NEUTRINOLESS DOUBLE BETA DECAY MEDIATED BY THE NEUTRINO MAGNETIC MOMENT*

Marek Góźdź, Wiesław A. Kamiński

Department of Informatics, Maria Curie-Skłodowska University
Akademicka 9, 20-033 Lublin, Poland

(Received November 27, 2015; revised version received January 26, 2016)

Neutrinoless double beta decay is a hypothetical nuclear process actively developed both on theoretical and experimental grounds. In the present paper, we extend the idea discussed in [*Phys. Rev. D*, **89**, 113005 (2014)] where a new channel of this decay has been proposed. In this scenario, neutrinos not only oscillate inside the nucleus but also interact with an external non-uniform magnetic field. We assume that the field rotates about the direction of motion of the neutrino and show that for a certain rotation speed the half-life of the $0\nu2\beta$ decay can be significantly shortened. While the presentation in the reference mentioned above was limited to a simplified two-neutrino case, in this work, we investigate the realistic three-neutrino case and perform a detailed numerical study of this process.

DOI:10.5506/APhysPolB.47.1245

1. Introduction

Neutrinos, although weakly, interact with other particles, and, therefore, propagation and oscillation of these particles in vacuum differs from that in matter. This is known as the Mikheyev–Smirnov–Wolfenstein effect (MSW) [1] and has recently been observed by the Super-Kamiokande Collaboration as an asymmetry in the oscillation rate between zenith and nadir neutrinos [2]. This effect is based on the fact that the components of 'ordinary' matter, *i.e.*, electrons, protons, and neutrons, interact with electron neutrinos via charged as well as neutral currents. Muon and tau neutrinos, on the other hand, cannot interact with the electrons, thus participate in the neutral current processes only. This results in an asymmetry in the forward scattering amplitude of different neutrino flavours, effectively changing the neutrino oscillation parameters. Therefore, regular matter distinguishes between electron and other neutrino flavours.

Weak interactions are not the only factors that may affect neutrino propagation and oscillations. Despite being electrically neutral, neutrinos, according to the Standard Model, should exhibit electromagnetic properties. In the second-order, 1-loop process $\nu \leftrightarrows W^{\pm}\ell^{\mp}$ neutrino magnetic moment has been estimated by Fuijkawa and Shrock to be $3.2\times 10^{-19}(m_\nu)\mu_{\rm B}$ [3]. For $m_\nu = 0.05$ eV its value reads $1.6\times 10^{-20}\mu_{\rm B}$, $\mu_{\rm B}$ being the Bohr magneton [4]. Another estimations, *e.g.* by Kayser [5], provide qualitatively similar results $\mu \sim 10^{-18}\mu_{\rm B}$. This value can be larger in various scenarios of physics beyond the Standard Model [6] and for certain ranges of non-standard parameters, it can reach the experimental limit of roughly $10^{-11}\mu_{\rm B}$ [7]. Due to the CPT theorem, Majorana neutrinos can have only transition (in the flavour basis) magnetic moments, while Dirac neutrinos can have also diagonal magnetic moments. It is important to note that the neutrino–photon effective interaction vertex may be constructed in such a way that it violates the lepton number by two units. This can be realized if one adds right-handed currents to the Standard Model in the R-parity violating supersymmetric models and others. In such a situation, the transition magnetic moments change neutrinos into antineutrinos of different flavour, while the diagonal magnetic moments change neutrinos into antineutrinos of the same flavour. It has also been pointed out [8, 9] that an external non-uniform magnetic field acts differently on neutrinos and antineutrinos, which is due to different helicities of these particles. This observation has been used to show that under special conditions, Pontecorvo oscillations [10] $\nu_\alpha \to \bar{\nu}_\alpha$ are possible [11, 12].

The neutrinoless double beta decay ($0\nu 2\beta$) is a hypothetical second-order process in which some lepton number violating non-standard mechanism accounts for the neutrinos not being released. This process is of the most importance because, if observed, will qualify neutrinos as Majorana particles, which is the contents of the famous Schechter–Valle black-box theorem [13]. To be more exact, as shown in [14], the black-box theorem states that the $0\nu 2\beta$ decay implies a Majorana-like contribution to the neutrino mass matrix, but does not exclude other contributions, also Dirac-like, which, in principle, could be even dominant. Nevertheless, for a pure Dirac neutrino, this decay is strictly forbidden. In this paper, we assume neutrinos to be Majorana particles.

The simplest and most often discussed mechanism of the $0\nu 2\beta$ decay is the so-called mass mechanism in which left-handed Majorana neutrinos of non-zero mass are produced in the beta vertex as a negative-helicity state with a small positive-helicity admixture. This positive-helicity admixture is responsible for the possibility of the neutrino being absorbed in the second beta vertex. Of course, the intermediate electron neutrino propagates between the beta vertices as a superposition of three mass eigenstates and, therefore, the inverse half-life of the decay depends on the so-called effective neutrino mass $\langle m\rangle_{0\nu}$. Assuming the exchange of light neutrino and the same chirality in both beta vertices, the neutrino part of the process is described by

$$\sum_{i=1,2,3} U_{ei}\frac{m_i}{p^2-m_i^2}U_{ei}^* \approx \frac{1}{p^2}\sum_{i=1,2,3}|U_{ei}|^2 m_i = \frac{1}{p^2}\langle m\rangle_{0\nu}\,, \tag{1}$$

where p is the neutrino momentum, U_{ei} are the elements of the first row of the neutrino mixing matrix, and we have used the approximation of small, comparing to p, neutrino mass. The factor $1/p^2$ is then absorbed by the nuclear matrix element as a part of the energy denominator. Other mechanisms involve different intermediate particles, such as the pions, supersymmetric particles, the Majoron, and others. In this paper, we describe another mechanism of the neutrinoless double beta decay based on the two-step Pontecorvo oscillations. It has been shown [11, 12] that this mechanism has a resonant-like behaviour and its application to the solution of the solar neutrino puzzle has been discussed. In [15], the very same mechanism has been discussed in the context of the $0\nu2\beta$ decay for a simplified two-neutrino case. This paper presents the realistic three-neutrino case together with a numerical analysis to show that the new mechanism may, under proper conditions, significantly shorten the half-life of the decay in the resonance region.

2. Neutrinos in nuclear matter

Our goal is to describe the nuclear process of the neutrinoless double beta decay. We start, therefore, with the discussion of the neutrino behaviour in the nuclear matter. Neutrino interactions and oscillations inside the nucleus are omitted in the standard approach to the $0\nu2\beta$ decay [16, 17], as it is argued that neutrinos travel a very short distance between the nucleons. Despite this fact, the process of flavour oscillations is vital for the proposed here mechanism.

Neutrinos travelling through matter undergo a phase shift due to their interactions with electrons, neutrons and protons via neutral and charged weak currents. A recent discussion of this problem [18] takes into account also a specific 4-fermion interaction of neutrinos and quarks, but we will not consider this possibility here.

In the most typical case of the MSW effect, matter is electrically neutral and contains neutrons and an equal amount of electrons and protons. The charged current Standard Model reactions occur between charged leptons and the corresponding neutrinos, so typically electrons and electron neutrinos. Since electrons are absent in the nuclear medium, this interaction is not present inside the nucleus. The neutral current contributions coming from the electrons and protons have the same magnitudes but different signs due to the opposite electric charges of these particles, resulting in mutual cancellation. In the case of nuclear matter, however, there are no electrons and the proton contribution will not be cancelled.

We closely follow the textbook approach presented in [19]. Writing the Hamiltonian of neutrinos in vacuum in the mass basis $(\nu_1, \nu_2, \nu_3)^T$

$$H = \mathrm{diag}(E_1, E_2, E_3) \tag{2}$$

and using the relation for light relativistic particles $E = \sqrt{p^2 + m^2} \approx p + m^2/2p$, we get

$$H = p + \frac{1}{2p}\mathrm{diag}\left(m_1^2, m_2^2, m_3^2\right) , \tag{3}$$

where the symbol diag represents the diagonal matrix and $p \equiv |\vec{p}|$ is the value of the neutrino momentum. The neutrino mass eigenstates evolve in time according to the Schrödinger-like equation

$$i\frac{\mathrm{d}}{\mathrm{d}t}\begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix} = H \begin{pmatrix} \nu_1 \\ \nu_2 \\ \nu_3 \end{pmatrix} . \tag{4}$$

The transformation to the flavour basis $(\nu_e, \nu_\mu, \nu_\tau)^T$ is defined by the unitary Pontecorvo–Maki–Nakagawa–Sakata (PMNS) matrix U as

$$\begin{aligned} H \to UHU^\dagger &= p + \frac{1}{2p}\left[U\mathrm{diag}\left(m_1^2, m_2^2, m_3^2\right)U^\dagger\right] \\ &= p + \frac{1}{2p}\mathcal{M}^2 , \end{aligned} \tag{5}$$

where $\mathcal{M}^2$ denotes the square of the neutrino mass matrix in the flavour basis.

The energy levels of the flavour states are corrected in the nuclear matter by their possible interactions via the neutral currents with neutrons and protons [19],

$$V_{\mathrm{nc}} = \sqrt{2}G_{\mathrm{F}} \sum_{f=n,p} n_f \left(I_3^{(f)} - 2q^{(f)}\sin^2\theta_{\mathrm{W}}\right) , \tag{6}$$

I_3 being the third component of the weak isospin, and q is the electric charge. Explicitly, the neutron and proton contributions read

$$V_{\mathrm{nc}}^{(n)} = \sqrt{2}G_{\mathrm{F}}\left(-\frac{1}{2}\right)n_n , \tag{7}$$

$$V_{\mathrm{nc}}^{(p)} = \sqrt{2}G_{\mathrm{F}}\left(\frac{1}{2} - 2\sin^2\theta_{\mathrm{W}}\right)n_p , \tag{8}$$

where G_{F} is the Fermi constant, θ_{W} is the Weinberg mixing angle, and $n_{n,p}$ are the neutron and proton number densities. We recall that due to the absence of electrons inside the nucleus, the charged-current contribution to the energy of electron neutrino is zero. Therefore, all flavour eigenstates are affected by the presence of nuclear matter in the same way and the contribution in this specific case takes the form of a constant shift of the neutrino energy levels. As the neutrino oscillations are sensitive to the differences of masses squared, this will not affect the oscillation rate.

3. Neutrinos in an external magnetic field

As it was already mentioned in Introduction, neutrinos, even within the Standard Model, possess a non-zero magnetic moment generated in second-order processes. In the case of Majorana neutrinos, this gives a non-zero probability of the transition between a predominantly left-handed neutrino ν and its right-handed counterpart $\bar{\nu}$ of different flavour, which is triggered by the effective interaction with an external photon.

The main mechanism leading to the $0\nu2\beta$ decay is based on the conversion between different helicities of an electron neutrino. So, if one combines the interaction via the magnetic moment with the flavour oscillations, one gets two possible chains which will satisfy the required conditions ($\alpha = \mu, \tau$):

$$\nu_e \;\to\; \bar{\nu}_\alpha \to \bar{\nu}_e \,, \tag{9}$$

$$\nu_e \;\to\; \nu_\alpha \to \bar{\nu}_e \,. \tag{10}$$

Both of these chains lead to the $0\nu2\beta$ decay, but their amplitudes cancel each other exactly. It is due to the fact that the main part of the amplitudes are the propagators of $\bar{\nu}_\alpha$ and ν_α. In normal circumstances, these particles have the same masses, so the propagators are equal, but from the antisymmetricity of the magnetic moment, $\mu_{e\alpha} = -\mu_{\alpha e}$, follow opposite signs of the final amplitudes and their cancelation. We have assumed, however, that there is an external magnetic field with which neutrinos interact, and behaviour of Majorana neutrinos in these conditions must be examined.

In the Standard Model, neutrinos are massless, which implies that definite helicity is assigned to a chiral state, so that the left-handed neutrino has negative helicity, while the right-handed antineutrino has positive helicity. Such an assignement is in agreement with experiments, which forces us to define the antineutrino as the CP-conjugate of the neutrino. Massive left-handed neutrinos, on the other hand, are a mixture of negative and positive helicity states, where the latter admixture is proportional to the term m_ν/E, thus being heavily suppressed for relativistic neutrinos. Therefore, practical reasons, for massive neutrinos can be treated as being predominantly in the negative helicity state and massive antineutrinos being predominantly in the positive helicity state.

The presence of the magnetic field B has a two-fold effect. Firstly, transitions between neutrino states of different helicities are possible via the neutrino magnetic moment. For Majorana neutrinos, the magnetic moment is antisymmetric $\mu_{\alpha\beta} = -\mu_{\beta\alpha}$, $\alpha, \beta = \{e, \mu, \tau\}$, *i.e.*, only transitions of the form

$$\nu_\alpha \leftrightarrows \bar{\nu}_\beta \,, \qquad \alpha \neq \beta \tag{11}$$

are possible. The strength of this interaction has the form $B\mu_{\alpha\beta}$. The second effect is that if the magnetic field changes along the neutrino path, the neutrinos of different helicities will obtain different corrections to their effective masses. This follows from the fact that a frame of reference with spin $\vec{s}$, which is rotating with the angular velocity $\vec{\omega}$, gains energy $(-\vec{s}\cdot\vec{\omega})$. Since left-handed neutrinos ($s = -1/2$) and their right-handed counterparts ($s = +1/2$) have opposite helicities, the degeneracy of their energy levels is lifted in the presence of an external rotating magnetic field. The component of the rotating field that is parallel to $\vec{s}$ does not contribute to this effect, so we denote by $B \equiv |\vec{B}_\perp|$ the magnitude of the perpendicular component of the magnetic field, with the angle $\phi = \phi(t)$ indicating the direction of $\vec{B}_\perp$, and switch to a reference frame which rotates with the field. We can write the magnetic field angular velocity as $\omega = \mathrm{d}\phi(t)/\mathrm{d}t \equiv \dot{\phi}(t)$. The correction to the energy coming from the field rotation is $+\dot{\phi}(t)/2$ for left-handed particles and $-\dot{\phi}(t)/2$ for right-handed particles, and the immediate consequence of the lifting of the degeneracy of Majorana neutrinos of different helicities masses is that the amplitudes of the chains (9)–(10) do not cancel each other.

3.1. The two-flavour case

In the two neutrinos case [11, 12, 15], the results can be presented in a concise analytical form and they exhibit all the key features of the realistic three-flavour case, which will be described in the next section. Choosing the basis

$$(\nu_e, \nu_\mu, \bar{\nu}_e, \bar{\nu}_\mu)^T \,, \tag{12}$$

the mixing matrix depends on one vacuum mixing angle θ only

$$U = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} . \tag{13}$$

We neglect here, for simplicity, the possible CP-violating phases, but will discuss them in the three-neutrino case. The Hamiltonian, diagonal in the mass basis, becomes non-diagonal in the flavour basis. Taking into account matter and magnetic field corrections, it takes the block form

$$H = \begin{pmatrix} H_\nu + \frac{\dot\phi}{2} & [B\mu]_2 \\ -[B\mu]_2 & H_\nu - \frac{\dot\phi}{2} \end{pmatrix}, \tag{14}$$

where

$$H_\nu = p + V_{\rm nc}^{(n)} + V_{\rm nc}^{(p)} + \frac{1}{2p}\mathcal{M}^2\,, \tag{15}$$

with

$$\mathcal{M}^2 = \begin{pmatrix} m_1^2\cos^2\theta + m_2^2\sin^2\theta & \frac{\Delta m^2}{2}\sin 2\theta \\ \frac{\Delta m^2}{2}\sin 2\theta & m_1^2\sin^2\theta + m_2^2\cos^2\theta \end{pmatrix}, \tag{16}$$

$\Delta m^2 = m_2^2 - m_1^2$, $V_{\rm nc}^{(n)}$ and $V_{\rm nc}^{(p)}$ being given by (7) and (8), and

$$[B\mu]_2 = B\begin{pmatrix} 0 & \mu_{e\mu} \\ -\mu_{e\mu} & 0 \end{pmatrix}. \tag{17}$$

Here, $\mu_{e\mu}$ is the antisymmetric Majorana neutrino transition magnetic moment.

Hamiltonian (14) can be diagonalized and its eigenvalues take the form

$$p + V_{\rm nc}^{(n)} + V_{\rm nc}^{(p)} + \frac{1}{2p}\begin{pmatrix} m_1'^2 \\ m_2'^2 \\ \bar m_1'^2 \\ \bar m_2'^2 \end{pmatrix}, \tag{18}$$

where the overbar indicates masses of the antiparticles. Explicitly, the masses squared are given by

$$m'^2_{1,2} = \frac{1}{2}\left(m_1^2 + m_2^2 \pm \sqrt{(4pB\mu_{e\mu})^2 + \left(2p\dot\phi + \Delta m^2\right)^2}\right), \tag{19}$$

$$\bar m'^2_{1,2} = \frac{1}{2}\left(m_1^2 + m_2^2 \pm \sqrt{(4pB\mu_{e\mu})^2 + \left(2p\dot\phi - \Delta m^2\right)^2}\right). \tag{20}$$

(There is a typo in Eq. (10) of Ref. [15]: the factor 2 is missing in $2p\dot\phi$.) We notice that the term $\dot\phi$ lifts the degeneracy between the mass eigenstates of neutrinos of different helicities, $m'^2_{1,2} \neq \bar m'^2_{1,2}$. Also, in the absence of the magnetic field, $B = 0$, $\dot\phi = 0$, we arrive at the expected result $m'^2_{1,2} = m^2_{1,2} = \bar m'^2_{1,2}$.

In the general case, the mixing angles of neutrino mass eigenstates in vacuum and in matter differ, and the source of this difference lies in the interaction of electron neutrinos with electrons via the charged current $V_{\rm cc}$. The corrected mixing angle is given by

$$\tan 2\theta' = \frac{\Delta m^2 \sin 2\theta}{\Delta m^2 \cos 2\theta - 2pV_{\rm cc}}\,, \tag{21}$$

where $V_{\rm cc} \sim n_e$, the electron number density in matter. In our case, however, $n_e = 0$ and, therefore, the U matrix remains unchanged. As mentioned earlier, one can take into account the $\nu\bar{\nu}q\bar{q}$ interaction [18] to refine the shape of the U matrix. We leave this possibility to be included in future work.

The nuclear matter effect on neutrino propagation manifests itself as a constant shift of the mass eigenstates. This shift induces a constant phase factor which does not affect the oscillation probabilities, but has an influence on the neutrino propagator.

3.2. The three-flavour case and the neutrinoless double beta decay

We present the generalization to the realistic three-neutrino case by expanding the flavour basis of neutrinos and antineutrinos to the form

$$(\nu_e, \nu_\mu, \nu_\tau, \bar{\nu}_e, \bar{\nu}_\mu, \bar{\nu}_\tau)^T \tag{22}$$

and taking into account the presence of matter and an external non-constant magnetic field in the way described in the previous section. The three-flavour neutrino Hamiltonian takes the form

$$H = \left(\begin{array}{cc} H_\nu + \frac{\dot{\phi}}{2} & [B\mu]_3 \\ -[B\mu]_3 & H_\nu - \frac{\dot{\phi}}{2} \end{array} \right), \tag{23}$$

where

$$[B\mu]_3 = B \left(\begin{array}{ccc} 0 & \mu_{e\mu} & \mu_{e\tau} \\ -\mu_{e\mu} & 0 & \mu_{\mu\tau} \\ -\mu_{e\tau} & -\mu_{\mu\tau} & 0 \end{array} \right). \tag{24}$$

H_ν is given by Eq. (15) with $\mathcal{M}^2$ defined in (5). We use the standard parametrization for the PMNS matrix,

$$\begin{aligned} U \;=\; & \left(\begin{array}{ccc} c_{12}c_{13} & s_{12}c_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}c_{23} - s_{12}s_{23}s_{13}e^{i\delta} & s_{23}c_{13} \\ s_{12}s_{23} - c_{12}c_{23}s_{13}e^{i\delta} & -c_{12}s_{23} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{array} \right) \\ & \times \mathrm{diag}\left(1, e^{i\phi_2}, e^{i\phi_3}\right), \end{aligned} \tag{25}$$

where $s_{ij} \equiv \sin\theta_{ij}$, $c_{ij} \equiv \cos\theta_{ij}$, and θ_{ij} is the mixing angle between the mass eigenstates m_i and m_j. The δ is the CP-violating Dirac phase and ϕ_2, ϕ_3 are CP-violating Majorana phases. We will discuss the impact of these phases in Section 4.

For the neutrinoless double beta decay, the essential neutrino transition is that of $\nu_e \leftrightarrows \bar{\nu}_e$. The off-diagonal block of the Hamiltonian, which describes the mixing of neutrinos of different helicities in our setup, has the form of (24) and, therefore, a direct transition between ν_e and $\bar{\nu}_e$ is forbidden. This transition, however, is possible, as it was pointed out in [11, 12], in a two-step processes given by (9) and (10). In the neutrinoless double beta decay, the chains of transitions have to be realized between two beta vertices and may be described by a factor, call it χ, which is related to the half-life of the $0\nu2\beta$ decay

$$\left(T^{0\nu}_{1/2}\right)^{-1} = G^{0\nu}\left|M^{0\nu}\right|^2 |B\chi|^2\,, \tag{26}$$

where $G^{0\nu}$ is the exactly computable phase-space factor and $M^{0\nu}$ contains the hadronic and electron parts of the amplitude. The neutrino is interchanged between interaction vertices and as a virtual particle, it is not bound by the mass-shell relation. It is also transfering momentum between the outgoing electrons and the external photon, which is attached in the magnetic moment vertex. Its own 4-momentum has to be integrated out. The full discussion of this problem, which involves the choice of an appropriate nuclear model to describe the hadronic part, is beyond the scope of the present manuscript. In what follows, we propose and discuss an approximate form of the χ term.

The neutrino part of the amplitude is proportional to two propagators of the intermediate neutrino and two transition probabilities, *i.e.*, the magnetic moment and a product of the U matrices. In order to proceed, we have to make some approximations. Firstly, we neglect the 4-momenta of the electrons and the photon. The product of two propagators of neutrinos with a common 4-momentum q reduces then to

$$\frac{\not{q} + \bar{m}'_i}{q^2 - \bar{m}'^2_i}\frac{\not{q} + m'_j}{q^2 - m'^2_j} \to \frac{q^2 + \bar{m}'_i m'_j}{\left(q^2 - \bar{m}'^2_i\right)\left(q^2 - m'^2_j\right)} \tag{27}$$

because there are left chiral projectors P_{L} in each beta vertex, as the W bosons couple to the left-handed fields only, and $P_{\mathrm{L}}\not{q}P_{\mathrm{L}} = 0$. At this point, one usually uses the light neutrino approximation, see Eq. (1), which is not applicable here, because m' and $\bar{m}'$ are functions of $\dot{\phi}$ and this parameter can be tuned. We notice, however, that for masses significantly smaller than the 4-momentum, expression (27) is proportional to $1/q^2$, which is a small number, while it explodes for massess $\bar{m}'^2, m'^2 \approx q^2$. This means that in the region close to the pole, one may retain the dominant terms only and write the approximate expression for χ in the form of

$$\chi = \sum_{i,j} \sum_{\alpha,\beta} \frac{q^2 + \bar{m}'_i m'_j}{\left(q^2 - \bar{m}'^2_i\right)\left(q^2 - m'^2_j\right)} U_{ei} U^*_{\alpha i} \mu_{\alpha\beta} U_{\beta j} U^*_{ej} \,, \tag{28}$$

where $i,j = 1,2,3$ number the mass eigenstates and $\alpha,\beta = e,\mu,\tau$ denote the flavour eigenstates.

We notice that since the factor $B\mu$ enters the Hamiltonian directly, it has the unit of mass and, therefore, $B\chi$ has the unit 1/eV and it cannot be directly compared with the effective neutrino mass $\langle m \rangle_{0\nu}$. We discuss the possibility of relating Eq. (28) to the expression describing the mass mechanism of the $0\nu 2\beta$ decay in Section 4.3.

The masses m' and $\bar{m}'$ are obtained from the diagonalization of the Hamiltonian (23). All possible choices of the intermediate flavour states are depicted in Fig. 1.

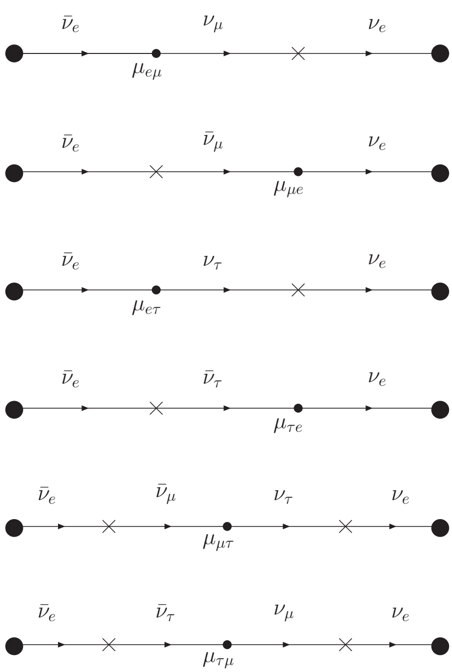

Fig. 1. Neutrino transitions that contribute to the neutrinoless double beta decay via the magnetic moment. The transitions take place in nuclear matter between two beta vertices in the presence of a non-stationary magnetic field. The small dots represent the magnetic moment insertions, while the crosses indicate flavour oscillations.

If the masses of the L-handed neutrino and the corresponding R-handed neutrino are the same, expression (28) yields zero and this contribution to the $0\nu 2\beta$ vanishes. However, the degeneracy is removed by different signs of the $\dot{\phi}$ term in Hamiltonian (23). What is more, by having the possibility of changing this term, we can arrive at the resonance $q^2 \approx m_i'^2$ or $q^2 \approx \bar{m}_j'^2$ boosting the χ significantly.

Another interesting observation is that in the case of CP-violation (see Section 4), non-zero phases in the matrix U, *c.f.* (25), appear and expression (28) will not be zero even if there is degeneracy among the masses. Similar situation occurs also when there is mixing between the standard model neutrinos and neutrinos from a fourth generation, in which case the matrix U will no longer be unitary.

4. Numerical analysis

In this section, we study the expression (28) numerically. To start with, we need to compute the corrected neutrino masses by diagonalizing the Hamiltonian (23). One of the newest compilations of neutrino oscillation parameters [20] gives the best-fit values:

$$\begin{aligned} \sin^2\theta_{12} &= 0.320\,, \qquad \sin^2\theta_{13} = 0.026\,, \qquad \sin^2\theta_{23} = 0.490\,, \\ \Delta m^2_{21} &= 7.62\times 10^{-5}\ \mathrm{eV}^2\,, \qquad \Delta m^2_{31} = 2.53\times 10^{-3}\ \mathrm{eV}^2\,, \end{aligned} \tag{29}$$

where the normal ordering of neutrino masses ($m_1 < m_2 \ll m_3$) is assumed. At first, the CP-violating phases are set to zero. The mass of the lightest neutrino is arbitrarily set to $m_1 = 0.05$ eV. We have estimated the neutrino magnetic moments to be of the order of $10^{-15}\mu_{\mathrm{B}}$, the field $B = 1$ T, and the neutron and proton number densities to be $\sim 10^{-31}$ eV^3, which correspond to the ^{76}Ge nucleus.

The average neutrino momentum $\langle p\rangle$ in the $0\nu 2\beta$ decay can be assessed from the mean nucleon distance inside the nucleus and is of the order of 100 MeV [17]. In what follows, we assume that $q \sim \langle p\rangle = 10^8$ eV. It is this value that must be compensated by the $\dot{\phi}$ parameter in order to reach the resonance region. In the simplified case when due to small mass splitting ($m_1 \approx m_2$) one discusses only two neutrino oscillations, it is possible to assess analytically the resonance condition, which reads [15]

$$\dot{\phi} \approx 2q\,. \tag{30}$$

In the three-neutrino case, the exact formula is much more involved, but results in a very similar relation. This is illustrated in Fig. 2 in which a clear boost in the function $\chi(\dot{\phi})$ is visible around the value $\dot{\phi} \approx 2\times 10^8$ eV.

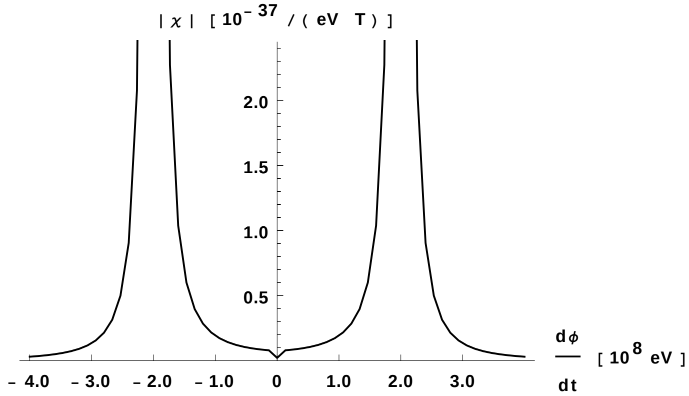

Fig. 2. The shape of the function $\chi(\dot{\phi})$ for the typical neutrino 4-momentum $q = 10^8$ eV. A clear resonance peak is visible around the value $\dot{\phi} \approx 2q$.

The figure is symmetric around zero, as the field B may rotate clockwise or anticlockwise.

One would expect that the peaks should appear for each mass eigenstate separately. This is indeed the case, but in Fig. 2, the detailed structure of the peaks is masked by the fact that the differences of neutrino masses are much smaller than neutrino momenta. The structure of the peak is visible in Fig. 3, where we have changed the scale of the horizontal axis. We notice also that being close to the pole, the value of the χ parameter increased by 20 orders of magnitude. One sees clearly that what appeared in Fig. 2 as a single peak has a double structure corresponding to $m'_{1,2}$ and m'_3 respectively.

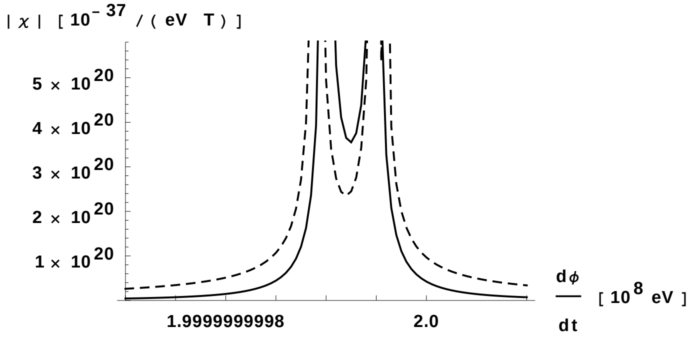

Fig. 3. A detailed view of the double structure of the peak from Fig. 2. The two separate peaks correspond to masses $m'_{1,2}$ and m'_3. Solid line: CP conserving case. Dashed line: the phase $\delta = \pi/2$.

4.1. CP-violating phases

Up till now, we have assumed full CP conservation in the neutrino mixing matrix and ignored the phases δ, ϕ_2, and ϕ_3 in (25). The phases are, in principle, uncorrelated and independently take values between 0 and 2π. Global fits from the neutrino oscillation experiments are rather non-conclusive and the best-fit values in the 1σ error range read [20]

$$\delta = \left(0.83\,{}^{+0.54}_{-0.64}\right)\pi \qquad \text{(normal hierarchy)}, \tag{31}$$

$$\delta = 0.07\,\pi \qquad \text{(inverted hierarchy)}, \tag{32}$$

where for the inverted hierarchy case, there is no preferred region in the parameter space. The 2σ error covers the whole range $(0, 2\pi)$. There are no data for the Majorana phases $\phi_{2,3}$.

The numerical scale of χ is dictated by the value of the 4-momentum q, therefore, small changes in the U matrix should not change the χ behaviour significantly. We have checked that the Majorana phases play basically no role, while the Dirac phase introduces a factor of 6 difference for $\delta = \pi/2, 3\pi/2$, see Fig. 4. In Fig. 3 (dashed line), we present the shape of the peak close to the pole for $\delta = \pi/2$. One sees that introducing CP violation does not change the picture qualitatively, while the quantitive changes are minimal.

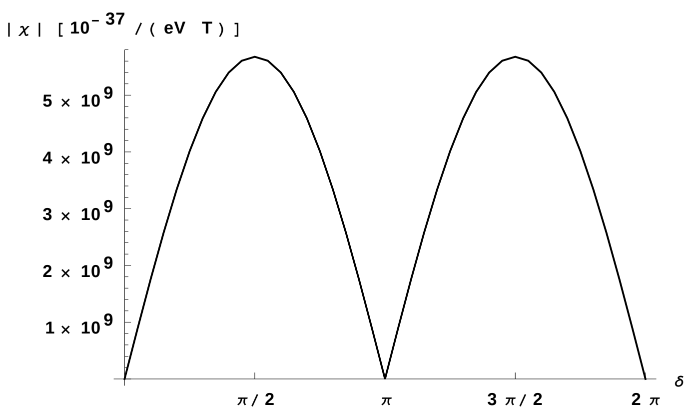

Fig. 4. The $|\chi|$ parameter as a function of the Dirac phase δ for $\dot{\phi} = \langle p \rangle = 100$ MeV. The Majorana phases are set to zero.

4.2. Neutrino magnetic moment

The value of the neutrino magnetic moment is bound by observations and theory. The upper limit comes from dedicated experiments [7] and is given roughly by $10^{-11}\mu_{\rm B}$. The Standard Model value is approximately equal to $10^{-19}\mu_{\rm B}$ [3–5] and everything in between will originate from some New Physics [6].

The neutrino magnetic moments μ enter expression (28) for $|\chi|$ indirectly, by affecting the masses m' and $\bar{m}'$, and directly under the sum. We have run a numerical test with the following parameters:

$$q = 100\ {\rm MeV}\,, \qquad B = 1\ {\rm T}\,, \qquad \delta = \pi/2\,, \qquad \mu_{\alpha\beta} = \left(10^{-19} - 10^{-11}\right)\mu_{\rm B}\,. \tag{33}$$

The results for $|B\chi|$ are presented in Table I. All $\mu_{\alpha\beta}$ have been set to a common value μ. The resonance point is clearly present around $\dot{\phi} = 2E$. One sees also that the value of $|B\chi|$ scales linearly with μ which means that the main impact comes from the $\mu_{\alpha\beta}$ appearing explicitly in (28).

TABLE I

Approximate numerical values of $|B\chi|$ as a function of $\dot{\phi}$ and the neutrino magnetic moment, for $B = 1$ T, $q = 100$ MeV, and $\delta = \pi/2$. The resonance points are close to the value $\dot{\phi} = 2E$.

$\dot{\phi}$	$\mu = 10^{-19}\mu_{\rm B}$	$\mu = 10^{-11}\mu_{\rm B}$
$1.8E$	5×10^{-31}	5×10^{-23}
$1.9E$	1×10^{-30}	1×10^{-22}
$2.0E$	2×10^{-21}	2×10^{-13}
$2.1E$	1×10^{-30}	1×10^{-22}

4.3. Comparison with the mass mechanism

It is desirable to compare the presented mechanism and the mass mechanism of the neutrinoless double beta decay. One has to bear in mind, however, that the nuclear matrix elements (NMEs) should be calculated for each mechanism separately.

Following [21], the part of the NME containing neutrino propagator in the mass mechanism of the $0\nu2\beta$ decay has the form presented in Eq. (1). The factor $1/p^2$ is successively integrated out with the help of the assumptions that $p \approx 100$ MeV and that the average neutrino momentum is much larger than the energy of the intermediate states (light neutrino approximation). This procedure allows to factor the expression for the inverse half-life in the well-known form of a product of the phase-space factor $G^{0\nu}$, the nuclear matrix element $M'^{0\nu}$, and the effective neutrino mass $\langle m\rangle_{0\nu}$:

$$\left(T^{0\nu}_{1/2}\right)^{-1} = G^{0\nu}|M'^{0\nu}|^2|\langle m\rangle_{0\nu}|^2 \,. \tag{34}$$

Similar factorization is possible in our case leading to Eq. (26), but the term $M^{0\nu}$ is not the same as the NME $M'^{0\nu}$. It follows, that one should compare for a given double beta emitter the quantities $|M'^{0\nu}\langle m\rangle_{0\nu}|$ and $|M^{0\nu}B\chi|$. The numerical values of $M^{0\nu}$, however, are not known and their calculations are beyond the scope of this work.

In order to obtain a crude estimation of the importance of the new mechanism, we propose the following approximation:

$$M'^{0\nu} \approx \frac{1}{\langle p\rangle^2} M^{0\nu} \,, \tag{35}$$

which allows us to compare the quantities $|B\chi|$ and $\langle m\rangle_{0\nu}/\langle p\rangle^2$.

The neutrino effective mass is given by

$$\langle m\rangle_{0\nu} = \sum_i |U_{ei}|^2 m_i \,, \tag{36}$$

i.e., it is the sum of mass eigenvalues weighed by the mixing matrix entries, see Eq. (25). Using Eq. (29) and setting the CP phase $\delta = 0$, one obtains the following limiting values:

$$4\times 10^{-3}\ \text{eV} \lesssim \langle m\rangle_{0\nu} \lesssim 3\times 10^{-1}\ \text{eV} \,, \tag{37}$$

which correspond to the mass of the lightest neutrino being 0 and 0.3 eV, respectively. The latter value is motivated by cosmological observations, which suggest that $\sum m_i \lesssim 1$ eV. We obtain in this way a rather conservative bound on the possible values of the effective neutrino mass. It is depicted in Fig. 5 as the shaded horizontal band, which has been rescaled by a factor $\langle p\rangle^{-2} = 10^{-16}\ \text{eV}^{-2}$. The factor $|\chi|$ is a function of the lightest neutrino mass, the B field, $\dot\phi$, and the neutrino magnetic moment. In Fig. 5, we have set $m_1 = 0.05$ eV, $q = p = 100$ MeV, and other parameters as in Eq. (29). On the horizontal axis of Fig. 5, the neutrino magnetic moment in Bohr magnetons is given and the whole range between $10^{-11}\mu_{\rm B}$ (the experimental limit) and $10^{-19}\mu_{\rm B}$ (the Standard Model limit) is shown. For simplicity, we have set all three magnetic moments to the same value. The straight lines correspond to $\dot\phi = 1.9q$ (lower line), and $\dot\phi \approx 2.0q$ (upper line). One can see that close to the resonance the magnitude of the $|\chi|$ factor is comparable or exceeds the rescaled effective neutrino mass for the magnetic moment close

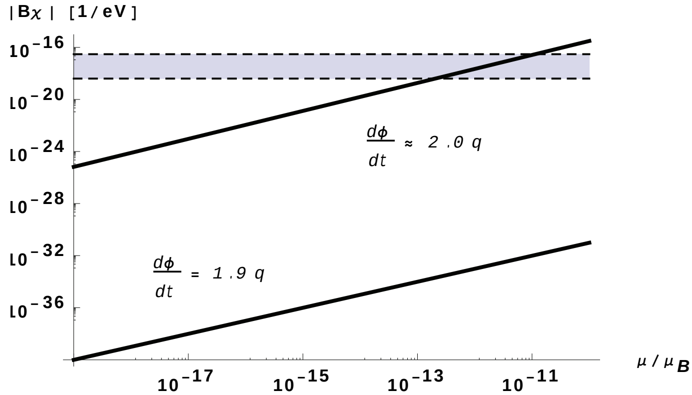

Fig. 5. Comparison of the rescaled effective neutrino mass (shaded region) with the values of the parameter $|B\chi|$, for $B = 1$ T, away and close to the resonance. See the text for more details.

to the experimental limit. We notice also that the stronger the field B, the shorter is the expected half-life. Theoretically, in our approximate approach, an arbitrary low value can be achieved by fine-tuning the $\dot{\phi}$ parameter.

As mentioned earlier, a more reliable discussion should include the recalculation of the NMEs specifically for the presented mechanism. In the neutrino part, represented here by the χ parameter, this should take into account the momenta of the electrons and the photon, a proper regularization of the propagators, and integration over the neutrino 4-momentum q. Finishing this discussion, we would like to stress that the mass mechanism may be realized spontaneously. Our mechanism, however, requires certain additional conditions, which have been described in this paper, and can be realized in a specific environment only, like in the vicinity of rotating neutron star or magnetar, in which fast changing magnetic fields of the order of 10^{10} T have been observed.

5. Conclusions

We have developed a description of a new channel of the neutrinoless double beta decay, first proposed in our previous work [15]. In this scenario, electron neutrino emitted in one beta vertex reacts through its induced magnetic moment with an external non-uniform magnetic field, which results in a helicity-flip and flavour change. A subsequent oscillation back to the electron flavour allows for an absorption of the resulting particle in the second beta vertex. This process may result in a $0\nu2\beta$ decay.

An interesting feature of this scenario is that the χ parameter, which describes the propagating neutrino, depends on the change of the direction of the external magnetic field, which was represented in our discussion by the parameter $\dot{\phi}$. The function $\chi(\dot{\phi})$ has a pole and for certain values of the argument, which is roughly equal to twice the neutrino 4-momentum, becomes large. Since the half-life of the discussed decay is proportional to χ^{-1}, it becomes short in the resonance region. This opens, at least theoretically, the possibility to induce the $0\nu2\beta$ decay by tuning the external magnetic field.

This work has been financed by the Polish National Science Centre under the decision number DEC-2011/01/B/ST2/05932.

REFERENCES

[1] L. Wolfenstein, *Phys. Rev. D* **17**, 2369 (1978); S.P. Mikheyev, A.Yu. Smirnov, *Nuovo Cim. C* **9**, 17 (1986).

[2] A. Renshaw *et al.* [Super-Kamiokande Collaboration], *Phys. Rev. Lett.* **112**, 091805 (2014).

[3] K. Fujikawa, R.E. Shrock, *Phys. Rev. Lett.* **45**, 963 (1980).

[4] R.E. Shrock, private communication.

[5] B. Kayser, *Phys. Rev. D* **26**, 1662 (1982).

[6] W.J. Marciano, A.I. Sanda, *Phys. Lett. B* **67**, 303 (1977); B.W. Lee, R.E. Shrock, *Phys. Rev. D* **16**, 1444 (1977); J. Schechter, J.W.F. Valle, *Phys. Rev. D* **24**, 1883 (1981); **25**, 283 (1982); J.F. Nieves, *Phys. Rev. D* **26**, 3152 (1982); R.E. Schrock, *Nucl. Phys. B* **206**, 359 (1982); L.F. Li, F. Wilczek, *Phys. Rev. D* **25**, 143 (1982); M. Góźdź, W.A. Kamiński, F. Šimkovic, A. Faessler, *Phys. Rev. D* **74**, 055007 (2006); M. Góźdź, W.A. Kamiński, F. Šimkovic, *Int. J. Mod. Phys. E* **15**, 441 (2006); M. Góźdź, W.A. Kamiński, *Phys. Rev. D* **78**, 075021 (2008); *Int. J. Mod. Phys. E* **18**, 109 (2009); **19**, 692 (2010); M. Góźdź, *Phys. Rev. D* **85**, 055016 (2012).

[7] Z. Daraktchieva *et al.* [MUNU Collaboration], *Phys. Lett. B* **564**, 190 (2003); **615**, 153 (2005); H.B. Li *et al.* [TEXONO Collaboration], *Phys. Rev. Lett.* **90**, 131802 (2003); G. Bellini *et al.* [Borexino Collaboration], *Phys. Lett. B* **696**, 191 (2011); A. Beda *et al.* [GEMMA Collaboration], *Adv. High Energy Phys.* **2012**, 350150 (2012).

[8] A.Yu. Smirnov, *Phys. Lett. B* **260**, 161 (1991).

[9] C. Broggini, C. Giunti, A. Studenikin, *Adv. High Energy Phys.* **2012**, 459526 (2012).

[10] B. Pontecorvo, *Sov. Phys. JETP* **26**, 984 (1968).

[11] E.Kh. Akhmedov, S.T. Petcov, A.Yu. Smirnov, *Phys. Lett. B* **309**, 95 (1993).

[12] E.Kh. Akhmedov, S.T. Petcov, A.Yu. Smirnov, *Phys. Rev. D* **48**, 2167 (1993).

[13] J. Schechter, J.W.F. Valle, *Phys. Rev. D* **25**, 2951 (1982).

[14] M. Duerr, M. Lindner, A. Merle, *J. High Energy Phys.* **1106**, 091 (2011).

[15] M. Góźdź, W.A. Kamiński, *Phys. Rev. D* **89**, 113005 (2014).

[16] H. Primakoff, S.P. Rosen, *Rep. Prog. Phys.* **22**, 121 (1959); M. Doi, T. Kotani, E. Takasugi, *Prog. Theor. Phys. Suppl.* **83**, 1 (1985).

[17] T. Tomoda, *Rep. Prog. Phys.* **54**, 53 (1991).

[18] S. Kovalenko, M.I. Krivoruchenko, F. Šimkovic, *Phys. Rev. Lett.* **112**, 142503 (2014).

[19] R.N. Mohapatra, P.B. Pal, *Massive Neutrinos in Physics and Astrophysics*, Singapore: World Scientific, 2004.

[20] M. Tórtola, *Fortschr. Phys.* **61**, 427 (2013).

[21] S.M. Bilenky, *Phys. Part. Nucl.* **41**, 690 (2010).