ON THE CONTRIBUTION OF THE DOUBLE DRELL–YAN PROCESS TO WW AND ZZ PRODUCTION AT THE LHC* **

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(Received January 20, 2015; revised version November 2, 2015)

In this paper, we investigate consequences of an assumption that the discrepancy of the predicted and observed W^+W^- production cross sections at the LHC is caused by the missing contribution of the double Drell–Yan process (DDYP). Using our simple model of DDYP [*Acta Phys. Pol. B* **45**, 71 (2014)], we show that inclusion of this production mechanism leads to a satisfactory, parameter-free description of the two-lepton mass distribution for 0-jet W^+W^- events and the four-lepton mass distribution for ZZ events. In such a scenario, the Higgs-boson contribution is no longer necessary to describe the data. An experimental programme to prove or falsify such an assumption is proposed.

DOI:10.5506/APhysPolB.47.1045

1. Introduction

The main motivation for our study presented in this paper was "the WW anomaly" at the LHC, *i.e.* deviations of the total cross sections for resonant W^+W^- production measured by the ATLAS [1, 2] and CMS [3, 4] experiments from their theoretical predictions, as shown in Table I. The cross

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** The work is partly supported by the program of the French–Polish co-operation between IN2P3 and COPIN No. 05-116, and by the Polish National Centre of Science grant No. DEC-2011/03/B/ST2/00220.

sections measured by ATLAS at the collision energies of 7 TeV and 8 TeV are higher respectively by factors of 1.16 and 1.22 than the theoretical predictions of the NLO QCD calculations obtained from the MCFM program [5]. The corresponding factors for the CMS measurements are 1.11 and 1.22.

TABLE I

Values of the total cross section for resonant W^+W^- production at the LHC measured by the ATLAS and CMS experiments and predicted by the theoretical NLO QCD calculations from the MCFM program.

	ATLAS	CMS	Theory $\sigma_{W^+W^-}$ [pb]	
$\sqrt{s}$	$\sigma_{W^+W^-}$ [pb]	$\sigma_{W^+W^-}$ [pb]	ATLAS	CMS
7 TeV	$51.9^{+2.0+3.9+2.0}_{-2.0-3.9-2.0}$	$52.4^{+2.0+4.5+1.2}_{-2.0-4.5-1.2}$	$44.7^{+2.0}_{-1.9}$	47.0 ± 2.0
8 TeV	$71.4^{+1.2+5.0+2.2}_{-1.2-4.4-2.1}$	$69.9^{+2.8+5.6+3.1}_{-2.8-5.6-3.1}$	$58.7^{+3.0}_{-2.7}$	$57.3^{+2.3}_{-1.6}$

Although each of these deviations does not exceed 2σ, the fact that they are present in four measurements and all exhibit excess of data with respect to theory may indicate that there exist some extra processes contributing to the measured cross sections that have not been taken into account in the theoretical predictions. Such processes may have an important impact on the significance of the Higgs-boson signals at the LHC.

Recently, the NNLO calculations for the W^+W^- production have been published [6]. They predict increase of the theoretical cross sections at these energies by about 9% with respect to the NLO results, so they get closer to the experimental measurements but do not remove the differences completely. These calculations have been done, however, for the total cross sections only and not for distributions considered by the LHC collaborations in the Higgs searches. The question if the increase of the cross section is in the Higgs-signal region or in the Higgs-monitoring region, or both, which is critical to our analysis, remains thus open. Therefore, we shall not use them in this paper. We can do this in the future when the ATLAS and CMS collaborations apply them in their data analyses.

Since the observed W^+W^- cross section discrepancies at 8 TeV are by a factor of ~ 3 higher than the expected contribution coming from the Standard Model (SM) Higgs-bosons decays, special measurement procedures were used by the ATLAS and CMS collaborations, see *e.g.* Refs. [7, 8], to increase their sensitivities to the Higgs signal. The normalisation of the predicted background was rescaled to fit the data in the monitoring region, where the Higgs contribution is negligible. Then, the rescaled background was used in the Higgs-search region to determine the strength of the Higgs signal.

In the case of the ATLAS experiment, the normalisation of the theoretical predictions for the background was multiplied by a factor of 1.22 for the 8 TeV analysis of the 0-jet $e\mu$ events [7], where the sensitivity to the Higgs boson decays is the highest. This value is compatible with the discrepancy of the measured and predicted total cross sections for the resonant W^+W^- production at 8 TeV, see Table I. The actual value of the rescaling factor for the CMS analysis is not explicitly quoted in the their papers. Therefore, the CMS data will not be used in the presented analysis[1].

In this paper, we shall make a bold *assumption* that the missing process accounting for the cross-section discrepancy, not considered so far in the calculations of the theoretical cross sections, is the double Drell–Yan process (DDYP) resulting from double-parton scattering (DPS) in proton–proton collisions. It should be stressed that the strength of DDYP can, at present, be neither directly constrained by experimental data nor predicted theoretically.

If one includes DDYP as a contributor to the W^+W^- production processes, it is bound to contribute as well to the ZZ production processes with a fully constrained strength. The immediate question which may be asked is if after adding the DDYP contribution to the Higgs-boson searches background, both in the W^+W^- and ZZ channels, there would still be a need to include the contribution coming from the Higgs-boson decays or, putting it alternatively, to which extent the DDYP contribution, on top of curing the WW anomaly, could mimic the Higgs-boson signal in the 125 GeV Higgs-sensitive phase-space regions.

To answer this question, we present a coherent analysis of the DDYP contribution to the $H \to W^+W^-$ and $H \to ZZ^*$ decay-channel backgrounds. We use the data corresponding to the total integrated luminosity collected so far at the LHC. Once the overall normalisation of the DDYP contribution is fixed to explain the WW anomaly, our DDYP model does not have any more free parameters, thus can be easily falsified by comparing its predictions to experimental data.

The DDYP contribution to the Higgs searches background in the $H \to ZZ^*$ decay channel was already analysed in our previous work [10]. Using a simplified model of DDYP, we have demonstrated the appearance of a peak in the four-lepton invariant mass, m_{4l}, distribution in the ~ 125 GeV

[1] A more general comment is obligatory for discerning unavoidable caveats of the analysis presented in this paper. The LHC collaborations publish very rarely their detector-effect unfolded distributions. The Higgs papers are no exceptions here. This preempts a fully irrefutable justification of any external analysis of these distributions, including the one presented in this paper. All we can do is to try to minimise the impact of the necessary underlying assumptions on the event selection efficiencies and detector smearing effects. Their remaining impact can be evaluated only by the relevant collaborations.

Higgs-signal region. This "Higgs-like" peak is not driven by the details of the DDYP model. It is generated by the interplay of a steeply falling m_{4l} distribution and the kinematical threshold effect driven by the experimental cuts on the outgoing leptons variables: the minimal transverse momenta of leptons and the minimal invariant mass of the opposite charge lepton pairs. These cuts are similar in the ATLAS and CMS analyses, and therefore result in similar DDYP peak positions within a 2 GeV interval.

It has to be stressed that the claim of the ATLAS and CMS collaborations that DDYP can be neglected as a potentially alarming source of background was based on the assumption of uncorrelated: (1) longitudinal momentum, (2) transverse position, (3) flavour, (4) charge and (5) spin of the partons taking part in DDYP, and on the assumption of the process-independent value of $\sigma_{\rm eff}$, governing the strength of the DPS processes. The above, in our view unjustified, assumptions lead to a significant underestimation of the contribution of DDYP to the Higgs searches background. As we argued in Ref. [10], its contribution must be, given the lack of the adequate theoretical calculations, determined experimentally, *e.g.* by using the experimental methods proposed therein and in Ref. [9].

There are three main reasons for writing this paper:

1. In Ref. [10], we applied our DDYP model only to the ATLAS data for the ZZ channel, while in this paper, we apply it to the ATLAS data for the WW channel, and after fitting the normalisation factor, we use it, parameter-free, in the ZZ channel.

2. The analysis in Ref. [10] was done only for the partial ATLAS data available at that time, here we use the full data collected by ATLAS in the LHC Run 1 to check if our DDYP model can still describe these data.

3. We propose new experimental methods of testing DDYP at the LHC, see Section 3.

The paper is organised as follows. In the next section, we present numerical results of our analysis. Section 3 includes a discussion of the results as well as a general discussion of the interplay between the DDYP and gluon–gluon scattering contributions. It contains a proposal of measurements capable to elucidate the role of DDYP in WW and ZZ production processes. Finally, Section 4 concludes our paper.

2. Results

The numerical results presented below have been obtained using the Monte Carlo event generator WINHAC [11–13] with the same model of DDYP and the same input parameters as in our previous paper [10].

The starting point to our quantitative studies in this paper is a shape analysis of the two-lepton mass, m_{ll}, distribution and its fit to the "$e\mu\nu\nu$ + 0-jets" final-state data presented by the ATLAS Collaboration in a wide m_{ll} range for the full data statistics at 8 TeV in Ref. [14].

In Fig. 1 (a), we present the results of the ATLAS Collaboration of Ref. [14] for the collision energy of 8 TeV. The background and the Higgs decays contributions are shown separately. This plot reflects the necessity of including the Higgs contribution to obtain a satisfactory description of the m_{ll} distribution in the region of its small values, where the sensitivity to the 125 GeV Higgs boson is the highest. However, we would like to stress again that the original background prediction was, in this case, rescaled by the ATLAS Collaboration by the factor of 1.22 in order to get a good description of data in the monitoring region.

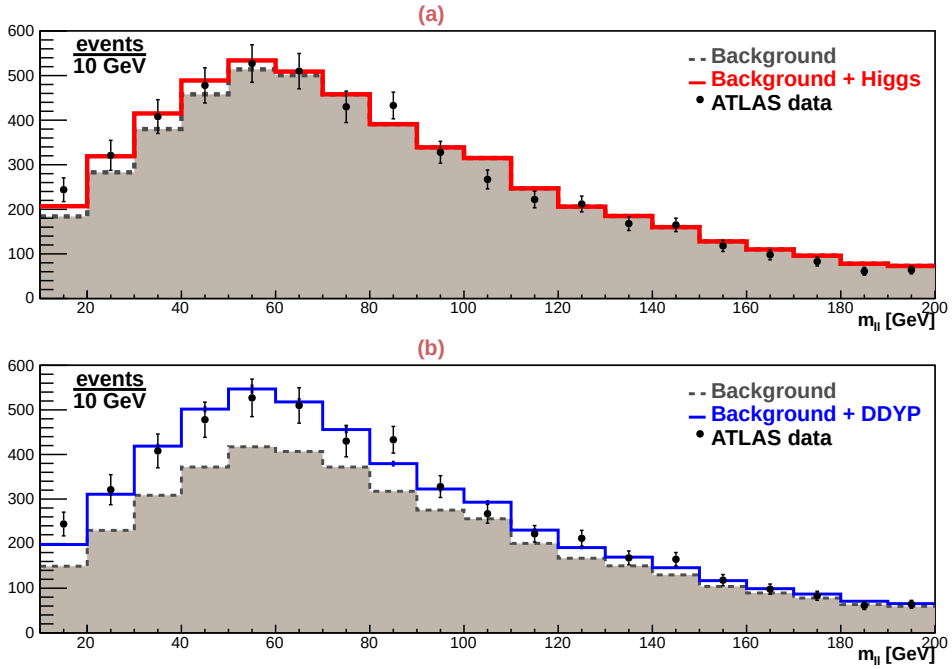


Fig. 1. The m_{ll} distributions for the $H \to W^+W^-$ searches at $\sqrt{s} = 8$ TeV: (a) the rescaled (ATLAS) background without (grey histogram) and with (solid/red line) the SM Higgs boson contribution compared with the ATLAS data (black dots) [14], and (b) the canonical background without (grey histogram) and with (solid/blue line) the DDYP contribution compared with the same ATLAS data (black dots).

For the plot presented in Fig. 1 (b), we re-normalise back the background distribution to its original, canonical theoretical predictions by dividing the background shown in Fig. 1 (a) by a factor of 1.22, and subsequently add the DDYP contribution with a normalisation of its prediction determined by minimisation of the χ^2/d.o.f. in the fit of the sum of the DDYP and theoretical background contributions to the data. The Higgs contribution is no longer necessary to obtain a satisfactory description of the data by the canonical background model if the DDYP contribution is included.

Indeed, the values of χ^2/d.o.f. corresponding to the m_{ll} distributions in Fig. 1 are 1.2 for the "rescaled background plus the SM Higgs" scenario (with the p-value equal to 0.26) and 0.8 for the "canonical background plus the DDYP" scenario (with p-value equal to 0.70). Both descriptions of the data are acceptable on a statistical basis, although the obtained values of the likelihood test give some better preference to the "DDYP plus canonical background" predictions. We stress again that in the latter case, the theoretical predictions of the SM background do not need to be rescaled to describe the data, as it was done in Refs. [7, 8].

Having determined the normalisation of DDYP cross section using the W^+W^- channel, we can now study the corresponding DDYP contribution to the m_{4l} distribution for the ZZ channel. In our model, the dominant contribution to the DDYP cross section comes form the $q\bar{q}$ excitations of the proton sea. Thus, the relative normalisation of the DDYP contributions for the W^+W^- and ZZ channels is driven, on the modelling side, only by the relative strength of the $u\bar{u}$ and $d\bar{d}$ excitations, reflecting the ratio of the $\bar{u}$ and $\bar{d}$ PDFs (for more details, see Ref. [10]). On the experimental side, it reflects the relative efficiencies of selection of W^+W^- and ZZ events in their kinematical acceptance regions[2]. The DDYP model predictions for the ZZ channel are thus fully constrained.

Again, as for the W^+W^- channel, the ATLAS data at 8 TeV [14] are presented first in Fig. 2 (a) with the SM Higgs contribution and the canonical background contributions shown separately. In Fig. 2 (b), we replace the Higgs contribution by the parameter-free DDYP model predictions. The values of χ^2/d.o.f. corresponding to m_{4l} distributions in Fig. 2 are 0.65 for the background plus SM Higgs scenario (with the p-value equal to 0.94) and 0.95 for the background plus our DDYP predictions (with p-value equal to 0.55). Both descriptions of data are statistically acceptable, however in this case, the SM Higgs prediction is slightly more preferred.

[2] For the analysis of the W^+W^- and ZZ spectra, we select events according to the selection criteria used for the Higgs searches in the corresponding decay channels. We assume the same efficiency of the W^+W^- and ZZ event selection and neglect the detector smearing effects.

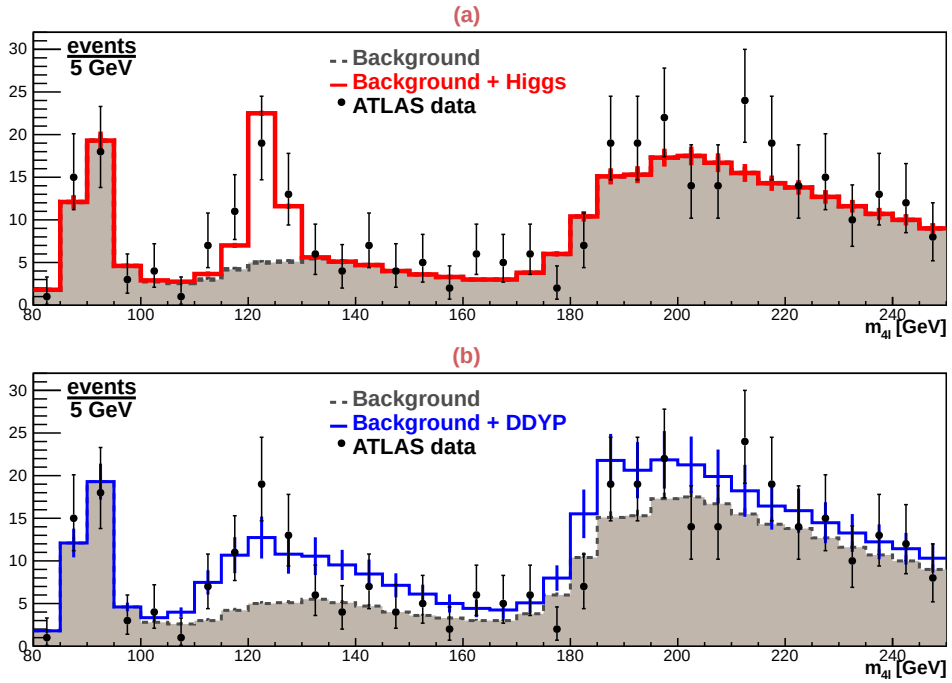


Fig. 2. The m_{4l} distributions for the $H \to ZZ$ searches at $\sqrt{s} = 8$ TeV: (a) without (grey histogram) and with (solid/red line) the SM Higgs-boson contribution compared with the ATLAS data (black dots) [14], and (b) the (ATLAS) background plus the DDYP contribution (solid/blue line) compared with the same ATLAS data (black dots).

In contrast to the SM Higgs scenario, DDYP contributes not only to the m_{4l} range near 125 GeV but also above the threshold of the on-shell ZZ production. We have calculated that with W^+W^--fixed normalisation, DDYP will result in 31 extra events in the range of 190 GeV $\leq m_{4l} \leq$ 250 GeV, which agrees at the 1σ level with the ATLAS data excess of 20 ± 13.5 events over the SM background [14]. CMS also observes an excess of the data with respect to the background in this kinematical region at a comparable level of $\sim 13\%$ [15].

3. Discussion

Our numerical results presented in the previous section show that once the DDYP contribution is normalised to account for the missing contribution to the W^+W^- cross section, one obtains a satisfactory predictions for the excess of events observed by the ATLAS experiment both in the m_{ll} and

m_{4l} distributions. These excesses of events were attributed by the ATLAS and CMS collaborations to the SM Higgs-boson decays to ZZ and W^+W^- with $m_H \approx 125$ GeV.

The presented results correspond to the two extreme scenarios explaining the excess of the experimental data over the SM background: the "SM-Higgs-only" and the "DDYP-only". There is, of course, a possibility that the two could contribute together. For example, in the case of the m_{4l} distribution, the SM Higgs contribution could better describe the peak width in the data in the 125 GeV region, while DDYP can add some events in this region but also in the higher mass range. This would lead to a different values of the Higgs couplings to the W and Z bosons with respect to those quoted in the canonical ATLAS and CMS analyses which neglect the DDYP contribution altogether.

The intriguing question is if the DDYP contribution could be sufficiently large to mimic fully the Higgs signal at the LHC. An equally important question is why it was not considered as important background source in the data analyses by the ATLAS and CMS collaborations. Let us start with the answer to the latter question before addressing the former one.

The DDYP contributions were studied by both collaborations with the help of the standard parton shower Monte Carlo generators and were found to be negligible. The reason why these generators predict such small cross sections for DDYP is related to the assumption of a completely uncorrelated two single-parton scattering (SPS) modelling of two Drell–Yan processes in a single proton–proton collision. The canonical DPS models assume the value of the so-called effective double-parton scattering cross section: $\sigma_{\rm eff} \approx 15$ mb. This parameter normalises the DPS cross section with respect to the product of two SPS cross sections, see *e.g.* [10].

The above value has been measured, but only for the cases of DPS involving at least one gluon in a pair of colliding partons. There are at least three reasons why the value of $\sigma_{\rm eff}$ may be significantly lower for the same flavour, opposite-charge and spin quark–antiquark pairs relevant to the ZZ and W^+W^- production processes:

— The quark–antiquark excitations of the proton involves partons of the same flavour. For example, if an s-quark takes part in the production of one of the vector boson, its $\bar{s}$ partner is already present in the wave function of the colliding proton. This enhances by a large factor, with respect to the canonical picture in which the PDFs of both partons are considered as uncorrelated, the probability that it may take part in the production of the second vector boson.

— The transverse–plane correlation length for the same flavour and opposite charge $q\bar{q}$ pairs is significantly smaller than for the gluon–gluon

pairs because of the dominance of the local charge and flavour conserving proton excitations — note that at the hardness scale of the vector-meson pair production processes and for the x-region relevant to the Higgs searches, there is less than one fixed-flavour $q\bar{q}$-pair excitation per incoming proton, while there is more than 100 gluon pairs covering uniformly, glueing together, the proton volume. No doubt, the strength of the gluonic DPS must reflect the transverse size of the proton, while DDYP should reflect rather the typical size of a quark–antiquark dipole.

— The long-lived $q\bar{q}$ excitations within the proton which do not lead to the significant effects modifying its overall spin involve quarks and antiquarks of opposite spins. Collisions of such pairs produce always vector-boson pairs with the total spin equal to zero, as in the case of the Higgs-boson decays.

All the above amplification effects cannot be calculated within the presently available QCD perturbative methods. This, however, does not mean that they do not exist. The LHC provides the unique opportunity to measure the respective quark–antiquark flavour, transverse plane, longitudinal momentum and spin correlations in the proton. The XY-pair production processes, where $X, Y \in \{\gamma^*, Z, W, J/\Psi, \Upsilon\}$, provide an excellent experimental testing ground for the quark–antiquark correlation models, in particular if: (1) data are collected at two or more collider energy settings (allowing to separate experimentally the quark and gluon originating processes [9]), (2) the data are collected not only in the pp but also in the pA and AA collision modes (to control the transverse–plane parton correlations) and, most importantly, (3) the DDYP effects are measured both in the Higgs-signal and monitoring regions. In addition, the measurement of the relative strengths of these processes provides a clear experimental test of the robustness of the SM Higgs interpretation of the data with respect to alternative mechanisms of the electroweak symmetry breaking. As long as such experimental studies are not made, the DDYP model should be, in our view, considered as equally plausible as the Higgs model in explaining the source of the excesses of events in the ZZ and W^+W^- channels observed at the LHC.

In this paper, we avoid giving any value of $\sigma_{\rm eff}$ for our DDYP, since, in general, DPS does not simply factorise into the product of two SPS processes, especially when the DPS contribution is sizeable, as it is in our case; examples of that are shown *e.g.* in Ref. [16]. Even if it were possible, the ATLAS and CMS experiments do not provide the necessary information (detector efficiencies and smearing effects) to translate our normalisation factor to $\sigma_{\rm eff}$. At the generator level, this normalisation factor is of the same order of magnitude as in Ref. [10].

Finally, let us address a more subtle question of the interplay between DDYP and the electroweak-boson pair production in the gluon–gluon fusion process. The latter processes are included in the MCFM program [5] as well as in the gg2WW [17] and gg2ZZ [18] generators which are used by the ATLAS and CMS collaborations for theoretical predictions of the respective SM background to the Higgs signal in its WW and ZZ decay channels. These calculations include also the so-called crossed-box contribution in which two incoming gluons split into quark–antiquark pairs and then a quark (antiquark) of one pair interacts with an antiquark (quark) of the second pair, leading to DDYP. One may, therefore, think that the processes we consider in this paper are already included in the theoretical predictions of the SM background to the respective Higgs signals.

In our opinion, the gluon–gluon fusion calculations include only partially the DDYP contribution and may even underestimate the included crossed-box diagram effects. Firstly, they take into account only on-shell incoming gluons which are purely left- or right-handed, while, as we argue in Ref. [10], one should consider all possible gluon polarisations. For the spin-zero vector boson pairs mimicking the Higgs signal, a particular care must be given to the full set of processes producing on-shell and off-shell $q\bar{q}$ pairs with compensating spins. Such pairs may be produced by longitudinally polarised gluons but also by the higher-twist effects. To our best knowledge these effects are not included in the existing theoretical calculations.

As was shown in a detail in Ref. [19], the crossed-box contributions exhibit a collinear singularity when the spins of the incoming on-shell gluons sum to zero. The typical collinear singularity is, within the leading-twist approach, damped in this case to the logarithmic (integrable) singularity due to the vector structure of QCD for the on-shell initial-state gluons, *i.e.* when they are purely left- or right-handed [19]. Off-shellness of longitudinally polarised gluons should, to some extent, damp the singularity, but will this reduce the enhancement due to the collinear quark–antiquark pair emission?

The question which remains to be answered is not only how frequently the incoming gluons are longitudinally polarised but also what is the probability for the $q\bar{q}$ pair propagating over large distances, before annihilating into a vector-boson pair, to become a spin-zero pair due to soft colour interactions with the medium. In both cases, the total spin of the colliding $q\bar{q}$ dipoles is zero, leading to a spin zero configuration of the final vector-boson pairs.

Another problematic issue is the question of the renormalisation and factorisation scales. In typical fixed order QCD calculations, as given in Refs. [5, 17–19], these scales are set equal to each other and taken as a hard-process scale. In the case of the processes under consideration, this scale is of the order of ~ 100 GeV. Is using such a high scale justified for processes in which incoming gluons split into almost collinear quark–antiquark pairs?

In our opinion it is not. In such cases, a better choice for the argument of the running QCD coupling may be not the hard process scale but rather the transverse momentum of the emitted quarks, see *e.g.* Ref. [20]. Generally, this kind of a scale is used in popular QCD parton shower generators. This is, however, often not the case in the fixed-order QCD calculations. The value of the running α_{s} between the scales of ~ 100 GeV and ~ 1 GeV is increased by a factor of ~ 5. Since the considered process involves α_{s}^2, we can easily get the enhancement factor of ~ 25. The factorisation scale for such collinear splittings should also be set to a similar value.

The effects discussed above should lead to enhancements in low p_{T} regions of produced electroweak bosons. This is hard to observe in the case of the W^+W^- production, as the transverse momenta of neutrinos from the leptonic W-boson decays cannot be measured individually, but could be seen rather easily in the $ZZ/Z\gamma^*$ processes through their charged lepton decay channels. Therefore, the latter processes can provide an important test of the interplay between the DDYP and gluon–gluon scattering contributions.

4. Conclusions

In this paper, we have investigated the consequences of the hypothesis that the double Drell–Yan process (DDYP) accounts for the excess of the measured W^+W^- cross section with respect to its theoretically predicted value. This assumption determines the absolute normalisation of the predictions of our simple DDYP model introduced in Ref. [10]. This normalisation factor is the only free parameter of the model.

We have demonstrated that adding the above absolutely normalised DDYP contribution to the canonical SM background is sufficient for a satisfactory description of the m_{4l} spectra for the ZZ final state and m_{2l} spectra for the W^+W^- final state, with a comparable fit quality as in the model assuming the existence of the SM Higgs boson.

We have argued that the DDYP contributions may indeed be significantly larger than that expected in the naive canonical DPS models because of the strong charge, flavour, longitudinal momentum, transverse position and spin correlations of the quark–antiquark pairs participating in DDYP. Such a possibility has not been excluded so far, neither by the theoretical calculations nor by the experimental measurements. We have presented some theoretical arguments in favour of such a DDYP contribution and proposed a measurement programme to test it experimentally at the LHC.

We would like to thank S.P. Baranov and J.R. Gaunt for useful discussions. We are indebted to the Academic Computer Centre CYFRONET AGH in Kraków, Poland, where our numerical simulations were performed with the use of the computing cluster Zeus.

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