SPLITTING FUNCTIONS FOR HIGH-ENERGY FACTORIZATION AT LEADING ORDER*

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We report on the recently calculated transverse-momentum-dependent (TMD) splitting functions for the quark channel at leading order (LO) in the high-energy factorization framework of QCD. Our calculations complement earlier results for the gluon channel, which makes a complete set of TMD splitting functions available at LO. They are required to formulate evolution equations for TMD parton distribution functions, to develop TMD parton shower algorithms, and for other applications.

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1. Introduction

The essential theoretical input for experimental findings at the Large Hadron Collider are parton distribution functions (PDFs) which describe momentum distributions of partons in the colliding hadrons in the presence of a hard scale. Together with factorization theorems and hard coefficient functions, PDFs allow to predict new phenomena and to describe existing data. A lot of recent activity in theory and phenomenology of QCD is devoted to the PDFs and factorization which include effects associated with a transverse momentum (for a review, we refer the Reader to [1]). While a rigorous formulation of the TMD factorization, valid for all kinematic regions, is still to be achieved (see *e.g.* [2]), it is possible to define TMD parton distributions for specific regions of the phase space, usually characterized by a hierarchy of scales [3–6]. One of those regions is the high-energy or small-x limit of perturbative QCD, characterized by the hierarchy $\sqrt{s} \gg M \gg \Lambda_{\mathrm{QCD}}$, where $\sqrt{s}$ denotes the center-of-mass energy of the process, M is the hard

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scale of the perturbative event, and $\Lambda_{\rm QCD}$ is the QCD characteristic scale of the order of a few hundred MeV. The underlying theoretical framework for TMD PDFs in this kinematic limit is usually referred to as the high-energy factorization (or $k_{\rm T}$-factorization) [7]. During the recent years, various hard processes, in particular those associated with the forward region of LHC detectors, characterized by large rapidities, have been studied within the high-energy factorization framework, such as forward jet and forward b-jet production [8–10] and forward Z production [11–13].

In the following, we are, in particular, interested in the evolution of TMD PDFs, which depend on the parton's longitudinal momentum fraction x, its transverse momentum $k_{\rm T}$, and the external hard scale. The evolution equation which has all these elements and is valid in angular ordered phase space in the gluon channel is provided by the Ciafaloni–Catani–Fiorani–Marchesini (CCFM) equation [14–17]. The key element of the evolution kernel of the CCFM equation is the P_{gg} splitting function. At leading order, it contains only the most singular pieces at low $z \to 0$ and large $z \to 1$, and appropriate form factors which resum virtual and unresolved real emissions in respectively low- and large-z regions.

The CCFM equation is restricted to the resummation of purely gluonic emissions. In particular, this implies that the moderate-x behavior of CCFM is not accurate and the formal moderate-z limit of the CCFM equation is incomplete, since it does not reduce to the matrix-valued DGLAP evolution equations. One of the observations based on the Monte Carlo implementation [18] of the CCFM equation is that the lack of such contributions leads indeed to non-negligible effects. Performing a fit to the proton structure function F_2 at both large and small x, it is likely that the gluon contribution is enhanced in regions where quarks in the evolution would contribute. While for inclusive observables, such as a structure function F_2, the overall fit turns out to be satisfactory, see *e.g.* [19], the predictions based on the gluon density are not satisfactory for exclusive observable, see *e.g.* [8]. While it is difficult to pinpoint the exact reason for this deficiency, DGLAP resummation definitely suggests that decoupled evolution of quarks and gluons is insufficient. This is further supported by application of the Kutak–Sapeta (KS) gluons densities [20, 21] which account for quark contribution in the evolution [22] and describes production of dijets in $p+p$ collisions at the LHC reasonably well [20, 23]. In order to be able to apply the CCFM evolution successfully, and to provide a full parton shower Monte Carlo description within CCFM, the ultimate goal must be, therefore, to arrive at a coupled system of equations which, in turn, requires a full set of $k_{\rm T}$-dependent splitting functions [24].

To arrive at a complete and consistent set of evolution equations, it is further necessary to include — apart from the quark splitting functions P_{gq} and P_{qq} — non-singular terms of the P_{gg} splitting function since these corrections are of the same order beyond leading order (LO) CCFM, *i.e.* beyond large- and small-z enhanced contributions. Note that in [18], it has been observed that inclusion of non-singular pieces of the DGLAP gluon splitting function into CCFM evolution strongly affects the solution of the evolution equation. One may, therefore, conclude that the effect of quarks in the evolution will be similarly significant.

A first step into this direction has been undertaken in [12], where the TMD gluon-to-quark splitting kernel P_{qg} obtained in [25] has been used to define a TMD sea-quark density within k_{T}-factorization. In the following, we report on the recently obtained results [26] for the real contributions to quark-to-quark and quark-to-gluon splitting functions.

From a technical point of view, the determination of TMD splitting functions is based on a generalization of the high energy factorization approach of Catani and Hautmann [25], which itself is based on the formulation of DGLAP evolution in terms of a two-particle-irreducible (2PI) expansion [27] (for overview and recent applications of the method, see [28–31]). To guarantee gauge invariance in presence of off-shell particles, we follow the proposal made in [12] and make use of the effective action formulation of the high-energy factorization in terms of reggeized quarks and gluons [32, 33]. In the case of the gluon channel, consistency of this formalism has been verified up to the 2-loop level through explicit calculations of the higher-order corrections [34–38] and has been recently used to determine the complete next-to-leading order corrections to the jet-gap-jet impact factor [39–41].

2. Overview of the method

The decomposition into 2PI diagrams, as introduced in [27], is based on the use of axial light-cone gauge, which allows to analyze collinear singularities on the graph-by-graph basis [42], in contrast to covariant gauges where such a rule is broken. Following [25], we will obtain TMD splitting functions which complete the set of already available evolution kernels. Unlike the case of the gluon-to-quark splitting treated in [25], the resulting splitting kernels have no direct definition as the coefficient of the BFKL Green's function (or it is equivalent in the case of t-channel quark exchange). While the TMD quark-to-quark splitting can be identified as a certain next-to-leading order contributions to the high-energy resummed non-singlet P_{qq} DGLAP splitting function, the TMD quark-to-gluon splitting is suppressed by a power of x w.r.t. the leading logarithmic small-x resummed P_{gq} DGLAP splitting function. Nevertheless, it is possible to attempt a definition of such quantities as matrix elements of reggeized quarks and conventional QCD degrees of freedom in light-cone gauge.

Following the framework set by [25, 27], the starting point for the definition of TMD splitting functions requires determination of the corresponding TMD splitting kernels

$$\hat{K}_{ij}\left(z, \frac{\boldsymbol{k}^2}{\mu^2}, \epsilon, \alpha_{\rm s}\right) = \int \frac{{\rm d}q^2 {\rm d}^{2+2\epsilon}\boldsymbol{q}}{2(2\pi)^{4+2\epsilon}} \Theta\left(\mu_{\rm F}^2 - q^2\right) \mathbb{P}_{j,\,\rm in} \otimes \hat{K}_{ij}^{(0)}(q,k) \otimes \mathbb{P}_{i,\,\rm out}\,. \tag{2.1}$$

Here, $\hat{K}_{ij}^{(0)}$, $i,j = q,g$ denotes the actual matrix element, describing the transition of parton j to parton i, which is defined to include the propagators of outgoing lines. In the case of gluons, these propagators are taken in $n\,A = 0$ light-cone gauge; a similar statement applies to the polarization of real emitted gluons. $\mathbb{P}_{i,\,\rm in/out}$ are, on the other hand, semi-projectors on incoming and outgoing lines. The symbol $\otimes$ represents contraction of indices and summation. $\mu_{\rm F}$ denotes the factorization and dimensional regularization in $d = 4+2\epsilon$ dimensions is employed with μ^2 the dimensional regularization scale. The Sudakov parametrization for incoming and outgoing momenta, k and q, reads

$$k^\mu = yp^\mu + k_\perp^\mu\,, \qquad q^\mu = xp^\mu + q_\perp^\mu + \frac{q^2 + \boldsymbol{q}^2}{2x^pn} n^\mu\,, \qquad \tilde{\boldsymbol{q}} = \boldsymbol{q} - z\boldsymbol{k}\,, \tag{2.2}$$

with $z = x/y$. The semi-projectors on outgoing lines, $\mathbb{P}_{j,\,\rm out}$, are directly taken from [27]

$$\mathbb{P}_{g,\,\rm out}^{\mu\nu} = -g^{\mu\nu}\,, \qquad \mathbb{P}_{q,\,\rm out} = \frac{\not{n}}{2^qn}\,. \tag{2.3}$$

While outgoing lines are at first treated in 1–1 correspondence to [27], the on-shell restriction on incoming lines is now relaxed. The corresponding semi-projectors, therefore, require a slight modification. With the original projectors $\mathbb{P}_{j,\,\rm in}$ of [27]

$$\mathbb{P}_{g,\,\rm in}^{[27]\,\mu\nu} = \frac{1}{m-2}\left(-g^{\mu\nu} + \frac{k^\mu n^\nu + n^\mu k^\nu}{^kn}\right)\,, \qquad \mathbb{P}_{q,\,\rm in}^{[27]} = \frac{\not{k}}{2} \tag{2.4}$$

are modified to

$$\mathbb{P}_{g,\,\rm in}^{\mu\nu} = \frac{k_\perp^\mu k_\perp^\nu}{\boldsymbol{k}^2}\,, \qquad \mathbb{P}_{q,\,\rm in} = \frac{y\,\not{p}}{2}\,. \tag{2.5}$$

While the modified gluon projector has been known for a long time [25], we emphasize that the modified quark projector follows directly from the decomposition of the high-energy projector. Its normalization is, on the other hand, fixed by requiring agreement with the corresponding projector of [27] in the collinear limit.

To ensure gauge invariance of the splitting functions in presence of off-shell momenta, it is further necessary to modify standard QCD vertices. The formalism which guarantees that gauge invariance holds is based on the reggeized quark formalism [32, 33, 43] (for more recent re-derivation in spin helicity formalism, see [44]). The modification is achieved through adding certain eikonal terms which then, in turn, arrange gauge invariance of the vertex. Apart from the conventional QCD quark–quark–gluon vertex, $\Gamma^{\mu}_{qqg} = igt^a\gamma^{\mu}$, we have for the off-shell vertex with one reggeized quark q^*

$$\Gamma^{\mu}_{q^*qg}(p_{q^*}, p_q, p_g) = igt^a\left(\gamma^{\mu} + \frac{p^{\mu}}{p p_g}\not{p}_{q^*}\right) \qquad \text{with} \qquad p_{q^*}\cdot p = 0\,. \tag{2.6}$$

Contracting the Lorentz index of this vertex with the gluon momentum yields $p_{g,\mu}\cdot\Gamma^{\mu}_{q^*qg} = -igt^a\not{p}_q$ which is equivalent to the corresponding expression for the conventional quark–quark–gluon vertex if the quark p_q^* is taken on the mass shell. Moreover, in case the second quark is on the mass shell, we have immediately $p_{g,\mu}\cdot\Gamma^{\mu}_{q^*qg}\bar{u}(p_q') = -igt^a\not{p}_q\bar{u}(p_q') = 0$ with $p^2_{q'} = 0$. We, therefore, find that using the generalized vertex Eq. (2.6), the current conservation holds despite of the quark with momentum p_{q^*} being off-shell.

3. Some results for TMD splitting functions

While the explicit angular momentum dependence of our results might be of interest for further Monte Carlo realizations which aim at description of exclusive final states, the evolution of TMD parton distribution functions generally requires only angle-averaged splitting functions. Furthermore, the splitting functions turn out to be divergent in certain regions of phase space, which will be identified below.

To arrive at a result similar to the one obtained in [25] for the TMD P_{qg}, it is further necessary to average over the azimuthal angle. With

$$\hat{K}_{ij}\left(z, \frac{\boldsymbol{k}^2}{\mu_{\mathrm{F}}^2}, \alpha_{\mathrm{s}}, \epsilon\right) = \frac{\alpha_{\mathrm{s}}}{2\pi} z \int\limits_0^{(1-z)\left(\mu_{\mathrm{F}}^2 - z\boldsymbol{k}^2\right)} \frac{\mathrm{d}\tilde{\boldsymbol{q}}^2}{\tilde{\boldsymbol{q}}^2}\left(\frac{\tilde{\boldsymbol{q}}^2}{\mu^2}\right)^{\epsilon} \frac{e^{-\epsilon\gamma_E}}{\Gamma(1+\epsilon)} P_{ij}^{(0)}\left(z, \frac{\boldsymbol{k}^2}{\tilde{\boldsymbol{q}}^2}, \epsilon\right), \tag{3.1}$$

which then defines the TMD splitting functions P_{ij}, we reproduce for the gluon-to-quark splitting the result of [25], also calculated in [12, 45] (here we show results for $\epsilon \to 0$ limit, for higher-order terms in ϵ, see [26])

$$P_{qg}^{(0)}\left(z, \frac{\boldsymbol{k}^2}{\tilde{\boldsymbol{q}}^2}\right) = T_f\left(\frac{\tilde{\boldsymbol{q}}^2}{\tilde{\boldsymbol{q}}^2 + z(1-z)\,\boldsymbol{k}^2}\right)^2\left[z^2 + (1-z)^2 + 4z^2(1-z)^2\frac{\boldsymbol{k}^2}{\tilde{\boldsymbol{q}}^2}\right]. \tag{3.2}$$

For the new TMD splitting functions, we obtain

$$P_{gq}^{(0)}\left(z,\frac{\boldsymbol{k}^2}{\tilde{\boldsymbol{q}}^2}\right) = C_f\left[\frac{2\tilde{\boldsymbol{q}}^2}{z\left|\tilde{\boldsymbol{q}}^2-(1-z)^2\boldsymbol{k}^2\right|}-\frac{\tilde{\boldsymbol{q}}^2\left(\tilde{\boldsymbol{q}}^2(2-z)+\boldsymbol{k}^2z\left(1-z^2\right)\right)}{\left(\tilde{\boldsymbol{q}}^2+z(1-z)\boldsymbol{k}^2\right)^2}\right], \tag{3.3}$$

$$P_{qq}^{(0)}\left(z,\frac{\boldsymbol{k}^2}{\tilde{\boldsymbol{q}}^2}\right) = C_f\left(\frac{\tilde{\boldsymbol{q}}^2}{\tilde{\boldsymbol{q}}^2+z(1-z)\boldsymbol{k}^2}\right)\left[\frac{\tilde{\boldsymbol{q}}^2+\left(1-z^2\right)\boldsymbol{k}^2}{(1-z)\left|\tilde{\boldsymbol{q}}^2-(1-z)^2\boldsymbol{k}^2\right|}\right.$$
$$\left.+\frac{z^2\tilde{\boldsymbol{q}}^2-z(1-z)\left(1-3z+z^2\right)\boldsymbol{k}^2}{(1-z)\left(\tilde{\boldsymbol{q}}^2+z(1-z)\boldsymbol{k}^2\right)}\right]. \tag{3.4}$$

As expected from our method to construct TMD splitting functions, we obtain in the collinear limit ($\boldsymbol{k}^2 \to 0$) the well-known real parts of the leading-order Altarelli–Parisi splitting functions in $d = 4 + 2\epsilon$ dimensions

$$P_{gq}^{(0)}(z,0) = C_f\frac{1+(1-z)^2}{z}, \tag{3.5}$$

$$P_{qg}^{(0)}(z,0) = T_f\left(z^2+(1-z)^2\right), \tag{3.6}$$

$$P_{qq}^{(0)}(z,0) = C_f\frac{1+z^2}{1-z}. \tag{3.7}$$

4. Summary and outlook

The extension [26] of the method developed by Catani and Hautmann [25] for the determination of transverse-momentum-dependent parton splitting functions to splittings of initial TMD quarks, based on factorization of cross sections in the high-energy limit has been overviewed. Gauge invariance of underlying amplitudes in presence of off-shell partons is achieved due to the reggeized quark calculus, which supplements conventional QCD vertices by certain eikonal contributions. While our approach is heavily based on the 2PI expansion in the light-cone gauge by Curci *et al.* [27], we have been able to verify that it is possible to generalize the employed projectors in a way, such that the choice of gauge for the sub-amplitudes, which underlie the derivation of our splitting kernels, becomes irrelevant *i.e.* our TMD splitting kernels are independent of the employed gauge. While our splitting kernels are, in this way, well-defined objects, they are not necessarily universal, since they cannot be directly defined as the coefficients of *e.g.* the high energy resummation of a certain TMD parton distribution function, such as the TMD gluon-to-quark splitting functions. They are merely constrained by the requirement to reduce in the collinear and high energy limit to the well-known exact expressions.

The current study determines only the real contribution to the TMD quark-to-quark and quark-to-gluon splitting kernels. Future studies will have to focus on the determination of the corresponding virtual corrections for the TMD quark-to-quark splitting function, the development of a coherent framework which allows for a systematic subtraction of singularities not canceled by virtual corrections and, finally, the formulation of appropriate coupled evolution equations for TMD parton distribution functions. As a long-term goal, a matching of TMD evolution based on factorization in the soft-collinear limit, see *e.g.* [46–48], is a task which needs to be addressed.

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