\mathrm{Tr}[S_{\mathrm{H}} G\boldsymbol{\Phi}]^2 \\ &\doteq \tfrac{1}{2}\int \boldsymbol{\Phi} D^{-1}\boldsymbol{\Phi} - \tfrac{1}{2}\int \boldsymbol{\Phi}\Pi_{\mathrm{H}}\boldsymbol{\Phi} \\ &\doteq \tfrac{1}{2}\int \boldsymbol{\Phi} D^{-1}_{\text{RPA}}\boldsymbol{\Phi}\,, \end{aligned} \tag{30}$$

where Π_{H} is the Hartree-dressed relativistic Lindhard function $\int \mathrm{d}x\mathrm{d}y\, \boldsymbol{S}(x-y)\boldsymbol{S}(y-x)$ (see [22]) and D_{RPA} is the RPA-dressed meson propagator (see [5] for details).

Finally, the third term in (27), namely

$$A^B[\boldsymbol{\Phi}](\beta)_{\text{int}} = -\,\mathrm{Tr}\sum_{n=3}^{\infty}\frac{(-1)^n}{n}[S_{\mathrm{H}} G\boldsymbol{\Phi}]^n \tag{31}$$

plays the role of an interaction term. Here, $\boldsymbol{G}$ is a 3-indices matrix containing the coupling constants, the indices corresponding to the two kinds of fermion a of kind of the boson. As a shortcut $\bar\psi_i(g_{ijk}\phi_k)\psi_j \equiv \bar{\boldsymbol{\Psi}}\boldsymbol{G}\boldsymbol{\Phi}\boldsymbol{\Psi}$.

Here, each $[SG\boldsymbol{\Phi}]^n \doteq \Pi^{(n)}\boldsymbol{\Phi}^n$ describes a (non-local) n-point vertex built by an n-point closed fermion loop with a factor $G\boldsymbol{\Phi}$ attached to any vertex. For the analytical properties of the $\Pi^{(n)}$s, the reader is referred to [23, 24].

In the above, the meson self-interaction have been neglected.

4. The mean field equations

The mean field equation $\bar{\boldsymbol{\Phi}}$ comes directly from Eq. (29). It has, however, different structures for scalar and vector mesons.

Expliciting the interaction, the motion equation for a (pseudo)scalar boson field reads

$$\left(\Box + \mu_{\mathrm{s}}^2\right)\bar\varphi_{\mathrm{s}} = \sum_{BB'} g_{\mathrm{s}BB'}T^i_{BB'}\langle\Phi_0|\bar B'\Gamma B|\Phi_0\rangle - U'_\sigma(\bar\sigma)\delta_{\mathrm{s}\sigma}\,, \tag{32}$$

where we have re-introduced the meson self-interaction. The index "s" runs over all (pseudo)scalar mesons.

The matrix element $\langle\Phi_0|\bar B'\Gamma B|\Phi_0\rangle$ is manifestly diagonal in the indices B, B'. Furthermore, parity conservation rules out pseudoscalar mesons. Thus, we are left with only scalar mesons having $T=0,1$ and $T_3=0$. Using translation invariance, we get

$$\mu_{\mathrm{s}}^2\bar\varphi^s = \sum_B \tilde g_{\mathrm{s}B}\left\langle\Phi_0|\bar BB|\Phi_0\right\rangle - U'_\sigma(\sigma)\delta_{\mathrm{s}\sigma} \tag{33}$$

[Remember: $\sigma \equiv \bar\varphi_\sigma$].

Note that $\bar{\varphi}^s$ depends upon the whole set of baryonic densities.

Introducing the scalar density

$$\rho_{\rm S}(k,M) = 2\int \frac{{\rm d}^3p}{(2\pi)^3}\frac{M}{\mathcal{E}(p,M)}\theta(k-p) = \frac{M}{2\pi^2}\left\{k\,\mathcal{E} - M^2\log\frac{k+\mathcal{E}}{M}\right\} \quad (34)$$

with $\mathcal{E}(k,M) = \sqrt{k^2+M^2}$, Eq. (33) becomes

$$\mu_{\rm s}^2\bar{\varphi}_{\rm s} = \sum_B \tilde{g}_{{\rm s}B}\rho_S - U'_\sigma(\sigma)\delta_{{\rm s}\sigma}\,. \quad (35)$$

To exemplify, take the case of the δ-meson. Applying our equation to the octet, we find

$$\mu_\delta^2\delta^0_{T=1} = \tfrac{1}{2}g_{\delta NN}\left[\rho_{\rm sp}-\rho_{\rm sn}\right] + g_{\delta\Sigma\Sigma}\left[\rho_{{\rm s}\Sigma^+}-\rho_{{\rm s}\Sigma^-}\right] + \tfrac{1}{2}g_{\delta\Xi\Xi}\left[\rho_{{\rm s}\Xi^0}-\rho_{{\rm s}\Xi^-}\right]. \quad (36)$$

The above shows explicitly that the δ may have a non-vanishing MF only if the isospin symmetry is broken by the structure of the ground state (for instance in the case of asymmetric nuclear matter, where $\rho_{\rm sp}\neq\rho_{\rm sn}$).

The motion equation for a vector boson is

$$\left(\Box + m_{\rm v}^2\right)\bar{A}^i_{{\rm v}\mu} = \sum_{BB'} g_{{\rm s}BB'}\bar{B}\gamma_\mu T^i_{BB'}B' - \frac{d}{3!}g^4_{\omega N}\bar{A}^i_{{\rm v}\mu}\bar{A}^j_{{\rm v}\lambda}\bar{A}^{j\lambda}_{\rm v}\delta_{{\rm v}\omega}\delta_{i0} \quad (37)$$

with the constraint

$$\partial^\mu\bar{A}^i_{{\rm v}\mu} = 0 \quad (38)$$

(Proca equations). The indices i, j denote the third component of the isospin, while "v" runs over the kind of vector mesons. As for the scalar mesons, the matrix element $\langle\Phi_0|\bar{B}\Gamma\gamma^\mu T^i_{BB'}B'|\Phi_0\rangle$ is meaningful only for $B=B'$. This, in turn, rules out all isospin components $\neq 0$. Further, in the infinite nuclear matter limit, we observe that $\langle\Phi_0|\bar{B}\Gamma\vec{\gamma}T^i_{BB'}B'|\Phi_0\rangle = 0$ by rotational invariance. Thus, the above equations for the 3-vector component of the meson become trivial and are solved by $\vec{A}^i_{\rm v} = 0$; the only non-trivial one is consequently $\bar{A}^{T_3=0}_{{\rm v},\mu=0}$.

So the relevant quantity in the r.h.s. of Eq. (37) is $\langle\Phi_0|\bar{B}\Gamma\gamma^0 T^i_{BB}B|\Phi_0\rangle$. Consequently, the meson cannot change the strangeness. Furthermore, the matrix element vanishes for pseudovector bosons. Thus, we are left with vector isoscalar or isovector mesons only.

Defining, in parallel with the scalar case,

$$\rho_{\rm V} \doteq \langle\Phi_0|B\gamma^0 B|\Phi_0\rangle = \frac{k^3_{{\rm F}_B}}{3\pi^2} \equiv \rho_B\,, \quad (39)$$

the equation of motion for the vector mesons reads

$$m_{\mathrm{v}}^2 A_{\mathrm{v}}^{\mu=0,i=0} = \sum_B \tilde{g}_{\mathrm{v}B}\rho_B - U'_\omega(\omega)\delta_{\mathrm{v}\omega} \tag{40}$$

restricted to non-strange bosons.

Observe finally, that at variance of $\rho_{\mathrm{s}B}$, $\rho_{\mathrm{v}B}$ is independent of the masses. The consequence is that the vector MF is kept out of the self-consistency game and of the definition of the baryon effective mass, as observed long time ago by the pioneering work of Lee and Wick [25].

To derive the equation of motion for a Dirac field, we take the action at the former level, before fermion integration (see Eq. (19)). This procedure is somehow incoherent, but on the other hand, a fully consistent treatment is unnecessarily cumbersome.

Explicitly, we find

$$\begin{aligned}(i\,\partial\!\!\!/ - M_B)\,B \;&=\; \sum_{\mathrm{s}B'} g_{\mathrm{s}BB'}\bar{\varphi}_{\mathrm{s}}^{\tau_3=0} T^0_{BB'} B' \\ &\quad + \sum_{\mathrm{v}B'} g_{\mathrm{v}BB'}\bar{A}_{\mathrm{v}}^{\mu=0,\tau_3=0}\gamma^0 T^0_{BB'} B'\,,\end{aligned} \tag{41}$$

where the meson involved can neither be strange nor carry odd parity.

Note that in (41), the r.h.s. may couple different baryons, provided they have the same quantum numbers. Luckily, this is not the case for the baryon octet, so that the system decouples. We are not able, however, to disentangle the nucleon dynamics from the one of a Roper resonance. This occurrence deserves a careful separated study.

Presently, however, the non-diagonal terms in (41) vanish and we are left with

$$(i\,\partial\!\!\!/ - M_B)\,B = -\sum_{\mathrm{s}} \tilde{g}_{\mathrm{s}B}\bar{\varphi}_{\mathrm{s}}^{\tau_3=0} B + \sum_{\mathrm{v}} \tilde{g}_{\mathrm{v}B}\bar{A}_{\mathrm{v}}^{\mu=0,\tau_3=0}\gamma^0 B\,. \tag{42}$$

It is then natural to define an effective mass

$$M_B^* = M_B - \sum_{\mathrm{s}} \tilde{g}_{\mathrm{s}B}\bar{\varphi}_{\mathrm{s}}^0 \tag{43}$$

so that

$$(i\,\partial\!\!\!/ - M_B^*)\,B = \sum_{\mathrm{v}} \tilde{g}_{\mathrm{v}B}\bar{A}_{\mathrm{v}}^{\mu=0,\tau_3=0}\gamma^0 B\,. \tag{44}$$

Note that the effective mass depends upon the full set of baryon indexes. This is because the evaluation of the effective mass entails an average over the ground state, which, in general, is *not* isospin-invariant.

We also notice that *any* effective mass depends upon the whole set of Fermi momenta $\{k_{\mathrm{F}_B}\}$.

The presence of the effective masses implies a self-consistency requirement in Eq. (35), that now reads

$$\mu_{\mathrm{s}}^2 \bar{\varphi}_{\mathrm{s}}^0 = \sum_B \tilde{g}_{\mathrm{s}B} \rho_{\mathrm{S}}(M_B^*, k_{\mathrm{F}_B}) - U'_\sigma(\sigma)\delta_{\mathrm{s}\sigma} \tag{45}$$

with M_B^* that depends, in turn, upon the MF values of the scalar bosons through Eq. (43). Putting together Eqs. (35), (43) and (45), we get the self-consistent equation

$$\mu_{\mathrm{s}}^2 \bar{\varphi}_{\mathrm{s}}^0 + U'\left(\bar{\varphi}_{\mathrm{s}}^0\right)\delta_{\mathrm{s}\sigma} = \sum_B \tilde{g}_{\mathrm{s}B} \rho_{\mathrm{S}}\left(M_B - \sum_u \tilde{g}_{uB} \bar{\varphi}_u^0, k_{\mathrm{F}_B}\right) \tag{46}$$

that can be solved numerically using the standard numerical methods based on the fixed point theorem.

5. Mean field observables

Now, we need the expressions of the observables at the MF level. The formal derivation is sometimes cumbersome but essentially straightforward; thus we simply list the results. Actually, this section is the couterpart of Subsec. 3.2.

Binding energy: The binding energy per particle reads

$$\begin{aligned}\frac{\mathrm{BE}}{N} &= \frac{1}{\rho}\sum_B \rho_B E_B^* \\ &\quad + \frac{1}{\rho}\left\{\frac{1}{2}\sum_{\mathrm{s}} \mu_{\mathrm{s}}^2 (\bar{\varphi}_{\mathrm{s}})^2 + \frac{1}{2}\sum_{\mathrm{v}} m_{\mathrm{v}}^2 \left(\bar{A}_{\mathrm{v}}^0\right)^2 + U_\sigma(\sigma) + U_\omega(\omega)\right\},\end{aligned} \tag{47}$$

where E_B^* is the energy of a Fermi sphere for baryons with effective mass M_B^*

$$E_B^* = \frac{d_B}{\rho_B}\int \frac{\mathrm{d}^3 k}{(2\pi)^3}\, \mathcal{E}\left(k, M_B^*\right)\theta(k_{\mathrm{F}_B} - k)\,. \tag{48}$$

Remark: we have not subtracted here the nucleon mass for ease of notation. The quantity customarily referred to as binding energy is $\mathrm{BE}/N - M$.

Chemical potential: The chemical potential for the fermion B reads

$$\mu_B = \mathcal{E}^*_B + \sum_{\rm v} \tilde{g}_{{\rm v}B} \frac{\sum_C \tilde{g}_{{\rm v}C}\rho_C}{m^2_{\rm v} + U''_\omega(\omega)\delta_{{\rm v}\omega}}\,, \tag{49}$$

where we have introduced the shortcut $\mathcal{E}^*_B = \mathcal{E}(k_{{\rm F}_B}, M^*_B)$.

Pressure:

$$\begin{aligned} p \;=\; & \frac{1}{4}\sum_B \rho_B \mathcal{E}^*_B - \frac{1}{4}\sum_B \rho_{{\rm S}B} M^*_B - \sum_{\rm s}\left[\frac{1}{2}\mu^2_{\rm s}\varphi^2_{\rm s} + U_\sigma(\sigma)\delta_{{\rm s}\sigma}\right] \\ & - \sum_{\rm v}\left[\frac{1}{2}m^2_{\rm v}\left(\bar{A}^0_{\rm v}\right)^2 + U_\omega(\omega)\delta_{{\rm v}\omega}\right] + \frac{\left(\sum_B \tilde{g}_{{\rm s}B}\rho_{{\rm S}B}\right)^2}{m^2_{\rm v} + U''_\omega(\omega)\delta_{{\rm v}\omega}}\,. \end{aligned} \tag{50}$$

Compressibility: The expression for the compressibility is more involved:

$$\begin{aligned} \mathcal{K} \;=\; & -\frac{\rm d}{6}\sum_B \frac{\rho_B k^2_{{\rm F}_B}}{\mathcal{E}^*_B} - \sum_{\rm v}\frac{\left(\tilde{g}_{{\rm v}B}\rho_B\right)^2}{m^2_\omega + U'_\omega(\omega)\delta_{\omega{\rm v}}} + \frac{\rm d}{6}\sum_B \frac{\tilde{g}_{\sigma B}\rho_B M^*_B}{\mathcal{E}^*_B} D\sigma \\ & + \frac{U^{(3)}_\omega(\omega)\left(\sum_B \tilde{g}_{\omega B}\rho_B\right)^2}{3\left(m^2_\omega + U_\omega\right)^2} D\omega\,, \end{aligned} \tag{51}$$

$$D\sigma \;=\; \sum_B k_{{\rm F}_B}\frac{\partial\sigma}{\partial k_{{\rm F}_B}} = \frac{\frac{3{\rm d}}{2}\sum_B \frac{\tilde{g}_{\sigma B}\rho_B M^*_B}{\mathcal{E}^*_B}}{\mu^2_\sigma - \frac{3{\rm d}}{2}\sum_B \frac{\tilde{g}^2_{\sigma B}\rho_B}{\mathcal{E}^*_B} + 3{\rm d}\sum_B \frac{\tilde{g}^2_{\sigma B}\rho_B}{M^*_B} + U''_\sigma(\sigma)}\,, \tag{52}$$

$$D\omega \;=\; \sum_B k_{{\rm F}_B}\frac{\partial\omega}{\partial k_{{\rm F}_B}} = \frac{3\sum_B \tilde{g}_{\sigma B}\rho_B M^*_B}{m^2_\omega + U''_\omega(\omega)}\,. \tag{53}$$

6. The Ur-QHD

6.1. The equations of the Ur-QHD

In the initial form, QHD contained only σ, ω and nucleons. As a first step, we consider an oversimplified model neglecting meson self-couplings and only dealing with symmetric nuclear matter (SNM).

Observe that the nucleon dynamics at the MF level is only affected by the tadpoles (Hartree contributions) associated to the σ and ω, that have the structure $g^2_{\sigma NN}/\mu^2_\sigma \rho_{\rm S}$ and $g^2_{\omega NN}/m^2_\omega \rho$. This suggests to introduce, consistently with the past literature, the notation

$$f_i = \left(\frac{\tilde{g}_{iN}}{\mu_i}\right)^2 = \frac{\tilde{f}_i}{M^2_N}\,, \tag{54}$$

where the index "i" runs over all mesons, μ_i being the corresponding mass.

In SNM, it is natural to replace $k_{\rm F}$ with the true Fermi momentum of the system defined in (17), namely $p_{\rm F} = k_{\rm F}/\sqrt[3]{2}$. Then the self-consistent equation reads

$$\varsigma(p_{\rm F}) = 2f_\sigma \rho_{\rm S}(M^*(p_{\rm F}), p_{\rm F}) - \frac{\sqrt{f_\sigma}}{\mu_\sigma} U'_\sigma \left(\frac{\varsigma(p_{\rm F})}{\sqrt{f_\sigma}\mu_\sigma} \right) , \tag{55}$$

where the rescaled field is $\varsigma(p_{\rm F}) = m_\sigma \sigma(p_{\rm F})$.

The effective masses for the nucleons read

$$M^*(p_{\rm F}) = M_N - \sqrt{f_\sigma}\varsigma(p_{\rm F}) \,. \tag{56}$$

The $0^{\rm th}$ component of the ω meson is

$$A^0(p_{\rm F}) = 2f_\omega \rho - \frac{\sqrt{f_\omega}}{m_\omega} U'_\omega \left(\frac{A^0(p_{\rm F})}{\sqrt{f_\omega} m_\omega} \right) \tag{57}$$

being $A^0(p_{\rm F}) = m_\omega \omega^0(p_{\rm F})$.

6.2. Lee and Wick model

The model studied by Lee and Wick is even simpler and contains only the σ-meson with $U_\sigma = 0$. The self-consistent equation

$$\frac{p_{\rm F}\mathcal{E}^*}{M^*} - \log\left| \frac{p_{\rm F} + \mathcal{E}^*}{M^*} \right| = \frac{\pi^2}{f_\sigma} \frac{M - M^*}{(M^*)^3} \tag{58}$$

determines M^*. Note that Eq. (58) in the limit $p_{\rm F} \to \infty$ entails $M^* \to 0$, *i.e.*, chiral symmetry may be restored. The binding energy takes the form of

$$\left(\frac{\rm BE}{N} \right)_{\rm LW} = \frac{(M - M^*)(2M - M^*)}{4f_\sigma \rho} + \frac{3}{4} \mathcal{E}^* \tag{59}$$

and the pressure is

$$p_{\rm LW} = \frac{1}{4}\rho\,\mathcal{E}^* - \frac{(M - M^*)(2M - M^*)}{4f_\sigma} \,. \tag{60}$$

The model at this stage has only one parameter, namely f_σ. To exemplify, in Fig. 1 we have reported the binding energy for different coupling constants. This plot clearly shows the inadequacy of the model, because either (low coupling limit) no saturation point occurs or (mild values of g_σ) the system is over-bound at exceedingly high density.

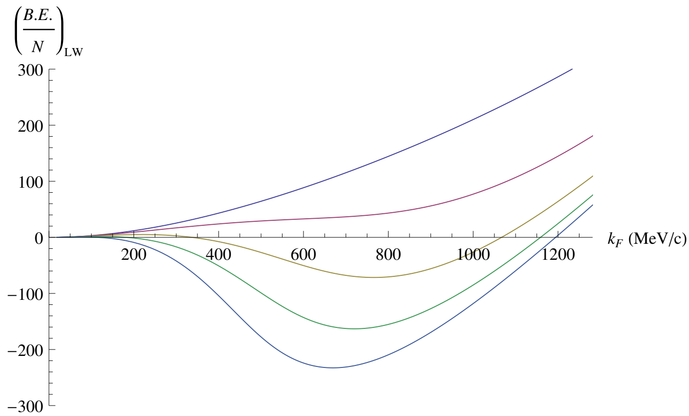

Fig. 1. The binding energy of the Lee–Wick for different coupling constants. We have put $g_\sigma = 1, 2, 3, 4, 5$ (in descending order) with $m_\mu = 550$ MeV. The nucleon mass is subtracted.

6.3. Corrections to the Lee–Wick model

This outcome was expected indeed since the Dirac phenomenology imposes the introduction of the ω. According to the content of Subsec. 6.1, the biding energy and pressure will be corrected as follows:

$$\left(\frac{\mathrm{BE}}{N}\right)_{\sigma\omega} = \left(\frac{\mathrm{BE}}{N}\right)_{\mathrm{LW}} + \frac{f_\omega \rho}{2}\,, \tag{61}$$

$$p_{\sigma\omega} = p_{\mathrm{LW}} + \frac{f_\omega \rho^2}{2}\,, \tag{62}$$

$$\mu = \mathcal{E}^* + f_\omega \rho\,. \tag{63}$$

Since the model has now two parameters, we can impose the two conditions that fix the position of the stability point, namely a binding of -16 MeV at $p_{\mathrm{F}} = 1.36\ \mathrm{fm}^{-1}$. The binding energy is shown in Fig. 2. All the remaining properties of the nuclear matter are then fixed. For instance, the effective mass at the saturation point is $M^* = 0.55\,M$, a value considerably lower than all the reasonable available evaluations and the compression modulus is $\mathcal{K}_\infty = 543$ MeV, twice as much as the expected result.

The minimal correction added to the Lee–Wick model is still unsatisfactory. Thus, as usual in the literature, we account for the quadrilinear self-interacting terms:

$$U_\sigma(x) = \tfrac{1}{4} c(\tilde{g}_{\sigma N} x)^4\,, \tag{64}$$

$$U_\omega(x) = 0\,. \tag{65}$$

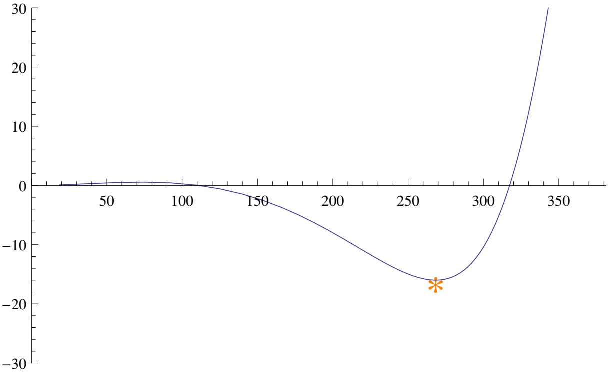

Fig. 2. The binding energy for the two-parameters model.

Using these definition, we write the observables as

$$\frac{\mathrm{BE}}{N} = \left(\frac{\mathrm{BE}}{N}\right)_{\mathrm{LW}} + \frac{f_\omega \rho}{2} + \frac{cM(M-M^*)^3}{4\rho}\,, \tag{66}$$

$$p = p_{\mathrm{LW}} + \frac{f_\omega \rho^2}{2} - \frac{1}{4} cM(M-M^*)^3\,, \tag{67}$$

$$\mu = \mathcal{E}^* + f_\omega \rho\,. \tag{68}$$

The expression of $\mathcal{K}$ is more cumbersome, although elementary.

To fix the parameters, we impose the equilibrium conditions so that we are faced with Eqs. (66) and (67) plus the self-consistency equation that, after suitable manipulations, takes the form

$$\rho_{\mathrm{S}} = \frac{(M-M^*)\left(cf_\sigma (M-M^*)^2+1\right)}{f_\sigma} \tag{69}$$

(all equations are taken at the equilibrium density).

Equation (69) is then invoked to determine the effective mass. So we are faced with four unknown, namely f_σ, f_ω, c and M^*. It is convenient instead to leave M^* free: the other parameters are now fixed by the relations

$$f_\sigma = -\frac{(M-M^*)^2}{\rho(\rho_{\mathrm{S}} M/\rho - 2M + 2V_0 + \mathcal{E})}\,, \tag{70}$$

$$f_\omega = -\frac{-M+V_0+\mathcal{E}}{\rho}\,, \tag{71}$$

$$c = \frac{\rho(\rho_{\mathrm{s}}/\rho(2M-M^*) - 2M + 2V_0 + \mathcal{E})}{(M-M^*)^4}\,. \tag{72}$$

Thus, if we leave the effective mass at the equilibrium as a free parameter, the model is analytically solvable.

In Fig. 3, the parameters are plotted as functions of M^*, together with the compressibility. In f_σ a striking singularity appears. Nevertheless, it does not affect the observables, since in all of them the limit $f_\sigma \to \infty$ is regular.

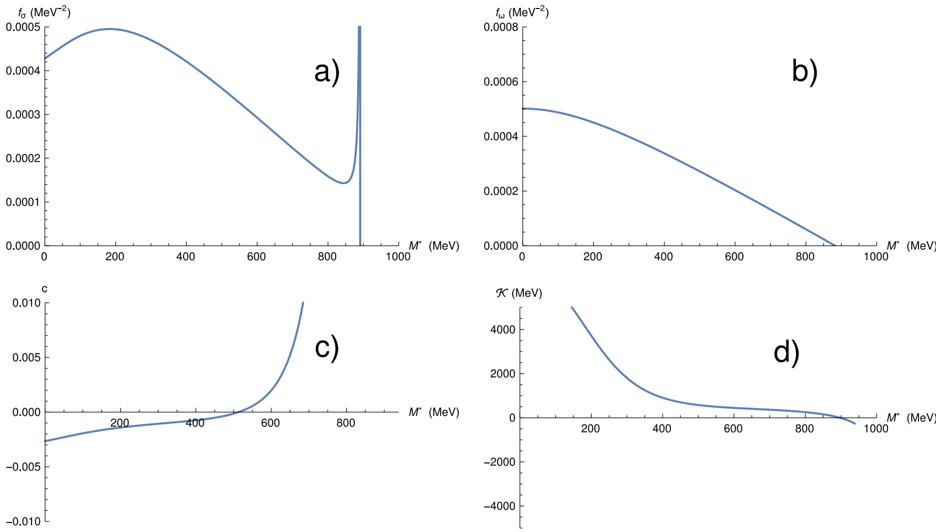

Fig. 3. Panel (a): the parameter f_σ as a function of M^*; Panel (b): the parameter f_ω as a function of M^*; Panel (c): the parameter c as a function of M^*; Panel (d): the compressibility as a function of M^*.

Also, the parameter c changes the sign for $M^* = 513.14$ MeV. We remind that the region $c < 0$ corresponds to an unstable system (the energy is not bounded from below). Thus, this model provides the constraint $M^* > 513.14$ MeV.

The other interesting quantity, for which we have some experimental indication, is the compression modulus (panel (d) in Fig. 3). This plot deserves some attention, as it displays a liquid-vapor phase transition when $\mathcal{K} \to 0$, *i.e.*, at $M^* = 896.92$ MeV.

In Fig. 4, we plot the binding energy, still keeping the equilibrium point fixed, for different effective masses. The results of the right panel are not physically sound, as they refer to unrealistically high values of the effective masses, but are nevertheless amusing since they display the behavior of the binding energy in proximity of the critical point of the phase transition. To complete this analysis, we list the parameters corresponding to the more likely values of the effective mass, together with the corresponding compressibility at the equilibrium in Table II. The masses of the mesons are $\mu_\sigma = 550$ MeV and $m_\omega = 783$ MeV. The trend is clearly an overestimate of the compression modulus or of the effective mass. Further, with increasing M^*, the repulsion is provided more by the self-interaction term and less by the ω-meson exchange.

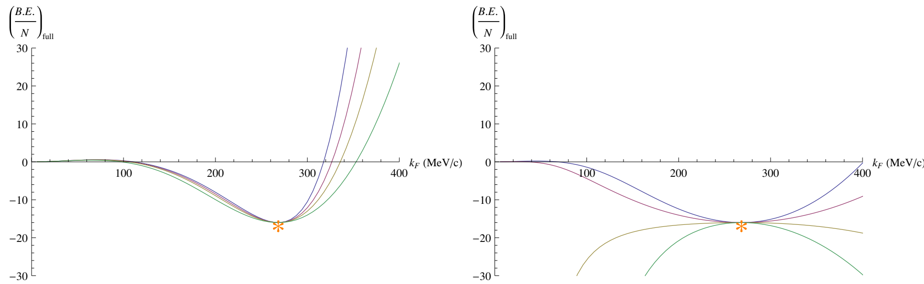

Fig. 4. The binding energy per particle as a function of the effective mass. Left panel: $M^*/M = 0.55, 0.65, 0.75, 0.85$ in descending order; right panel: $M^*/M = 0.92, 0.94, 0.96, 0.98$ in descending order.

TABLE II

Parameters and compressibility modulus at the equilibrium for different effective masses.

M^*/M	f_σ	f_ω	g_σ	g_ω	c	$\mathcal{K}$
0.65	2.856×10^{-4}	1.962×10^{-4}	9.29	10.97	2.42×10^{-3}	547.9
0.75	2.206×10^{-4}	1.297×10^{-4}	8.17	8.92	1.46×10^{-2}	438.1
0.85	1.6×10^{-4}	6.2×10^{-5}	6.96	6.16	0.137	257.6

7. Numerical results: full model

In this section, the dynamics is enlarged to include the isovector doublet δ (scalar) and ρ (vector) so to ensure cancellation just as in the case of the isoscalar doublet (σ,ω)

7.1. Asymmetric nuclear matter

The first step is the study of the SNM, and in this context the Ur-QHD model acts as a starting point, to which we add the δ and the ρ.

However, as shown in Eq. (36), the driving term of the equation of motion for the δ is proportional to $\rho_p - \rho_n$ plus terms carrying strangeness. Thus, in the case of SNM the equation for the δ becomes homogeneous, and the corresponding field vanishes. The ρ–meson does the same, since the coefficients in (36) stem only from isospin. Thus, no effect is expected at this level.

Of course, the cancellation is not complete in the ANM case. We choose the coupling constant of the δ according to [6] (up to some cumbersome transformations of the units), *i.e.*, $g_{\delta NN} = 7.85$, while the coupling of the ρ is $g_{\rho NN} = 7.02$. We assume further $\rho_n = 0.6$ and $\rho_p = 0.4$ (that is very near to the asymmetry of Pb^{208}) and its opposite $\rho_n = 0.4$ and $\rho_p = 0.6$. A plot of the BE/N is meaningless, because the curves turn out to be almost superimposed. A numerical calculation provides the following bindings:

$$\begin{cases} -14.4472 & \text{for } \rho_n = 0.6\,, \quad \rho_p = 0.4\,, \\ -15.3699 & \text{for } \rho_n = 0.5\,, \quad \rho_p = 0.5\,, \\ -14.6896 & \text{for } \rho_n = 0.4\,, \quad \rho_p = 0.6 \end{cases}$$

that is, a small repulsive contribution is added. A comment is in order: the expected behavior like $(\rho_n - \rho_p)^2$ is not reproduced simply because our calculations have been performed with the true masses of n and p as in PDG, so breaking the expected symmetry.

7.2. Strange nuclear matter

Next, we consider a nuclear system with a strangeness component. We assume, to exemplify, a nucleus with 5 protons (boron), 4 neutrons and one Λ. This corresponds to $\rho_p = 0.5$, $\rho_n = 0.4$ and $\rho_\Lambda = 0.1$. We observe first of all that the vertices $\delta\Lambda\Lambda$ and $\rho\Lambda\Lambda$ are not permitted by isospin conservation. Hence, only the Ur-QHD dynamics matters.

We thus repeat the previous calculations, getting the results of Fig. 5 — dashed line (red). We stress once more that the dynamics is too poor to draw reliable conclusions, but it is worth noticing that the order-of-magnitude of

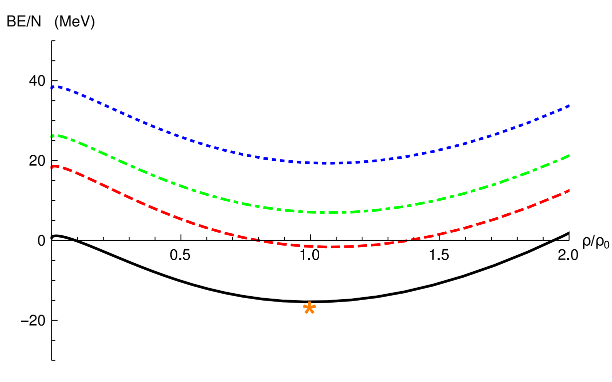

Fig. 5. BE/N for SNM and (Σ, Ξ, Λ)-nuclei. Solid line (black): SNM in the $\sigma\omega$ model, dotted line (blue): nuclear matter with a 10% of Ξ, dot-dashed line (green): nuclear matter with a 10% of Σ, dashed line (red): nuclear matter with a 10% of Λ.

the jump from SNM to a Λ nucleus is about $(m_\Lambda - m_N)/N$ (in our case less or about 17 MeV) and that the model predicts a weakly bound state around the equilibrium density of SNM.

Next, we come to the Σ-nuclei. Again, δ and ρ are decoupled from $\Sigma_0^\dagger \Sigma_0$. Thus, along the same scheme as for the Λ we get, for $\rho_{\Sigma_0} = 0.1$, the dot-dashed line (green) of Fig. 5.

Finally, we consider a Ξ-nucleus. The ground state energy is shown by the dotted line (blue) of Fig. 5.

8. Outlook

The present paper provided us with the following outcomes:

1. A detailed theoretical frame for the construction of well-behaved approximation, *i.e.*, that may be regarded as power expansions of one or more parameters. The consequence is that, under suitable conditions (almost always fulfilled), general theorems and sum rules, usually proved in the general case, hold order-by-order in the power expansion. Of course, the expansion parameter may be set to 1 after the expansion, as is the case of the loop expansion, where the parameter is $\hbar$. In the case of BLE, we multiply by α the term

$$\mathrm{Tr} \sum_{n=3}^{\infty} \frac{(-1)^n}{n} [SG\boldsymbol{\Phi}]^n$$

   of Eq. (31) and put $\alpha \to 1$ after having carried out the power expansion. Note that the sum starts from $n = 3$ because we want to single out and embody in the leading order all the terms quadratic in the meson field.

2. The MF is described by means of the σ, ω, δ and ρ exchange. A reasonable description of the nuclear static properties is obtained, without introducing instabilities in the system (the σ self-interaction has a stable minimum for $\sigma = 0$, while in other parametrizations the above point is unstable and a proper handling of the model leads to a collapse). Further, we have shown that once the stability point of the BE is fixed, then the model can be described analytically as a function of the effective mass. The description of the strange matter is also reasonable, although the poor dynamics introduced so far does not allow a reliable description of its phase diagram (in practice, we should introduce at least the K meson, that at the MF is ineffective).

As already explained in the introduction, these achievements are the starting points for new calculations, namely the next-to-leading order in BLE.

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