

MASSES OF HEAVY AND LIGHT SCALAR TETRAQUARKS IN A NON-RELATIVISTIC QUARK MODEL*

ZAHRA GHALENOVI^a, FRANCESCO GIACOSA^{b,c}, DIRK H. RISCHKE^c

^aDepartment of Physics, Kosar University of Bojnord, Bojnord, Iran

^bInstitute of Physics, Jan Kochanowski University, 25-406 Kielce, Poland

^cInstitute for Theoretical Physics, Goethe University
Max-von-Laue-Str. 1, 60438 Frankfurt, Germany

(Received January 19, 2016)

Scalar tetraquark states are studied within the diquark–antidiquark picture in a non-relativistic approach. We consider two types of confining potentials, a quadratic and a linear one, to which we also add spin–spin, isospin–isospin, and spin–isospin interactions. We calculate the masses of the scalar diquarks and of the ground state open and hidden charmed and bottom scalar tetraquarks. Our results indicate that the scalar resonances $D_0^*(2400)$ and $D_s(2632)$ have a sizable tetraquark amount in their wave function, while, on the other hand, it turns out that the scalar states $D_{s0}^*(2317)$ and $X(3915)$ should not be considered as being predominantly diquark–antidiquark bound states. We also investigate the masses of light scalar diquarks and tetraquarks, which are comparable to the measured masses of the light scalar mesons.

DOI:10.5506/APhysPolB.47.1185

1. Introduction

One of the most important problems in modern hadron physics is to determine the structure and the properties of the newly discovered X, Y, Z states as well as other enigmatic mesons, such as $D_{s0}^*(2317)$, $D_0^*(2400)$, $D_{s1}^*(2460)$, *etc.*, see *e.g.* Refs. [1–4] and references therein. These states cannot be accommodated within the simple quark–antiquark picture and are, therefore, of special interest.

One possibility is to interpret (some) of these enigmatic mesons as tetraquark states, where the constituent objects are a diquark and an antidiquark. Namely, although a diquark cannot be a color singlet, the attraction between

* Funded by SCOAP³ under Creative Commons License, CC-BY 3.0.

two quarks can be strong, as various approaches based on one-gluon exchange processes [5], instantons [6], lattice calculations [7], and quark–diquark models for the nucleon [8] and for baryons in general [9] have shown. Thus, the diquark is an important object for the understanding of baryon structure and is also potentially important for understanding of unconventional mesons, most notably tetraquarks. In particular, in this work, we are interested in scalar diquarks: these are ‘good diquarks’ in Jaffe’s terminology [10], with vanishing spin and angular momentum and an antisymmetric flavor wave function of the type $[q, q']$, where $q, q' = u, d, s, c, b$ (a similar antisymmetric combination is realized in color space).

The masses of heavy tetraquarks as diquark–antidiquark bound states were studied in the presence of spin–spin interactions in Refs. [11, 12] and later in the comprehensive study of Ref. [13]. The masses of tetraquarks were also calculated in a quark model employing a potential derived from the AdS/QCD correspondence [14], by using a confining interaction and a meson-exchange potential in a non-relativistic approach [15], by implementing the Glozman–Riska (flavor–spin) interaction Hamiltonian and SU(3) flavor symmetry breaking [16], and in the framework of a non-relativistic potential model which includes a three-body quark interaction [17].

In this paper, we continue along these lines and calculate masses of (hidden and open) charmed and bottom ground-state *scalar* tetraquarks using two potential models in the non-relativistic limit. As a four-body system, a tetraquark state is quite different from a conventional $q\bar{q}$ meson and we solve the problem in a two-step procedure: first, we use a quark–quark interaction Hamiltonian in order to obtain the mass of a constituent ‘good diquark’ of the type $[q, q']$. Second, we regard the diquarks as point-like objects and use a diquark–antidiquark interaction Hamiltonian in order to obtain the tetraquark masses. In both steps, we solve the two-body Schrödinger equation by performing a Taylor expansion [18–20] or by using a variational method. We compare the values of the heavy tetraquark masses with the values obtained in previous works and discuss some possible experimental candidates.

Finally, we focus on the light scalar mesons $f_0(500)$, $K_0^*(800)$, $f_0(980)$, and $a_0(980)$. These states have been, and still are, in the center of a vivid debate concerning their nature: there is now a consensus that they are not predominantly quark–antiquark objects [21], but that they emerge either as dynamically generated molecular-type states, see *e.g.* [22–26] and references therein, and/or as tetraquark states as proposed some decades ago by Jaffe [27, 28] and further investigated in Refs. [9, 11, 13–17, 29–36]. (Note that the quark–antiquark states appear in the spectrum but are heavier, since they lie above 1 GeV [37, 38]). We apply the very same two-step approach described above for a system made of two light diquarks. We eval-

uate the masses of light scalar diquarks and tetraquarks and investigate to what extent the light scalar resonances can be described as scalar tetraquark objects.

The paper is organized as follows. In Sec. 2, we introduce the two potential models and present methods to solve the Schrödinger equation in the presence of hyperfine interactions. Our predictions for diquarks and scalar tetraquark masses obtained in the two models are presented and discussed in Sec. 3. Finally, a summary and discussion are presented in Sec. 4.

2. The models

2.1. The Hamiltonian

The interaction Hamiltonian for the quark–quark interaction leading to the formation of diquarks is given by

$$H^{qq}(x) = V^{qq}(x) + H_{\text{hyp}}^{qq}, \quad (1)$$

where the potential $V^{qq}(x)$ consists of three parts:

$$V^{qq}(x) = V_{\text{conf}}(x) - \frac{\tau}{x} - C. \quad (2)$$

The first term $V_{\text{conf}}(x)$ is a confining potential (see the next subsections), the second term $-\tau/x$ is a Coulomb-like potential due to one-gluon exchange processes, and C is a constant. The variable x is the relative quark–quark coordinate. The quantity H_{hyp}^{qq} is the hyperfine interaction given by

$$H_{\text{hyp}}(x) = H_{\text{S}}(x) + H_{\text{I}}(x) + H_{\text{SI}}(x), \quad (3)$$

where $H_{\text{S}}(x)$, $H_{\text{I}}(x)$, and $H_{\text{SI}}(x)$ are spin–spin, isospin–isospin, and spin–isospin interactions, respectively. They read explicitly [39–44]

$$H_{\text{S}} = A_{\text{S}} \left(\frac{1}{\sqrt{\pi}\sigma_{\text{S}}} \right)^3 \exp \left(-\frac{x^2}{\sigma_{\text{S}}^2} \right) (\vec{s}_1 \cdot \vec{s}_2), \quad (4)$$

$$H_{\text{I}} = A_{\text{I}} \left(\frac{1}{\sqrt{\pi}\sigma_{\text{I}}} \right)^3 \exp \left(-\frac{x^2}{\sigma_{\text{I}}^2} \right) (\vec{t}_1 \cdot \vec{t}_2), \quad (5)$$

$$H_{\text{SI}} = A_{\text{SI}} \left(\frac{1}{\sqrt{\pi}\sigma_{\text{SI}}} \right)^3 \exp \left(-\frac{x^2}{\sigma_{\text{SI}}^2} \right) (\vec{s}_1 \cdot \vec{s}_2) (\vec{t}_1 \cdot \vec{t}_2), \quad (6)$$

where s_i and t_i are the spin and isospin operators of the i^{th} quark, respectively, while A_k and σ_k with $k = \text{S, I, SI}$ are constants. Note that the operator t_z has eigenvalue $+\frac{1}{2}$ for the u quark, $-\frac{1}{2}$ for the d quark, and zero

for all other quark flavors. Following Refs. [40–44], the spatial dependence of the hyperfine interaction terms is not modeled by a Dirac δ function, but by a smooth Gaussian function. The hyperfine Hamiltonian is treated as a perturbation which slightly modifies the energy levels.

Next, we turn to the diquark–antidiquark potential. First, we recall that the one-gluon exchange potential is such that the quark–antiquark potential and quark–quark potentials are related by $V_{q\bar{q}} = 2V_{qq}$ (this is due to the product of Gell-Mann matrices $\vec{\lambda}_i \cdot \vec{\lambda}_j$, for details see Refs. [45–49]). When turning to the interaction between a good diquark and a good antidiquark, we assume the same form as for a quark–antiquark pair [10]. Thus, taking into account the factor 2, we get for a diquark–antidiquark system

$$H^{D\bar{D}}(x) = V^{D\bar{D}}(x) + H_{\text{hyp}}^{D\bar{D}}, \quad (7)$$

where the potential $V^{D\bar{D}}(x)$ reads

$$V^{D\bar{D}}(x) = 2V_{\text{conf}}(x) - \frac{2\tau}{x} - C. \quad (8)$$

The variable x is now the relative diquark–antidiquark coordinate and $H_{\text{hyp}}^{D\bar{D}}$ has the same form as H_{hyp} in Eq. (3). When applied to (good) diquarks, the isospin operator t_z has eigenvalue $+\frac{1}{2}$ for the diquark $[u, q]$ (with $q = s, c, b$), $-\frac{1}{2}$ for the diquark $[d, q]$ (with $q = s, c, b$), and zero for the diquarks $[u, d]$ and $[q, q']$ (with $q, q' = s, c, b$).

2.2. Quadratic confinement

In this work, we consider both quadratic and linear potentials in order to model confining interactions. First, we study the confining potential in Eq. (1) between two quarks as given by (Model 1)

$$V_{\text{conf}}(x) = ax^2, \quad (9)$$

where a is a positive constant. Since the potential is assumed to depend on x only, one can factor out the angular part of the two-body wave function. The remaining radial part of the wave function for the two-body problem with the unperturbed potential $V^{qq}(x)$ is then determined by the Schrödinger equation

$$\left[\frac{d^2}{dx^2} + \frac{2}{x} \frac{d}{dx} - \frac{l(l+1)}{x^2} \right] \psi_l(x) = -2m[E_l - V^{qq}(x)] \psi_l(x), \quad (10)$$

where $\psi_l(x)$ is the radial wave function, l is the angular quantum number, and m is the reduced mass of the two-body system,

$$m = \frac{m_1 m_2}{m_1 + m_2}, \quad (11)$$

with m_1 and m_2 being the constituent quark (and, subsequently, diquark) masses. Now, we solve the radial Schrödinger equation for the two-body interaction potential (2). The transformation

$$\psi_l(x) = x^{-1}\varphi_l(x) \quad (12)$$

reduces Eq. (10) to the form

$$\frac{d^2}{dx^2}\varphi_l(x) + \left[\epsilon_l - 2\max^2 + \frac{2m\tau}{x} - \frac{l(l+1)}{x^2} \right] \varphi_l(x) = 0. \quad (13)$$

The radial wave function $\varphi_l(x)$ is a solution of the reduced Schrödinger equation for the wave function of two identical particles with mass m and interaction potential (2), where

$$\epsilon_l = 2m(E_l + C). \quad (14)$$

The effective potential $U_l(x)$ reads

$$U_l(x) = 2\max^2 - \frac{2m\tau}{x} + \frac{l(l+1)}{x^2}. \quad (15)$$

In order to solve Eq. (13), we perform a Taylor expansion of $U_l(x)$ around $x = x_l$,

$$U_l(x) \approx U_l(x_l) + \Omega_l^2(x - x_l)^2, \quad (16)$$

where x_l is such that $dU_l(x)/dx|_{x=x_l} = 0$ and

$$\Omega_l^2 = \frac{1}{2!} \left. \frac{d^2 U_l(x)}{dx^2} \right|_{x=x_l}. \quad (17)$$

Substituting Eq. (16) for U_l , Eq. (15) into Eq. (13) we find

$$\frac{d^2}{dx^2}\varphi_l(x) - \Omega_l^2(x - x_l)^2\varphi_l(x) = -[\epsilon_l - U_l(x_l)]\varphi_l(x), \quad (18)$$

which is the well-known equation for a one-dimensional harmonic oscillator. Namely, for a particle with mass m , oscillation frequency ω' , energy eigenvalues $\varepsilon' = (n + \frac{1}{2})\hbar\omega'$, and spatial wave function $\phi(x)$, the one-dimensional harmonic oscillator equation reads

$$\frac{d^2}{dx^2}\phi(x) - \frac{m^2\omega'^2x^2}{\hbar^2}\phi(x) = -\frac{2m\varepsilon'}{\hbar^2}\phi(x). \quad (19)$$

We consider here the ground state of the scalar diquarks ($l = n = 0$). In this way, upon a comparison of Eq. (18) with Eq. (19), we have

$$\Omega_0 = \frac{m\omega'}{\hbar}, \quad \varepsilon_0 - U_0(x_0) = \frac{2m\varepsilon'}{\hbar^2}. \quad (20)$$

Finally, the ground-state energy eigenvalue E_0 is obtained using Eq. (14)

$$E_{0,q\bar{q}} = -C + \frac{1}{2m} [U_0(x_0) + \Omega_0] \quad (21)$$

with the corresponding ground-state wave function

$$\varphi_0(x) = \sqrt{\frac{2\Omega_0}{\sqrt{\pi}}} e^{-\frac{1}{2}\Omega_0 x^2}, \quad (22)$$

where the constant term in front is a normalization constant.

The very same mathematical problem needs to be solved for the diquark–antidiquark state by treating (anti)diquarks as point particles under the influence of the potential (8). The energy eigenvalue $E_{0,D\bar{D}}$ of the tetraquark ground state $n = l = 0$ is then calculated in the same way.

2.3. Linear confinement

We also model confinement via a linearly rising potential (Model 2)

$$V_{\text{conf}}(x) = ax. \quad (23)$$

The potential (2) is now the well-known Cornell potential. Similarly to the potential of model 1, we can factorize the angular part of the Schrödinger equation. Upon substituting the potential (23) into Eq. (10) and using the transformation (12) we obtain

$$\frac{d^2}{dx^2} \varphi_l(x) + \left[\epsilon_l - 2\max + \frac{2m\tau}{x} - \frac{l(l+1)}{x^2} \right] \varphi_l(x) = 0. \quad (24)$$

We use a variational method to solve the Schrödinger equation for the case $l = 0$ using the normalized test function

$$\varphi_0(x) = \sqrt{\frac{16p^3}{\sqrt{2\pi}}} x e^{-p^2 x^2}, \quad (25)$$

where p is the variational parameter. By minimization of the energy of the system, we calculate the energy and also the wave function of the system (for further details of this approach, see Ref. [42]). Also in this case, the approach can be easily extended to the calculation of the ground-state energies of diquark–antidiquark objects.

3. Diquark and tetraquark masses

In this section, we present the results for diquark and tetraquark masses. We first focus on diquarks and then on the corresponding tetraquarks containing at least one heavy quark. Finally, we turn to light diquarks and tetraquarks.

3.1. Diquarks

The diquark masses obtain a contribution from the constituent quark masses as well as from the confining and the spin–isospin-dependent interactions

$$M_{\text{diquark}} = m_{q1} + m_{q2} + E_{0,qq} + \langle H_{\text{hyp}} \rangle, \quad (26)$$

where m_{qi} is the mass of i^{th} quark and $E_{0,qq}$ is the ground-state energy calculated in the previous section. The first-order energy correction from the non-confining potential $\langle H_{\text{hyp}} \rangle$ is calculated using the unperturbed wave function obtained in Secs. 2.2 and 2.3

$$\langle H_{\text{hyp}} \rangle = \int d^3x \varphi H_{\text{hyp}} \varphi. \quad (27)$$

For the numerical evaluation, we use for Model 1 the light and heavy quark masses and the parameter τ from Ref. [45], while the parameter a and the hyperfine potential parameters are taken from Refs. [41, 44]. In Model 2, the light and heavy quark masses are still taken from Ref. [45] but the potential parameters are from Ref. [50]. In both models, the parameter C is obtained by fitting it to the experimental mass of the ρ meson. The parameters of both models are summarized in Table I.

TABLE I

Parameters used in our models.

Parameter	Model 1	Model 2
σ_S	2.87 fm	2.87 fm
A_S	67.4 fm ²	67.4 fm ²
σ_{SI}	2.31 fm	2.31 fm
A_{SI}	−106.2 fm ²	−106.2 fm ²
σ_I	3.45 fm	3.45 fm
A_I	51.7 fm ²	51.7 fm ²
$m_u = m_d$	277 MeV	280 MeV
m_s	553 MeV	569 MeV
m_c	1816 MeV	1840 MeV
m_b	5206 MeV	5213 MeV
a	0.73 fm ^{−3}	1.23 fm ^{−2}
τ	0.424	0.287
C	−2.684 fm ^{−1}	−2.08 fm ^{−1}

The scalar diquark masses obtained by Models 1 and 2 are shown in Table II and compared with the theoretical works [13, 30, 51]. We note that the predictions of the two models are similar to each other as well as to previous theoretical calculations.

TABLE II

Diquark masses (in MeV).

Diquark	Model 1	Model 2	Ref. [13]	Ref. [30]	Ref. [51]
$[qq]$	406	527	710	395	441
$[qs]$	678	784	948	590	659
$[qc]$	1918	2012	1973	1933	1980
$[sc]$	2147	2213	2091	—	2120
$[qb]$	5296	5371	5359	—	5140
$[sb]$	5523	5563	5462	—	5210

3.2. Heavy scalar tetraquarks

Once the diquark masses are calculated, we can evaluate the tetraquark masses by following the same steps. The explicit expression reads

$$M_{\text{tetraquark}} = m_{\text{diquark}} + m_{\text{antidiquark}} + E_{0,D\bar{D}} + \left\langle H_{\text{hyp}}^{D\bar{D}} \right\rangle. \quad (28)$$

The results for open charmed and bottom tetraquarks are listed in Table III and compared with other theoretical predictions [12, 13] and experimental candidates [52, 53]. The masses of the tetraquarks are indeed very similar in all theoretical approaches, with the exception of $[cs\bar{q}\bar{s}]$, which, in our case, turns out to be heavier than in Ref. [13]. Our results show that the scalar resonance $D_0^*(2400)$ may contain a sizable tetraquark amount in its wave function. On the other hand, the resonance $D_{s0}^*(2317)$ is too light to be interpreted as a $[cq\bar{q}\bar{s}]$ tetraquark (see also Ref. [54] for a discussion concerning conventional quark–antiquark charmed scalar states). Another interesting but still controversial state is the so-called $D_{s0}^*(2632)$ meson observed by SELEX [53], the mass of which fits well to our theoretical predictions.

TABLE III

Masses of open charmed and bottom tetraquarks (in MeV).

Tetraquark	Model 1	Model 2	Ref. [13]	Ref. [12]	Exp. [52, 53]
$[cq\bar{q}\bar{q}]$	2398	2426	2399	—	$D_0^*(2400)$
$[cq\bar{q}\bar{s}]$	2618	2600	2619	2371	$D_{s0}^*(2317)$, $D_{s0}^*(2632)$
$[cs\bar{q}\bar{s}]$	2855	2798	2753	—	—
$[bq\bar{q}\bar{q}]$	5763	5748	5758	—	—
$[bq\bar{q}\bar{s}]$	5980	5901	5997	—	—
$[bs\bar{s}\bar{q}]$	6217	6103	6108	—	—

The results for hidden charmed and bottom tetraquarks are listed in Table IV and compared with the theoretical predictions of Refs. [12, 55–57]. Also here, the theoretical results are compatible with each other.

TABLE IV

Masses of double-hidden charmed and bottom scalar tetraquarks and masses of open charmed and bottom scalar tetraquarks (in MeV).

Tetraquark	Model 1	Model 2	Ref. [55, 57]	Ref. [12]	Ref. [56]
$[cq\bar{c}\bar{q}]$	3807	3662	3812	3723	—
$[cq\bar{c}\bar{s}]$	4043	3862	3922	—	—
$[cs\bar{c}\bar{s}]$	4268	4050	4051	—	—
$[bq\bar{b}\bar{q}]$	10521	10044	10471	—	—
$[bq\bar{b}\bar{s}]$	10747	10228	10572	—	—
$[bs\bar{b}\bar{s}]$	10973	10412	10662	—	—
$[cq\bar{b}\bar{q}]$	7162	6908	—	—	7177
$[cq\bar{b}\bar{s}]$	7399	7106	—	—	7282
$[cs\bar{b}\bar{q}]$	7397	7096	—	—	7294
$[cs\bar{b}\bar{s}]$	7623	7285	—	—	7398

Quite interestingly, the by now established scalar resonance $X(3915)$ turns out to be too heavy to be a $cq\bar{c}\bar{q}$ state and too light to be a $cs\bar{c}\bar{s}$ state. It is then compatible with being a conventional $\chi_{c0}(2P)$ quarkonium state.

3.3. Light scalar tetraquarks

Finally, we apply our formalism to the calculation of the masses of light tetraquarks. The aim is to understand if the resonances $f_0(500)$, $K(800)$, $f_0(980)$, and $a_0(980)$ contain a sizable tetraquark amount or not (for experiments concerning these states see Refs. [58–61] and for theoretical works concerning the tetraquark hypothesis Refs. [27–30, 33, 62]). In this framework, the scalar diquarks behave under flavor (and also color) transformations as antiquarks,

$$[u, d] \leftrightarrow \bar{s}, \quad [d, s] \leftrightarrow \bar{u}, \quad [s, u] \leftrightarrow \bar{d}, \quad (29)$$

therefore, one can construct a nonet of tetraquarks where the lightest state is the $[ud][\bar{u}\bar{d}]$ and corresponds to $f_0(500)$, the second lightest are the kaonic-like states $[sq][\bar{u}\bar{d}]$, $[\bar{s}\bar{q}][ud]$ (with $q = u, d$ and $I = 1/2$) to be identified with $K_0^*(800)$ and, finally, the four tetraquarks $([sq][\bar{s}\bar{q}])$ with $I = 0, 1$ which correspond to $f_0(980)$ and $a_0(980)$.

Using the parameters of Table I and the diquark masses of Table II generates tetraquark masses which are 100–200 MeV heavier than the states $f_0(500)$, $K_0^*(800)$, $f_0(980)$, and $a_0(980)$. In order to investigate whether a better agreement is possible, we re-fit the parameter C of Eq. (2) for both models, using the well-known tetraquark state $a_0(980)$ as an input and obtain the light and strange diquark masses as following: (a) Model 1 [$C = -3.507 \text{ fm}^{-1}$]: $M_{[qq]} = 244 \text{ MeV}$, $M_{[qs]} = 515 \text{ MeV}$; (b) Model 2 [$C = -3.045 \text{ fm}^{-1}$]: $M_{[qq]} = 330 \text{ MeV}$, $M_{[qs]} = 592 \text{ MeV}$. Santopinto and Galata [62] have considered a diquark–antidiquark picture of the light scalar tetraquarks in the non-relativistic limit, where the masses of the scalar diquarks were obtained as $M_{[qq]} = 275 \text{ MeV}$, $M_{[sq]} = 492 \text{ MeV}$.

Using the new diquark masses, we obtain the masses of the light scalar tetraquark nonet listed in Table V. Our predictions for the masses of the light scalar tetraquarks are in good agreement with the experimental data and also with the results obtained in Refs. [62] and [63]. In our model, a small difference between the masses of $a_0(980)$ and $f_0(980)$ arises from the isospin-dependent hyperfine interaction.

TABLE V

Masses of light tetraquark states (in MeV).

Resonance	Flavor content	$I(J^P)$	Model 1	Model 2	Ref. [62]	Ref. [63]	Exp. [52]
$f_0(500)$	$[ud][\bar{u}\bar{d}]$	$0(0^+)$	546	614	550	596	400–550
$K(800)$	$[ud][\bar{s}\bar{d}]$	$1/2(0^+)$	765	804	767	730	653–701
$f_0(980)$	$[us][\bar{u}\bar{s}] + [ds][\bar{d}\bar{s}]$	$0(0^+)$	962	962	984	992	970–990
$a_0(980)$	$[us][\bar{u}\bar{s}] - [ds][\bar{d}\bar{s}]$	$1(0^-)$	984	984	984	992	983.5–985.9

In the context of light scalar states, it should be stressed that the role of loop corrections to the self-energy is surely non-negligible for the masses of these states [21, 23–25]. Namely, light scalars have a strong coupling to pseudoscalar mesons and a diquark–antidiquark configuration can easily transform into a meson–meson one. Moreover, our approach is non-relativistic, thus its application to the light scalar sector must be treated with care. Yet, our study shows once more that the light scalar mesons are not simple quark–antiquark states but may have a sizable four-quark component. In conclusion, we mention that light scalar mesons also play an important role at non-zero temperature [35] and at non-zero density [64].

4. Summary

In this work, we have calculated the masses of the ground-state heavy and light scalar tetraquarks in the framework of a non-relativistic approach with two types of confining potentials, a quadratically and a linearly rising one, as well as (iso)spin–(iso)spin interactions. The results for the scalar diquarks are shown in Table II, while the heavy scalar tetraquarks are summarized in Tables III and IV. The results of both models are compatible with each other, showing only a mild influence of the particular form of the confining potential. Moreover, the results are in agreement, apart from a few exceptions, with previous theoretical calculations of Refs. [11–13].

Our results for the masses show that the resonance $D_{s0}^*(2317)$ is too light to be predominantly a tetraquark state of the type $cq\bar{q}s$, while the hidden charmed state $X(3915)$ is too heavy to be $cq\bar{c}\bar{q}$ and too light to be $cs\bar{c}\bar{s}$. On the other hand, the state $D_0^*(2400)$ and the putative $D_{s0}^*(2632)$ can contain an important tetraquark component in their flavor wave function ($cq\bar{q}\bar{q}$ and $cq\bar{q}s$, respectively). In addition to already existing experimental candidates, we also made predictions for the masses of scalar tetraquark states which can be discovered in the future (see Tables III and IV). Namely, some of the X , Y , and Z states could turn out to be scalar objects. Finally, we have also studied the light scalar sector of QCD and found that the masses of light scalar mesons $f_0(500)$, $K_0^*(800)$, $f_0(980)$, and $a_0(980)$ can be described well in the tetraquark picture (see Table V).

In this work, the masses of the tetraquarks are calculated by using a static approach. Mass shifts take place as soon as interactions and quantum fluctuations are taken into account. These modifications are typically small for hadrons which are (i) narrow and (ii) are far from any decay threshold. For what concerns point (i), the ratio $\Gamma/(M - E_{\text{th}})$, where Γ is the decay width, M the mass of a hadron, and E_{th} the lowest decay threshold of the state, is an important quantity to estimate the role of loops [26]. This ratio is indeed large for the light scalar mesons $f_0(500)$ and $K_0^*(800)$ (see the discussion in Sec. 3.3), thus loop corrections are surely also an important element towards their understanding. Even when this ratio is relatively small (as it usually is for mesons in the charmonium region), one should then consider point (ii): namely, when M is close to one of the decay threshold (not necessarily the lowest), distortion of the spectral functions, mass shifts and sizable meson–meson amounts in the wave function of the unstable meson may occur. This is the case of the light scalar mesons $f_0(980)$ and $a_0(980)$: both of them are fairly distant from the lowest threshold ($\pi\pi$ and $\pi\eta$, respectively), but very close to the KK threshold. Similarly, the state $D_{s0}^*(2317)$ is pretty close to the DK threshold. In general, many of the newly discovered X , Y , and Z resonances are close to one of their intermediate threshold, thus care is definitely needed since the role of loops can be very important.

In view of this discussion, it must be stressed that also the calculation of decay widths should be performed in the future. Namely, it is possible that some of the predicted tetraquark states are, due to a ‘fall apart’ decay mechanics, *too wide* to be measured and that, therefore, will never be seen in experiments. This possibility would explain why only some of the (many possible) tetraquark states are actually detectable, that is when the energy threshold of the main decay channel is not too far from the mass of the tetraquark state, in such a way that the kinematic suppression balances the large decay amplitude. Indeed, this pattern takes place for the light scalar mesons, where $f_0(500)$ and $K_0^*(800)$ are very broad, while $f_0(980)$ and $a_0(980)$ are narrow due to the nearby kaon–antikaon threshold.

Another (indeed related) improvement is to go beyond the two-step calculations performed in this work. Surely, it is much easier to solve two two-body problems than a four-body problem, but a general feature of our model (as well as of other tetraquark models) is that the diquarks have a dimension which is comparable to that of the tetraquark (about 1 fm). In this respect, there are also strong quark–antiquark correlations within the tetraquark, because the diquarks cannot be considered as point-like objects. Just as mentioned above, the interchange of a diquark–antidiquark $(qq)(\bar{q}\bar{q})$ bound state with a more molecular-like quark–antiquark $(q\bar{q})(q\bar{q})$ surely takes place (and is related to the decay of the tetraquark in ordinary mesons). Thus, the view of a pure diquark–antidiquark bound state serves as a simple (albeit useful) approximation of the problem, but in the future one should also go beyond it and solve a (relativistic) four-body problem.

In addition to the listed needed improvements, we also mention that our approach can be extended to other quantum numbers as well, thus being potentially interesting to investigate further up to now not yet understood mesons.

REFERENCES

- [1] S. Godfrey, S.L. Olsen, *Annu. Rev. Nucl. Part. Sci.* **58**, 51 (2008) [arXiv:0801.3867 [hep-ph]].
- [2] E.S. Swanson, *Phys. Rep.* **429**, 243 (2006) [arXiv:hep-ph/0601110].
- [3] N. Brambilla *et al.*, *Eur. Phys. J. C* **71**, 1534 (2011) [arXiv:1010.5827 [hep-ph]].
- [4] E. Braaten, C. Langmack, D.H. Smith, *Phys. Rev. D* **90**, 014044 (2014) [arXiv:1402.0438 [hep-ph]]; G.T. Bodwin *et al.*, arXiv:1307.7425 [hep-ph].
- [5] A. De Rujula, H. Georgi, S.L. Glashow, *Phys. Rev. D* **12**, 147 (1975); T.A. DeGrand, R.L. Jaffe, K. Johnson, J.E. Kiskis, *Phys. Rev. D* **12**, 2060 (1975).

- [6] E.V. Shuryak, *Nucl. Phys. B* **203**, 93 (1982); T. Schafer, E.V. Shuryak, *Rev. Mod. Phys.* **70**, 323 (1998) [arXiv:hep-ph/9610451]; E. Shuryak, I. Zahed, *Phys. Lett. B* **589**, 21 (2004) [arXiv:hep-ph/0310270].
- [7] M.G. Alford, R.L. Jaffe, *Nucl. Phys. B* **578**, 367 (2000) [arXiv:hep-lat/0001023]; N. Mathur *et al.*, *Phys. Rev. D* **76**, 114505 (2007) [arXiv:hep-ph/0607110]; F. Okiharu, H. Suganuma, T.T. Takahashi, *Phys. Rev. D* **72**, 014505 (2005) [arXiv:hep-lat/0412012].
- [8] G. Hellstern, R. Alkofer, H. Reinhardt, *Nucl. Phys. A* **625**, 697 (1997) [arXiv:hep-ph/9706551]; G. Hellstern, R. Alkofer, M. Oettel, H. Reinhardt, *Nucl. Phys. A* **627**, 679 (1997) [arXiv:hep-ph/9705267]; M. Oettel, R. Alkofer, L. von Smekal, *Eur. Phys. J. A* **8**, 553 (2000) [arXiv:nucl-th/0006082]; A. Bender, C.D. Roberts, L. Von Smekal, *Phys. Lett. B* **380**, 7 (1996) [arXiv:nucl-th/9602012]; P. Maris, C.D. Roberts, *Int. J. Mod. Phys. E* **12**, 297 (2003) [arXiv:nucl-th/0301049]; U. Vogl, W. Weise, *Prog. Part. Nucl. Phys.* **27**, 195 (1991).
- [9] D. Ebert, T. Feldmann, C. Kettner, H. Reinhardt, *Int. J. Mod. Phys. A* **13**, 1091 (1998) [arXiv:hep-ph/9601257].
- [10] R.L. Jaffe, *Phys. Rep.* **409**, 1 (2005) [arXiv:hep-ph/0409065].
- [11] L. Maiani, F. Piccinini, A.D. Polosa, V. Riquer, *Phys. Rev. D* **72**, 031502 (2005) [arXiv:hep-ph/0507062].
- [12] L. Maiani, F. Piccinini, A.D. Polosa, V. Riquer, *Phys. Rev. D* **71**, 014028 (2005) [arXiv:hep-ph/0412098].
- [13] D. Ebert, R.N. Faustov, V.O. Galkin, *Phys. Lett. B* **696**, 241 (2011) [arXiv:1011.2677 [hep-ph]].
- [14] M.V. Carlucci *et al.*, *Eur. Phys. J. C* **57**, 569 (2008) [arXiv:0711.2014 [hep-ph]].
- [15] J. Vijande, F. Fernandez, A. Valcarce, *Phys. Rev. D* **73**, 034002 (2006) [Erratum *ibid.* **74**, 059903 (2006)] [arXiv:hep-ph/0601143].
- [16] V.B. Jovanovic, *Fortsch. Phys.* **56**, 462 (2008).
- [17] Z. Fan Yong *et al.*, *Chin. Phys. C* **32**, 515 (2008).
- [18] H. Hassanabadi, M. Hamzavi, S. Zarrinkamar, A.A. Rajabi, *Few-Body Syst.* **48**, 53 (2010).
- [19] O. Ozer, H. Koklu, *Adv. Math. Phys.* **2014**, 537563 (2014).
- [20] R. Koc, E. Olgar, arXiv:1008.0697 [math-ph].
- [21] J.R. Peláez, G. Rios, *Phys. Rev. Lett.* **97**, 242002 (2006) [arXiv:hep-ph/0610397]; J.R. Peláez, *Phys. Rev. Lett.* **92**, 102001 (2004) [arXiv:hep-ph/0309292].
- [22] O. Krehl, J. Speth, *Nucl. Phys. A* **623**, 162c (1997); *Pisma Zh. Eksp. Teor. Fiz.* **23**, 369 (1976); V. Baru *et al.*, *Phys. Lett. B* **586**, 53 (2004) [arXiv:hep-ph/0308129]; T. Branz, T. Gutsche, V.E. Lyubovitskij, *Phys. Rev. D* **78**, 114004 (2008) [arXiv:0808.0705 [hep-ph]]; S. Krewald, R.H. Lemmer, F.P. Sassen, *Phys. Rev. D* **69**, 016003 (2004) [arXiv:hep-ph/0307288]; V. Baru *et al.*, *Phys. Lett. B* **586**, 53 (2004) [arXiv:hep-ph/0308129].

- [23] J.A. Oller, E. Oset, *Nucl. Phys. A* **620**, 438 (1997) [arXiv:hep-ph/9702314]; J.A. Oller, E. Oset, J.R. Peláez, *Phys. Rev. Lett.* **80**, 3452 (1998) [arXiv:hep-ph/9803242]; *Phys. Rev. D* **59**, 074001 (1999) [Erratum *ibid.* **60**, 099906 (1999); **75**, 099903 (2007)] [arXiv:hep-ph/9804209]; E. van Beveren *et al.*, *Z. Phys. C* **30**, 615 (1986) [arXiv:0710.4067 [hep-ph]]; E. van Beveren, D.V. Bugg, F. Kleefeld, G. Rupp, *Phys. Lett. B* **641**, 265 (2006); W. Heupel, G. Eichmann, C.S. Fischer, *Phys. Lett. B* **718**, 545 (2012) [arXiv:1206.5129 [hep-ph]].
- [24] D. Morgan, M.R. Pennington, *Phys. Rev. D* **48**, 1185 (1993); N.A. Törnqvist, *Z. Phys. C* **68**, 647 (1995) [arXiv:hep-ph/9504372]; N.A. Törnqvist, M. Roos, *Phys. Rev. Lett.* **76**, 1575 (1996) [arXiv:hep-ph/9511210v1]; F.E. Close, N.A. Törnqvist, *J. Phys. G* **28**, R249 (2002) [arXiv:hep-ph/0204205]; M. Boglione, M.R. Pennington, *Phys. Rev. Lett.* **79**, 1998 (1997) [arXiv:hep-ph/9703257].
- [25] T. Wolkanowski, F. Giacosa, D.H. Rischke, *Phys. Rev. D* **93**, 014002 (2016) [arXiv:1508.00372 [hep-ph]]; T. Wolkanowski, M. Soltysiak, F. Giacosa, arXiv:1512.01071 [hep-ph].
- [26] F. Giacosa, G. Pagliara, *Phys. Rev. D* **76**, 065204 (2007) [arXiv:0707.3594 [hep-ph]]; F. Giacosa, T. Wolkanowski, *Mod. Phys. Lett. A* **27**, 1250229 (2012) [arXiv:1209.2332 [hep-ph]].
- [27] R.L. Jaffe, *Phys. Rev. D* **15**, 267 (1977); *Phys. Rev. Lett.* **38**, 195 (1977).
- [28] R.L. Jaffe, *Phys. Rev. D* **15**, 281 (1977).
- [29] R.L. Jaffe, *Phys. Rep.* **409**, 1 (2005) [arXiv:hep-ph/0409065].
- [30] L. Maiani, F. Piccinini, A.D. Polosa, V. Riquer, *Phys. Rev. Lett.* **93**, 212002 (2004) [arXiv:hep-ph/0407017].
- [31] S.M. Gerasyuta, V.I. Kochkin, *Phys. Rev. D* **78**, 116004 (2008) [arXiv:0804.4567 [hep-ph]].
- [32] J. Vijande, F. Fernandez, A. Valcarce, B. Silvestre-Brac, *Eur. Phys. J. A* **19**, 383 (2004) [arXiv:hep-ph/0310007].
- [33] F. Giacosa, *Phys. Rev. D* **74**, 014028 (2006) [arXiv:hep-ph/0605191].
- [34] F. Giacosa, *Phys. Rev. D* **75**, 054007 (2007) [arXiv:hep-ph/0611388].
- [35] A. Heinz, S. Struber, F. Giacosa, D.H. Rischke, *Phys. Rev. D* **79**, 037502 (2009) [arXiv:0805.1134 [hep-ph]].
- [36] W. Heupel, G. Eichmann, C.S. Fischer, *Phys. Lett. B* **718**, 545 (2012) [arXiv:1206.5129 [hep-ph]].
- [37] S. Gallas, F. Giacosa, D.H. Rischke, *Phys. Rev. D* **82**, 014004 (2010) [arXiv:0907.5084 [hep-ph]]; D. Parganlija, F. Giacosa, D.H. Rischke, *Phys. Rev. D* **82**, 054024 (2010) [arXiv:1003.4934 [hep-ph]]; S. Janowski, D. Parganlija, F. Giacosa, D.H. Rischke, *Phys. Rev. D* **84**, 054007 (2011) [arXiv:1103.3238 [hep-ph]].
- [38] D. Parganlija *et al.*, *Phys. Rev. D* **87**, 014011 (2012) [arXiv:1208.0585 [hep-ph]]; S. Janowski, F. Giacosa, D.H. Rischke, *Phys. Rev. D* **90**, 114005 (2014) [arXiv:1408.4921 [hep-ph]].
- [39] Z. Ghalenovi, A.A. Rajabi, S.-x. Qin, D.H. Rischke, *Mod. Phys. Lett. A* **29**, 1450106 (2014) [arXiv:1403.4582 [hep-ph]].

- [40] Z. Ghalei, A.A. Rajabi, A. Tavakolinezhad, *J. Phys.: Conf. Ser.* **347**, 012015 (2012).
- [41] Z. Ghalei, A.A. Rajabi, A. Tavakolinezhad, *Int. J. Mod. Phys. E* **21**, 1250057 (2012).
- [42] Z. Ghalei, A.A. Rajabi, M. Hamzavi, *Acta Phys. Pol. B* **42**, 1849 (2011).
- [43] H. Hassanabadi, A.A. Rajabi, S. Zarrinkamar, *Mod. Phys. Lett. A* **23**, 527 (2008).
- [44] M.M. Giannini, E. Santopinto, A. Vassallo, *Prog. Part. Nucl. Phys.* **50**, 263 (2003) [arXiv:nuc1-th/0301017].
- [45] B. Silvestre-Brac, *Few-Body Syst.* **20**, 1 (1996).
- [46] L.A. Blanco, F. Fernandez, A. Valcarce, *Phys. Rev. C* **59**, 428 (1999).
- [47] B. Keren-Zur, *Ann. Phys.* **323**, 631 (2008) [arXiv:hep-ph/0703011].
- [48] C. Helminen, D.O. Riska, *Nucl. Phys. A* **699**, 624 (2002) [arXiv:nuc1-th/0011071].
- [49] W. Lucha, F.F. Schoberl, arXiv:hep-ph/9601263.
- [50] A. Valcarce, P. Gonzalez, F. Fernandez, V. Vento, *Few-Body Syst. Suppl.* **8**, 340 (1995) [arXiv:nuc1-th/9510018].
- [51] B. Chakrabarti, A. Bhattacharya, S. Mani, A. Sagari, *Acta Phys. Pol. B* **41**, 95 (2010).
- [52] K.A. Olive *et al.* [Particle Data Group], *Chin. Phys. C* **38**, 090001 (2014).
- [53] A.V. Evdokimov *et al.* [SELEX Collaboration], *Phys. Rev. Lett.* **93**, 242001 (2004) [arXiv:hep-ex/0406045].
- [54] W.I. Eshraim, F. Giacosa, D.H. Rischke, *Eur. Phys. J. A* **51**, 9 (2015) 112 [arXiv:1405.5861 [hep-ph]].
- [55] D. Ebert, R.N. Faustov, V.O. Galkin, *Phys. Atom. Nucl.* **72**, 178 (2009).
- [56] D. Ebert, R.N. Faustov, V.O. Galkin, W. Lucha, *Phys. Rev. D* **76**, 114015 (2007) [arXiv:0706.3853 [hep-ph]].
- [57] D. Ebert, R.N. Faustov, V.O. Galkin, *Eur. Phys. J. C* **58**, 399 (2008) [arXiv:0808.3912 [hep-ph]].
- [58] A. Aloisio *et al.* [KLOE Collaboration], *Phys. Lett. B* **537**, 21 (2002) [arXiv:hep-ex/0204013].
- [59] E.M. Aitala *et al.* [Fermilab E791 Collaboration], *Phys. Rev. Lett.* **86**, 770 (2001) [arXiv:hep-ex/0007028].
- [60] M. Ablikim *et al.* [BES Collaboration], *Phys. Lett. B* **633**, 681 (2006) [arXiv:hep-ex/0506055].
- [61] W.M. Yao *et al.* [Particle Data Group], *J. Phys. G* **33**, 1 (2006).
- [62] E. Santopinto, G. Galata, *Phys. Rev. C* **75**, 045206 (2007) [arXiv:hep-ph/0605333].
- [63] D. Ebert, R.N. Faustov, V.O. Galkin, *Eur. Phys. J. C* **60**, 273 (2009) [arXiv:0812.2116 [hep-ph]].
- [64] S. Gallas, F. Giacosa, G. Pagliara, *Nucl. Phys. A* **872**, 13 (2011) [arXiv:1105.5003 [hep-ph]]; A. Heinz, F. Giacosa, D.H. Rischke, *Nucl. Phys. A* **933**, 34 (2015) [arXiv:1312.3244 [nuc1-th]].