A NOTE ON THE ANOMALOUS MAGNETIC MOMENT OF THE MUON

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The anomalous magnetic moment of the muon is an important observable that tests radiative corrections of all three observed local gauge forces: electromagnetic, weak and strong interactions. High precision measurements reveal some discrepancy with the most accurate theoretical evaluations of the anomalous magnetic moment. We show in this note that the UV finite theory cannot resolve this discrepancy. We believe that more reliable estimate of the nonperturbative hadronic contribution and the new measurements can resolve the problem.

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1. Introduction and motivation

Since the most reliable Standard Model (SM) evaluations of the anomalous magnetic moment of the muon differs three to four standard deviations from the measurements [1], it might be useful to examine predictions for this observable of the theories beyond the SM.

It is well-known that the SM has three major obstacles: (1) massless neutrinos, (2) absence of a dark matter particle, and (3) absence of the lepton and baryon number violations. We show in Ref. [2] that the UV nonsingular theory, free of the SU(2) global anomaly (called the BY theory in [2]), is free of the SM deficiencies. Besides three heavy Majorana neutrinos as cold dark matter particle(s), the BY theory contains three light Majorana neutrinos with a relation on the three mixing angles, allowing the inclusion of the lepton CP-violating phase. The effects of the universal UV space-like cut-off are studied in various strong and electroweak processes in Refs. [3–5]. The cosmological consequences of the nonsingular Einstein–Cartan theory of gravity can be envisaged in Ref. [6]. The minimal distance in the Einstein–Cartan cosmology is compatible with the UV cut-off of the BY theory.

In this note, we want to inspect the impact of the UV cut-off on the radiative corrections of the anomalous magnetic moment of the muon. Next chapter deals with the explicit calculations, however, the relevant detailed formulae the reader can find in Appendix. The concluding section discusses the numerical results and their consequences.

2. One-loop corrections within the UV nonsingular theory

The anomalous magnetic moment of the muon is one of the most precisely measured observable to the uncertainty of $\mathcal{O}(10^{-9})$ [1]. It is calculated within the perturbative quantum field theory to the very high order.

The perturbation theory works well and is very accurate because of the very small fine structure constant. The anomalous magnetic moment formula can be cast into transparent perturbation series where it is possible to compare and study any modification of the Standard Model (SM) [7]. We evaluate one-loop corrections with virtual electroweak bosons within the UV finite BY theory [2]. Although this theory contains Majorana light neutrinos, we can safely neglect their Majorana character and their masses because of their smallness. The heavy Majorana neutrinos are coupled strongly to Nambu–Goldstone scalars at tree level, but not to electroweak gauge bosons. They are, therefore, decoupled in the evaluation of the anomalous magnetic moment.

Since the gauge invariant physical result cannot depend on the choice of gauge, we can freely perform our calculations in the 't Hooft–Feynman gauge with Nambu–Goldstone scalars instead in the unitary gauge, where Nambu–Goldstone scalars are decoupled from lepton doublets [8]. However, the scalar doublet does not contain the Higgs scalar [2] since the non-contractible space is a symmetry breaking mechanism in the BY theory and the UV cut-off is fixed at tree level by the weak boson mass $\Lambda = \frac{\pi}{\sqrt{6}}\frac{2}{g}M_W$, $e = g\sin\Theta_{\mathrm{W}}$, $\cos\Theta_{\mathrm{W}} = \frac{M_W}{M_Z}$ [2]. The Higgs scalar is decoupled from other particles in the BRST transformations [10] and does not play essential role in the proof of the renormalizability of the spontaneously broken gauge theories [11]. Consequently, the UV finite BY theory without the Higgs scalar is also renormalizable.

It is necessary to comment the claim that the recently discovered 125 GeV scalar resonance is the SM Higgs scalar [12]. Cea [13] proposed the most natural explanation of the 125 GeV resonance as a mixture of toponium and gluonium. The possible new 750 GeV heavy boson resonance at the LHC [14] might be a perfect candidate for a heavier scalar twin of the 125 GeV boson [15].

Let us go back to the description of the electroweak one-loop contributions to the anomalous magnetic moment. The SM one-loop result takes the form [1, 7–9] (m denotes the mass of the muon and $s_w = \sin\Theta_W$)

$$\begin{aligned} a_\mu &\equiv \frac{1}{2}(g_\mu - 2)\,, \\ a_\mu^\gamma &= \frac{1}{2}\frac{\alpha}{\pi} + \mathcal{O}\left(\left(\frac{\alpha}{\pi}\right)^2\right), \\ a_\mu^{\mathrm{EW}(1)} &= \frac{G_\mathrm{F}}{\sqrt{2}}\frac{m^2}{8\pi^2}\left\{\frac{10}{3} + \frac{1}{3}\left(1-4s_w^2\right)^2 - \frac{5}{3} + \mathcal{O}\left(10^{-5}\right)\right\}. \end{aligned}$$

We have to re-derive these results equipped with the SM Feynman rules in the 't Hooft–Feynman gauge. One can straightforwardly extract the contributions with one virtual photon [16] and one virtual W and Z bosons [8] in terms of the coefficient functions of the tensor and vector Green functions that can be deduced from the scalar Green functions-master integrals [17] (for definitions and explicit expressions, see Appendix)

$$\lim_{\Lambda\to\infty} m^2(C_{11}+C_{21})(\text{photon}) = \frac{1}{2}\,, \tag{1}$$

$$\lim_{\Lambda\to\infty} \frac{1}{4}(C_{11}-C_{21})(W\text{ boson}) = \frac{10}{3}\frac{1}{16M_W^2}\,, \tag{2}$$

$$\begin{aligned} &\lim_{\Lambda\to\infty}\left\{\frac{1}{2}\left[\frac{1}{4}\left(1-2s_w^2\right)^2 + s_w^4\right](2C_0+3C_{11}+C_{21}) + s_w^2\left(1-2s_w^2\right)\right. \\ &\left.\times(C_0+C_{11})\right\}(Z\text{ boson}) = \frac{1}{48M_Z^2}\left[\left(3-4c_w^2\right)^2-5\right]. \end{aligned} \tag{3}$$

The deviation between the SM and the BY theory could be found in the scalar master integrals. The UV cut-off Λ in the space-like domain of the Minkowski spacetime is introduced as a Lorentz and gauge invariant quantity. The analytical continuation to the time-like domain is performed on the Riemann's sheets, when necessary. By the symmetrization of the external momenta of the master integrals with the UV cut-off, we insure their translational invariance. Consequently, all the master integrals of the BY theory have a correct limit $\Lambda\to\infty$ of the standard QFT master integrals.

We left our presentation of the explicit results and comments to the next section.

3. Results and conclusions

The one-loop results for the BY theory follow from Eqs. (1)–(3) and the expansion formulas for scalar one-, two- and three-point functions in the SM and the BY theory presented in Appendix. Besides the difference between the SM and BY in the cut-off ($\Lambda = \infty$ or $\Lambda = \frac{\pi}{\sqrt{6}}\frac{2}{g}M_W$), one has to exclude the Higgs boson contribution for the BY theory ($M_H \to \infty$)[18]:

$$a_\mu^\gamma(\mathrm{BY})^{(1)} = \left(\frac{1}{2} - \frac{1}{4}\frac{m^2}{\Lambda^2} + \frac{5}{12}\frac{m^6}{\Lambda^6}\right)\frac{\alpha}{\pi}\,, \tag{4}$$

$$a_\mu^W(\mathrm{BY})^{(1)} = \frac{G_\mathrm{F}}{\sqrt{2}}\frac{m^2}{8\pi^2}\left\{\frac{10}{3} - \frac{41}{6}\frac{M_W^2}{\Lambda^2} + \frac{99}{8}\frac{M_W^4}{\Lambda^4}\right\}, \tag{5}$$

$$a_\mu^Z(\mathrm{BY})^{(1)} = \frac{G_\mathrm{F}}{\sqrt{2}}\frac{m^2}{8\pi^2}\left\{\frac{1}{3}\left(1-4s_w^2\right)^2 - \frac{5}{3} + \left(\frac{3}{2} + 2s_w^2 - 4s_w^4\right)\frac{M_Z^2}{\Lambda^2} + \frac{1}{48}\left(43 - 332s_w^2 + 664s_w^4\right)\frac{M_Z^4}{\Lambda^4}\right\}. \tag{6}$$

We evaluate numerically the difference with the following set of the well-established parameters (see also Fig. 1 for the cut-off dependence)

$$\begin{aligned} m(\mathrm{muon}) &= 105.658\ \mathrm{MeV}\,, & & M_Z = 91.188\ \mathrm{GeV}\,, \\ G_\mathrm{F} &= 1.166\times 10^{-5}\ \mathrm{GeV}^{-2}\,, & & \alpha = 1/137.036\,, \\ M_W &= 80.385\ \mathrm{GeV}\,, \quad s_w^2 = 0.22295\,, & & \Lambda = 326.2\ \mathrm{GeV}\,, \end{aligned}$$

$$\begin{aligned} \left[a_\mu^{\mathrm{BY}} - a_\mu^{\mathrm{SM}}\right]^{\gamma,1\ \mathrm{loop}} &= -6.09\times 10^{-11}\,, \\ \left[a_\mu^{\mathrm{BY}} - a_\mu^{\mathrm{SM}}\right]^{W,1\ \mathrm{loop}} &= -4.307\times 10^{-10}\,, \\ \left[a_\mu^{\mathrm{BY}} - a_\mu^{\mathrm{SM}}\right]^{Z,1\ \mathrm{loop}} &= +1.595\times 10^{-10}\,, \\ \left[a_\mu^{\mathrm{BY}} - a_\mu^{\mathrm{SM}}\right]^{\gamma+W+Z,1\ \mathrm{loop}} &= -3.321\times 10^{-10}\,. \end{aligned}$$

It is evident from the recent comparison between the experimental and the theoretical SM prediction in Ref. [1, 7]

$$\begin{aligned} a_\mu^{1+2\ \mathrm{loops}}(\mathrm{Higgs}) &= +\mathcal{O}\left(10^{-11}\right)\,, \\ a_\mu^{\mathrm{exp}} - a_\mu^{\mathrm{SM}} &= 2.7\times 10^{-9}\,, \end{aligned}$$

that the loop corrections of the BY theory cannot explain the deviation from the experimental value. It seems that more reliable estimates of the hadronic contributions are necessary. They should be studied by the non-perturbative methods supplemented by the experiments with hadrons [1]. The new experiments to measure the anomalous magnetic moment of the muon are planned in the USA and Japan with a good potential to further reduce the experimental error.

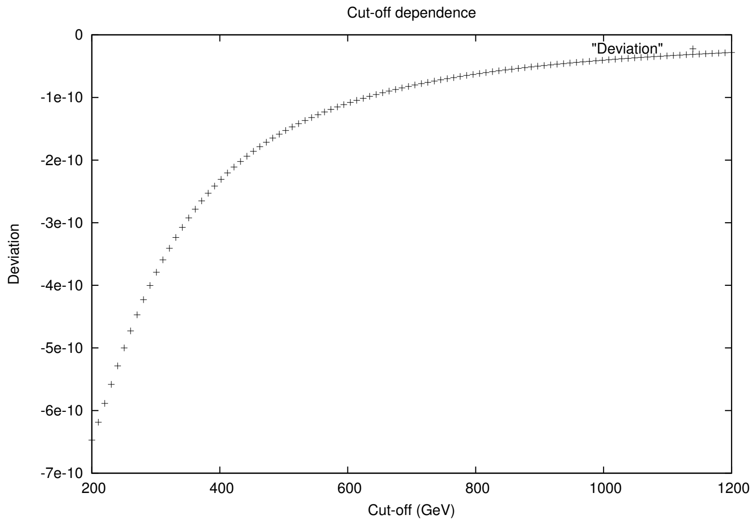

Fig. 1. Cut-off (Λ) dependence of the deviation $\left[a_\mu^{\mathrm{BY}} - a_\mu^{\mathrm{SM}}\right](\gamma + W + Z)$.

Appendix

The appendix is devoted to the exposure of the explicit formulas for scalar, vector and tensor functions, as well as for one-, two- and three-point scalar Green functions.

Let us start with definitions and conventions as in [17]

$$\frac{\imath}{16\pi^2} A\left(m^2\right) = \int \frac{\mathrm{d}^4 k}{(2\pi)^4} \frac{1}{k^2 - m^2}\,,$$

$$\frac{\imath}{16\pi^2} B_{0;\mu}\left(q^2; m_1^2, m_2^2\right) = \int \frac{\mathrm{d}^4 k}{(2\pi)^4} \frac{1; k_\mu}{\left[k^2 - m_1^2\right]\left[(k+q)^2 - m_2^2\right]}\,,$$

$$B_\mu = q_\mu B_1\,,$$

$$\frac{\imath}{16\pi^2} C_{0;\mu;\mu\nu}\left(p_1^2, p_2^2, p^2; m_1^2, m_2^2, m_3^2\right) =$$

$$\int \frac{\mathrm{d}^4 k}{(2\pi)^4} \frac{1; k_\mu; k_\mu k_\nu}{\left[k^2 - m_1^2\right]\left[(k+p_1)^2 - m_2^2\right]\left[(k+p_1+p_2)^2 - m_3^2\right]}\,,$$

$$C^\mu = p_1^\mu C_{11} + p_2^\mu C_{12}\,, \qquad p = -p_1 - p_2\,,$$

$$C^{\mu\nu} = g^{\mu\nu} C_{24} + p_1^\mu p_1^\nu C_{21} + p_2^\mu p_2^\nu C_{22} + \left(p_1^\mu p_2^\nu + p_2^\mu p_1^\nu\right) C_{23}\,.$$

The standard Green function can be found in Ref. [17], while the Green functions in the non-contractible space ($\Lambda < \infty$) have the form [3]:

$$\Re B_0^{\Lambda}\left(p^2; m_1, m_2\right) = \frac{1}{2}\left[\Re \tilde{B}_0^{\Lambda}(p^2; m_1, m_2) + \Re \tilde{B}_0^{\Lambda}\left(p^2; m_2, m_1\right)\right] ,$$

$$\Re \tilde{B}_0^{\Lambda}\left(p^2; m_1, m_2\right) = \left(\int_0^{\Lambda^2} \mathrm{d}y K\left(p^2, y\right) + \theta\left(p^2 - m_2^2\right) \int_{-\left(\sqrt{p^2} - m_2\right)^2}^{0} \mathrm{d}y \Delta K\left(p^2, y\right)\right) \times \frac{1}{y + m_1^2} ,$$

$$K\left(p^2, y\right) = \frac{2y}{-p^2 + y + m_2^2 + \sqrt{\left(-p^2 + y + m_2^2\right)^2 + 4p^2 y}} ,$$

$$\Delta K\left(p^2, y\right) = \frac{\sqrt{\left(-p^2 + y + m_2^2\right)^2 + 4p^2 y}}{p^2} ,$$

$$\begin{aligned}\Re C_0^{\Lambda}\left(p_1^2, p_2^2, p_3^2; m_1^2, m_2^2, m_3^2\right) = \frac{1}{3}\Big[&\Re \tilde{C}_0^{\Lambda}\left(p_1^2, p_2^2, p_3^2; m_1^2, m_2^2, m_3^2\right) \\ &+ \Re \tilde{C}_0^{\Lambda}\left(p_2^2, p_3^2, p_1^2; m_2^2, m_3^2, m_1^2\right) \\ &+ \Re \tilde{C}_0^{\Lambda}\left(p_3^2, p_1^2, p_2^2; m_3^2, m_1^2, m_2^2\right)\Big] ,\end{aligned}$$

$$\Re C_0^{\Lambda}\left(p_i, m_j\right) = \int_0^{\Lambda^2} \mathrm{d}q^2 \Phi\left(q^2, p_i, m_j\right) + \int_{\mathrm{TD}} \mathrm{d}q^2 \Xi\left(q^2, p_i, m_j\right) ,$$

$$\Re C_0^{\Lambda}(p_i, m_j) = \mathrm{Re} C_0^{\infty}(p_i, m_j) - \int_{\Lambda^2}^{\infty} \mathrm{d}q^2 \Phi\left(q^2, p_i, m_j\right) ,$$

$\Phi \equiv$ *function derived by the angular integration after Wick' rotation*,
$C_0^{\infty} \equiv$ *standard 't Hooft–Veltman scalar function*,
TD $\equiv$ *time-like domain of integration.*

We list the Green functions (real parts only) necessary for the one-loop contributions with a virtual W boson (photon and Z boson contributions proceed similarly):

$$C_0 = C_0\left(m^2, 0, m^2; 0, M_W^2, M_W^2\right) ,$$

$$\lim_{m^2 \to 0} C_{11} = \frac{1}{2}\left[\frac{\mathrm{d}B_0\left(m^2; 0, M_W^2\right)}{\mathrm{d}m^2}\left(m^2 = 0\right) - C_0\left(m^2 = 0\right)\right.$$

$$+ M_W^2 \frac{\mathrm{d}C_0}{\mathrm{d}m^2}\left(m^2=0\right)\Big] ,$$

$$\lim_{m^2\to 0} C_{21} = -\frac{C_{24}\left(m^2\right)}{\mathrm{d}m^2}\left(m^2=0\right) - \frac{1}{2}C_{11}\left(m^2=0\right) + \frac{M_W^2}{2}\frac{\mathrm{d}C_{11}}{\mathrm{d}m^2}\left(m^2=0\right)$$
$$+\frac{1}{2}\frac{\mathrm{d}B_1\left(m^2;0,M_W^2\right)}{\mathrm{d}m^2}(m^2=0) ,$$

$$\frac{\mathrm{d}C_{24}\left(m^2\right)}{\mathrm{d}m^2}\left(m^2=0\right) = \frac{1}{4}\left[C_{11}\left(m^2=0\right) - M_W^2\frac{\mathrm{d}C_{11}}{\mathrm{d}m^2}\left(m^2=0\right)\right] ,$$

$$C_{11}\left(m^2\right) = \frac{1}{2m^2}\left[B_0\left(m^2;0,M_W^2\right) - B_0\left(0;M_W^2,M_W^2\right) - \left(m^2-M_W^2\right)C_0\right] ,$$

$$B_1\left(m^2;0,M_W^2\right) = \frac{1}{2m^2}\left[-A\left(M_W^2\right)+A(0)+\left(M_W^2-m^2\right)B_0\left(m^2;0,M_W^2\right)\right] ,$$

$$B_0^\Lambda\left(0;0,M_W^2\right) = \ln\frac{\Lambda^2+M_W^2}{M_W^2} ,$$

$$\frac{\mathrm{d}B_0^\Lambda\left(m^2;0,M_W^2\right)}{\mathrm{d}m^2}\left(m^2=0\right) = \frac{1}{2}\frac{1}{M_W^2} - \frac{1}{4}\frac{M_W^2}{\left(\Lambda^2+M_W^2\right)^2} ,$$

$$\frac{\mathrm{d}^2B_0^\Lambda\left(m^2;0,M_W^2\right)}{\mathrm{d}\left(m^2\right)^2}\left(m^2=0\right) = \frac{1}{3}\frac{1}{M_W^4} + \frac{1}{3}\frac{M_W^2}{\left(\Lambda^2+M_W^2\right)^3} - \frac{1}{2}\frac{M_W^4}{\left(\Lambda^2+M_W^2\right)^4} ,$$

$$C_0^\Lambda\left(m^2\right) = C_0^\infty\left(m^2\right) + \Delta C_0^\Lambda , \qquad C_0^\infty\left(m^2\right) = \frac{1}{m^2}\ln\frac{M_W^2-m^2}{M_W^2} ,$$

$$\Delta C_0^\Lambda\left(m^2\right) = \frac{1}{3m^2}\left[\int_0^{1/\Lambda}\mathrm{d}y y^{-1}\frac{1-m^2y^2}{(1+M_W^2y^2)^2}\left(\sqrt{1+\frac{4m^2y^2}{\left(1-m^2y^2\right)^2}}-1\right)\right.$$
$$\left. + 2\int_0^{1/\Lambda}\mathrm{d}y y^{-1}\left(1-\frac{y^2\left(M_W^2-m^2\right)+1}{\sqrt{\left(1+y^2\left(M_W^2-m^2\right)\right)^2+4m^2y^2}}\right)\right] ,$$

$$C_0^\Lambda\left(m^2=0\right) = \frac{1}{M_W^2}\left(-1+x(1+x)^{-1}\right) , \qquad x = \frac{M_W^2}{\Lambda^2} ,$$

$$\frac{\mathrm{d}C_0^\Lambda}{\mathrm{d}m^2}\left(m^2=0\right) = \frac{1}{M_W^4}\left[-\frac{1}{2}+\frac{2}{3}\left(-(1+x)^{-2}+(1+x)^{-3}\right)\right] ,$$

$$\frac{\mathrm{d}^2C_0^\Lambda}{\mathrm{d}(m^2)^2}\left(m^2=0\right) = \frac{1}{M_W^6}\left[-\frac{2}{3}+\frac{4}{3}\left(-2(1+x)^{-5}+8(1+x)^{-4}-\frac{37}{3}(1+x)^{-3}\right.\right.$$
$$\left.\left.+9(1+x)^{-2}-3(1+x)^{-1}+\frac{1}{3}\right)\right] .$$

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